

## Special Session 73: Mathematical Models for Upwelling Ocean Currents and Related Phenomena

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In this mini symposium, we bring together a multidisciplinary group of mathematicians and scientists who use different perspectives to model fluid dynamics of upwelling and downwelling in ocean flow. Of a particular interest are related phenomena such as polynyas which are large persistent regions of open water where one would expect ice. Topics include a variety of techniques and aspects involved in the modeling of these phenomena.

### The effect of bathymetric profile on the structure of coastal ocean upwelling

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Pedlosky's (1978) dynamical model of coastal ocean upwelling is elegantly simple, yet complex enough to possess rich and subtle dynamics. This model continues to generate new insights into the dynamics of upwelling. Solutions to the Pedlosky model are compared with numerical simulations to study the effects of the bathymetric profile on the structure of velocity and density during upwelling. When the bathymetric profile has positive curvature, the deep onshore-directed flow is surface-intensified. When the bathymetric profile has negative curvature, such as over a continental shelf break, the deep onshore flow is bottom-intensified. For negative curvature of sufficient magnitude, the dynamical model solutions exhibit features of an obstacle problem, which is a free-boundary problem that arises in the study of elliptic partial differential equations. These features make the problem challenging to solve, either analytically or numerically. Recent progress toward solving the problem is discussed.

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### Locating coherent structures in turbulent flows using the geodesic theory of transport barriers

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We use the recently developed geodesic theory of transport barriers to locate a variety of Lagrangian Coherent Structures (LCSs) in two-dimensional turbulent flows. We review the numerical challenges in the implementation of the theory, and describe a numerical algorithm that addresses these challenges. The algorithm is in turn illustrated on direct numerical simulations of decaying and forced Navier–Stokes

turbulence. In particular, we identify hyperbolic barriers (generalized stable and unstable manifolds) and elliptic barriers (Lagrangian vortex boundaries) in the flow. The latter barriers enclose coherent vortices that are more robust and live longer than typical vortices in turbulence. We also identify a systematic difference in the size of Lagrangian eddies in forced and decaying turbulence.

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### Using Lagrangian coherent structures to understand coastal water quality

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The accumulation of pollutants near the shoreline can result in low quality coastal water levels with negative effects on human health. To understand the water quality of a particular beach requires the study of the Lagrangian circulation in the vicinity of the beach in question. Here we demonstrate the utility of dynamical systems methods in achieving such an understanding. The specific dynamical system notion considered is that of Lagrangian Coherent Structure (LCS). Hidden in the currents, the LCSs constitute the centerpieces of patterns formed by fluid particle trajectories. As such, the LCSs act as skeletons of the Lagrangian circulation. Particular focus is placed on Hobie Beach, a recreational marine beach located in Virginia Key in Miami, Florida. According to studies of water quality, Hobie Beach is characterized by high microbial levels. Possible sources of pollution in Hobie Beach include human bather shedding, dog fecal matter, and sand efflux at high tides. Consistent with the patterns formed by satellite-tracked drifter trajectories, the LCSs extracted from simulated currents reveal a Lagrangian circulation which favors the retention near the shoreline of pollutants released along the shoreline, which can help explain the low quality water levels registered at Hobie Beach.

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**Sea ice processes in Antarctic polynyas****Kenneth Golden**University of Utah, USA  
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The largest polynya observed in the Antarctic was the Weddell Polynya of 1975-77. The Antarctic Zone Flux Experiment (ANZFLUX) in 1994 was conducted to explore the dynamic region of the Southern Ocean where this polynya appeared. Large vertical heat fluxes from convective overturning and strong winter storms subject the sea ice pack to powerful forces that affect its growth and decay, as well as the robust ecosystems living within the ice. We will discuss various processes that are active in this region and the Southern Ocean at large, such as snow ice formation, nutrient replenishment, brine drainage, and enhanced thermal exchanges due to brine convection. Moreover, we will describe recent progress in the mathematics of sea ice that was spawned by our findings from ANZFLUX and subsequent expeditions.

