Special Session 72: Special Methods for Solving Systems of Non-linear Differential Equations and their Applications to Sciences and Engineering

Mufid Abudiab, Texas A&M University-Corpus Christi, USA

The purpose of this session is establishing a forum for the most effective special methods in solving systems of non-linear differential equations and their application in sciences and engineering. This forum will provide services for graduate students, post docs, and researcher in the field of non-linear differential equations. Also, it will enhance the view of scientists and engineers about systems of non-linear differential equation (dynamical systems) and provide them with very effective tools for modeling and hence solving a variety of problems in their fields.

Modeling of the dynamics of client-centered health care

Mufid Abudiab
Texas A&M University-Corpus Christi, USA
mufid.abudiab@tamucc.edu
Suzanne Beltz

For the purpose of providing and delivering safe, effective and appropriate health care services, patients need to play an integral role in the informed decision making of the process. the Institute of Medicine has called fpr transformation of the health care system in the United States and identified client-centered care as one of the major tenets of this transformation. Health care professional use mathematics on a daily basis to execute several medical functions ranging from data collection to regor statistical computations. A variety of mathematical modeling techniques can utilizedd to help improve the delivery of health care. In this paper, we use literature, statistical analysis, and mathematical modeling tools to shed light on the main factors that impact the delivery of health care services. Also, we explore the great variation in defining the construct of client-centered care and the lack of tools for measuring the construct. Finally, a conceptual model of the continuous interaction among the client's model of the world, self-care knowledge, self-care assets, self-care resources, and self-care action is presented and analyzed to optimize the way of providing client-center care.

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Ratio-dependent predator-prey model with infection in prey population

Inkyung Ahn
Korea University, Korea
ahnik@korea.ac.kr
Wonlyul Ko

We study a predator-prey system with a ratio-dependent functional response when a prey population is infected. First, all nonnegative equilibria is investigated, and then a condition which gives a stability at these equilibria shall be found. Especially, disease-free and biological control states are discussed in view of a biological interpretation. Lastly, under the Neumann boundary condition, the existence of nonconstant positive steady-states are studied. The employed methods are a comparison principle for a parabolic problem and Leray-Schauder Theorem.

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A spatio-temporal model for tumor-immune interaction and siRNA treatment

Seunghyeon Baek
Korea University, Korea
seunghyeon9@gmail.com
Inkyung Ahn

In this research, a spatio-temporal mathematical model consisting of nonlinear reaction-diffusion equations is examined that describes interactions among immune effectors and tumor cells. The system we consider involves the immuno-suppressive TGF-beta(Transforming growth factor-beta) and TGF-beta-suppressive siRNA. In particular, the density of tumor is assumed to disperse by diffusion together with directed movement towards more sufficient nutrition. Also the concentration of siTNA is assumed to have the oriented movement of cells in response to the one of TGF-beta due to the chemotaxis of TGF-beta. Tumors can evade immune surveillance by secreting immuno-suppressive factors such as TGF-beta. TGF-beta inhibits the activation of the effector cells and stimulates tumor growth by promoting angiogenesis. Our main goal is to investigate the spatial effects of tumor-immune interactions, as well as siRNA treatment which can control the tumor effectively by suppressing TGF-beta production.

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Global existence to the Navier-Stokes equations through a diffusive upper solution.

Guy Bernard
Midwestern State University, USA
guy.bernard@mwsu.edu

A global existence result is presented for the Navier-Stokes equations filling out all of three-dimensional Euclidean space. The initial velocity is required to satisfy a smallness condition in the form of an upper bound (bell-like) rational function. The method of proof is based on elementary symmetry transformations of the Navier-Stokes consisting of dilations and contractions of space, time, and velocity. The presented demonstration is a step method. A solution to the Navier-Stokes equations is shown to exist in a small time interval through a self-similar-like upper solution modeling the diffusive nature of the principal part of the Navier-Stokes equations (i.e. a heat equation). The local solution is then extended to an even larger time interval by transforming the N-S equations through a symmetry transformation which reduces this first extension to the first local solution. This process is repeated iteratively to produce a global solution. The symmetry transformations utilized exploit the diffusive and self-similar-like nature of the upper solution.

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A unique positive solution to a system of nonlinear elliptic equations

Diane Denny
Texas A&M University-Corpus Christi, USA
diane.denny@tamucc.edu

We study a system of nonlinear elliptic equations that arises from a predator-prey model. Previous related work proved uniqueness when a positive parameter in the system of equations is sufficiently large. We prove the existence of a unique positive solution to this system of equations for all positive values of this parameter.

