Special Session 78: Multiple Time Scale Dynamics With a View Towards Biological Applications

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Bursting is a well known mechanism of oscillation occurring in slow/fast systems. More recently the canard phenomenon has been identified as providing a complementary route to sensitive oscillatory dynamics. This special session will cover topics related to special orbits of multiple time scale systems (bursting, canards, torus canards, MMOs) and their relation to each other. The talks will present theoretical results and their applications in life sciences (neuronal dynamics, endocrinology, epidemiology, etc).

Torus canards in R^3

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Torus canards are a higher-dimensional generalization of the classical canard orbits familiar from planar systems and arise in fast-slow systems of ordinary differential equations in which the fast subsystem contains a saddle-node bifurcation of limit cycles. Torus canards are trajectories that pass near the saddle-node and subsequently spend long times near a repelling branch of slowly varying limit cycles. They can arise in computational models of neurons in a narrow region between bursting and fast-spiking behavior, similar to the way traditional canards separate small amplitude orbits from relaxation oscillations. It has been shown in an elementary third-order system that the relative speed of the two subsystems affects the importance of torus canards. In the regime of fast rotation, the torus canards behave much like their planar counterparts. In the regime of slow rotation, the phase dependence creates rich torus canard dynamics and dynamics of mixed mode type.

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Canard explosions in the templator model

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The templator is a nonlinear differential equation model for a self-replicating chemical system. During one of the reaction steps adimer acts as a catalyst (template) for its own production. Mixed modeoscillations and canard explosions have been found numerically in the model. We show how the model can be recast as a singular perturbation problem with different definitions of the small parameter in different parameter regimes. Asymptotic determination of the canard point agrees very well with the numerical result. We also discuss how the canard point may be determined without explicitly determining a small parameter.

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Canards and Hopf mechanisms in a model for perceptual rivalry

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Stimulus tuning in a firing rate model for perceptual rivalry leads to mixed-mode oscillations (MMOs), a temporal pattern featuring alternating small- and large-amplitude oscillations. Key ingredients for the generation of MMOs are mutual inhibition, slow negative-feedback in the form of adaptation, and nonlinearity of the gain function. By exploiting a normal form calculation, we show that MMOs occur due to the interaction of canard and singular Hopf mechanisms as the stimulus strength approaches a critical regime.

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Canards, inflection and excitability threshold

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In this talk, we will revisit the "inflection line method" introduced in the early 1990s to characterize canard explosions geometrically and study their dependence on the time scale ratio ε. We will then apply this idea to the context of planar neuronal dynamics, where we will show that the inflection set provide an approximation to the excitability threshold in both integrator and resonator type neuron models.

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Canards and MMOs in coupled oscillator systems

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We present a study of mixed-mode oscillations in systems of coupled oscillators with one slow and one fast dimension. We use the combination of the slow/fast and coupled oscillator structures to reduce some of the dimensions. For two coupled oscillators we can reduce the problem to the classical folded node/folded saddle node case. For more than two coupled oscillators we use the recently developed approach of Wechselberger to generalize the analysis. We also consider some degenerate cases.

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Shaping bursting by electrical coupling and noise

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We study a randomly perturbed slow-fast system of differential equations modeling electrical activity in networks of pancreatic beta cells. Our analysis reveals the interplay of the intrinsic properties of the cells, network topology, and noise in shaping network dynamics. In particular, we describe a transition from irregular spiking to synchronized bursting under the variation of the strength of coupling. We also provide a detailed analysis of synchronization in this model. Analytical results are illustrated by numerical simulations of conductance-based networks.

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Canards in slow-fast piecewise-linear planar systems

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In this talk, we will present recent results on the behavior of canard solutions in the context of planar piecewise-linear systems. We will focus on Liénard type systems, which is a sufficiently big class of interesting systems from the point of view of applications. We will show how periodic orbits of canard type appear in this context through a Hopf-like event and how his explosive evolution is related to a bifurcation at infinity. This will allow us to draw a parallel with the smooth case.

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Transitional isochron portraits in biological models with multiple time-scales

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Phase response is an important functional property of many oscillatory biological systems, and is frequently a key determinant of network behaviors such as synchronization. We have found that the phase response properties of multiple time-scale dynamical systems, e.g. neuronal and predator-prey models, depend heavily on the specific characteristics of their fast and slow subsystems, especially bifurcation structure. Computation of isochrons, manifolds of equivalent asymptotic phase, gives a global portrait of a system's intrinsic phase response properties (i.e. for perturbations of arbitrary strength and shape). In our analysis, we link isochronal curvature, phase response sensitivity, and time-scale separation. We illustrate our findings in neuronal and ecological models transitioning between activity modes, e.g. between spiking and bursting regimes.

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Bifurcations of bursting polyrhythms in plausible 3-cell motifs

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Motifs of three coupled cells are a common network configuration including models of biologicalcentral pattern generators. We describe a novel computational approach to reduce detailed models of central pattern generators to equation-less return mappings for the phase lags between the constituting bursting interneurons. Such mappings are studied geometrically as the model parameters, including coupling properties of inhibitory and excitatory synapses, or external inputs are varied. Bifurcations of the fixed points and invariant circles of the mappings corresponding to various types of rhythmic activity are examined. This reveals the organizing centers of emergent poly-rhythmic patterns and their bifurcations, as the asymmetry of the synaptic coupling is varied. These changes uncover possible biophysical mechanisms for control and modulation of motor-pattern generation. Our analysis does not require knowledge of the equations that model the system, and so provides a powerful new approach to studying detailed models, applicable to a variety of biological phenomena beyond motor control.

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The dynamics underlying pseudo-plateau bursting in a pituitary cell model

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Pituitary cells of the anterior pituitary gland secrete hormones in response to patterns of electrical activity. Several types of pituitary cells produce short bursts of electrical activity which are more effective than single spikes in evoking hormone release. These bursts, called pseudo-plateau bursts, are unlike bursts studied mathematically in neurons (plateau bursting) and the standard fast-slow analysis used for plateau bursting is of limited use. Using an alternative fast-slow analysis, with one fast and two slow variables, we show that pseudo-plateau bursting is a canard-induced mixed mode oscillation. Using this technique, it is possible to determine the region of parameter space where bursting occurs as well as salient properties of the burst such as the number of spikes in the burst. The information gained from this one-fast/two-slow decomposition complements the information obtained from a two-fast/one-slow decomposition.

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Mixed-mode oscillations in a model of hormone secretion: insight into the variability of GnRH surge-to-pulse transition

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The secretion of Gonadotropin Releasing Hormone (GnRH) by specific hypothalamic neurons plays a major role in the neuroendocrine control of the reproductive function in female mammals. The periodic back-and-forth transitions between the pulsatile secretion phase and the pre-ovulatory surge are among the most crucial mechanisms underlying this control, but they remain not completely understood on the biological level. I will present a model of the GnRH secretion and show how Mixed-Mode Oscillations arise in the surge-to-pulse transition. Often MMO solutions are quite regular, i.e. the number of small oscillations does not vary too much. I will show solutions that have very high variability of small oscillations in a same orbit. Such solutions exist in the GnRH model even when the singular parameter goes to 0.

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