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**Seasonality of the circulation on the West Florida Shelf****Yonggang Liu**University of South Florida, USA  
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Seasonal variations of the circulation on the West Florida Continental Shelf (WFS) are described using sustained (1998-2009) observations of velocity from an array of moored acoustic Doppler current profilers, plus various ancillary data. A robust seasonal circulation cycle is found, which varies in a dynamically sensible way across the shelf. Over most of the inner shelf these seasonal variations are primarily in response to local wind forcing, through Ekman-gestrophic spin-up, as previously found for the synoptic scale variability. Thus the inner shelf circulation is predominantly upwelling favorable during fall to spring months (October-April) and downwelling favorable during summer months (June-September). Seaward from about the 50 m isobath, where the buoyancy forcing becomes of increasing importance, the distinctive inner shelf seasonal variations blend into a continuum of variability. Over the outer shelf and near the southwestern end of the WFS, the seasonal circulations are obscured by the deep ocean effects of the Gulf of Mexico Loop Current and its eddies.

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**The connection between the Loop Current excursions and the Florida Red Tides****Grace Maze**University of Miami, USA  
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Red tides are found all over the world, however they are not all caused by the same organism. In the Gulf of Mexico the red tides are caused by a slow growing dinoflagellate, *Karenia brevis*. *K. brevis* is found in background concentration over most of the Gulf, but tends to produce the most intense and frequent blooms on the West Florida Shelf (WFS). *K. brevis* releases a suite of neurotoxins called brevetoxins. These toxins cause large kills in marine life and is also hazardous to humans who come in contact with the aerosolized toxin or eat contaminated shellfish.

Lots of work has been done to try to determine the physical mechanisms responsible for the red tides since the biological growth and reproduction rates are not enough to account for the rapid increase in concentration associated with a bloom. However, none of the research presented to date has come up with a definitive answer that accounts for the majority of recorded blooms in history.

This presentation will examine the influence of the position of the Loop Current and different wind regimes on the concentration of *K. brevis* on the WFS in conjunction with some of the theories of red tide initiation.

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**Lagrangian transport analysis of the surface ocean circulation in the Gulf of Mexico****Maria Olascoaga**RSMAS/UM, USA  
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Analysis of drifter trajectories in the Gulf of Mexico (GoM) has revealed the existence of a region on the southern portion of the West Florida Shelf (WFS) that is not visited by drifters that are released outside of the region. This so-called “forbidden zone” (FZ) suggests the existence of a persistent cross-shelf transport barrier on the southern portion of the WFS. Seven-year-long records of surface currents produced by a Hybrid-Coordinate Ocean Model simulation of the GoM are used to compute Lagrangian coherent structures (LCSs), which reveal the presence of a persistent cross-shelf transport barrier in approximately the same location as the boundary of the FZ. The location of the cross-shelf transport barrier follows an oscillation, being closer to the coast when the Loop Current is in its northernmost location. The analysis also suggests the existences of other two regions with similar isolated characteristics on the Texas-Louisiana Shelf and the Yucatan Shelf. Implications of the results for the dispersal of pollutants, such as oil, in the GoM are discussed.

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### Coastal variability and Lagrangian circulation in Todos Santos Bay and off Baja California during Spring-Summer 2007

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The circulation in Todos Santos Bay area (31.88N) and off Baja California is studied for period Spring-Summer 2007, when an intense toxic algal bloom occurred. High-resolution, numerical model simulations were carried out to study dynamical features along the coast of the northern Baja California (BC) Peninsula and within Todos Santos Bay (TSB), and also to be used in a three-dimensional Lagrangian analysis which provides information about the origin and distribution of the waters present in the Bay during the occurrence of the toxic bloom. The regional dynamics reproduced by the model include the poleward propagation of coastal trapped waves along the BC coast, the so-called Ensenada front, and a persistent cyclonic eddy formed northwest of TSB. The Lagrangian results show that these last two features drive the paths followed by the water parcels found in TSB in Spring 2007. Most of those waters come from locations west of TSB (even beyond the model's domain), stay within the Bay by about one month, and ultimately scatter south-southwestward along the BC coast.

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### Individual trajectory complexity methods & an upwelling flow example

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We consider the analysis of fluid flows in terms of the complexity of the individual trajectories. Specifically, we explore the complexity of the fluid particle trajectories using two measures - the correlation dimension and the ergodicity defect. The goal is to use these measures to reveal structures resembling Lagrangian coherent structures in the flow. We test the technique with upwelling flow data from the Oregon Coast during 2005. As these measures use properties of individual trajectories, and not separation rates between closely spaced trajectories, they may have advantages for the analysis of typical float and drifter data sets in which trajectories are widely and non-uniformly spaced.

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### Identification and tracking of coherent Agulhas Current rings

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We introduce a Lagrangian method, referred to as "geodesic method", for objectively identifying and tracking oceanic mesoscale eddies in altimetry sea-surface height (SSH) datasets. The geodesic method derives from the geodesic theory of transport barriers in two-dimensional flows by Haller and Beron-Vera (2012). Traditional Eulerian methods as the Okubo-Weiss parameter-based method and Chelton et al.'s (2011) SSH-based automated eddy identification procedure are also applied in the same region for comparison. Our findings indicate that unlike traditional Eulerian methods, the geodesic method is capable of correctly locating and tracing coherent Agulhas Current rings. Implications of the results for the computation of transport are discussed.

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### An overview of models for the opening of coastal polynyas

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Polynyas, polar oceanic regions with little sea ice cover during the winter, are recurrent features found in certain oceanic locations. They support rich marine ecosystems and they are important sites for the formation of dense water that contributes to the maintenance of the global thermohaline circulation.

During the past 20 years a mathematical framework for describing the opening, and closing, of coastal polynyas has been developed based on equations describing the thermodynamic production of sea ice and its subsequent transport offshore/onshore. These "flux models" are amenable to analytical treatment under certain assumptions. All flux models predict that under steady forcing (i.e. ice production rate and offshore wind stress) a coastal polynya will open to a steady-state width. This presentation will report on recent research that shows this behaviour is an artefact of a long-standing assumption in flux models about the behaviour of sea ice near the polynya edge. In this presentation a new model for the opening of a coastal polynya will be presented that relaxes this assumption. Whether or not a polynya reaches a steady-state width is shown to be dependent upon the rheological description of the sea ice.

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**Geostrophic adjustment in a polar basin****Andrew Willmott**National Oceanography Centre, England  
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The geostrophic adjustment of homogeneous fluid in a circular basin with idealised topography is addressed using a numerical ocean circulation model and analytical process models. When the basin is uniformly rotating, the adjustment takes place via excitation of boundary propagating waves, and when topography is present, via topographic Rossby waves. In the numerically derived solution, the waves are damped due to bottom friction, and a quasi-steady geostrophically balanced state emerges that subsequently spins-down on a long-time scale. It is demonstrated that the adjusted states emerging in a circular basin with a step escarpment or a top hat ridge, centred on a line of symmetry, are equivalent to that in a uniform depth semi-circular basin, for a given initial condition. These quasi-steady solutions agree well with linear analytical solutions for the latter case in the inviscid limit.

On the polar plane, the high latitude equivalent to  $\beta$  - plane, no quasi-steady adjusted state emerges from the adjustment process. In the intermediate time-scales, after the fast Poincaré/Kelvin waves are damped by friction, the solutions take the form of steady-state adjusted solutions on the f-plane. On the longer timescales, planetary waves control the flow evolution and the dynamics of these waves will be discussed.

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