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Solutions and conservation laws of a coupled Kadomtsev-Petviashvili system

Chaudry Khalique
North-West University, Mafikeng Campus, So Africa
Masood.Khalique@nwu.ac.za

In this talk we study the coupled Kadomtsev-Petviashvili system

$$(u_t + u_{xxx} + 3uu_x + 3ww_x)_x + u_{yy} = 0,$$
$$(v_t + v_{xxx} + 3vv_x + 3ww_x)_x + v_{yy} = 0,$$
$$\left(w_t + w_{xxx} + \frac{3}{2}(uw)_x + \frac{3}{2}(vw)_x\right)_x + w_{yy} = 0,$$

which has applications in many scientific fields. Exact solutions will be obtained using the Lie symmetry method along with the simplest equation method of the underlying system. In addition the conservation laws will be derived using the multiplier method.

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Conservation laws of some evolution equations via non variational approach

Muhammad Danish Khan
Lahore University of Management Science, Pakistan
k.danish@hotmail.com
Imran Naeem

We consider (1+1)-dimensional third order non linear modified Benjamin Bona Mahony (mBBM) equation and generalized regularized long wave (GRLW) equation which have their importance in many branches of physics. These equations arise in wave phenomena in fluid dynamics, elastic media, optic fibres and plasma etc. and are thus important. We compute the conservation laws of these equations via partial Lagrangian approach. Since this approach works only for those differential equations whose higher order terms are of even order so first we increase their order by assuming dependent variable to be the derivative of new variable by setting either $u = v_x$ or $u = v_t$, where u and v are old and new dependent variable respectively. The conservation laws for mBBM and GRLW equations are computed in both cases and it is found that the results obtained for both equations are very useful in their solution process.

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The enzyme-catalysed reaction-diffusion system

Wonlyul Ko
Korea university, Korea
kowl@korea.ac.kr

In this talk, we discuss the biochemical reaction-diffusion model with Michaelis-Menten type response velocity function, which captures the feature of an enzyme-catalysed reaction from glycolysis, and we mainly study the corresponding steady-state under the homogeneous Neumann-boundary condition. In particular, we provide some results on the existence and nonexistence of nonconstant stationary solutions.

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The qualitative study on a new special analytical method for solving wide classes of non-linear differential equations-reality, potential

Habibolla Latifizadeh
Shiraz University of Technology, Iran
h.latifi.62.math@sutech.ac.ir
Faramarz Tahamtani, Esmail Hesameddini

Nonlinear phenomena play a crucial role in applied mathematics, physics and engineering. Much effort was paid on existence, uniqueness and stability of the solution of the problem. Concerning this important topic over the last decades several analytical/approximate methods have been developed to solve nonlinear ordinary and partial differential equations. So at this paper a new analytical iterative procedure have been establish and proved. It may be concluded that the method is very powerful and efficient technique in finding an acceptable solution for wide classes of nonlinear problems. Also, it can be noted that there are many advantage of this proposed method that is based on Laplace transform, the main advantages are the fast convergence to the solutions, does not require discretizations of space and time variables, no need to solve nonlinear system of equations as in finite element methods and finite difference methods, then, no necessity of large computer memory. So to illustrate the aforesaid claims some nonlinear differential equations that has some applications in fields applied mathematics and engineering is analyzed. The obtained results are in good agreement with the existing ones in open literature and it is shown that the technique introduced here is robust, efficient and easy to implement.

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Approximate solutions for 1-D premixed flame propagation model

Christina Martinez
Texas A&M University-Corpus Christi, USA
Christina.Martinez4@tamucc.edu
Devanayagam Palaniappan, Lea-Der Chen

The one-dimensional boundary value problem of the premixed flame propagation in a condensed medium leads to a system of highly nonlinear ordinary differential equations. In this presentation, some explicit approximate solutions obtained via matched asymptotic expansion will be shown. A brief discussion of our numerical solutions to this model will be provided. Comparison of our results with the existing solutions shows an excellent agreement. Applications of our results in the context of transport mechanisms will also be demonstrated.

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Analysis of a system of reaction-diffusion PDE model

Devanayagam Palaniappan
Texas A&M University-Corpus Christi, USA
devanayagam.palaniappan@tamucc.edu
Mufid Abudiab

Reaction-diffusion systems are a set of semi-linear parabolic partial differential equations (PDEs). Such mathematical models can be used to describe several dynamical processes of chemical and non-chemical nature. In this presentation, we discuss a model that is applicable in biology, especially, in the determination of infected and noninfected cells. A direct similarity method - introduced originally by Clarkson and Kruskal (CK-method) for Boussinesq equation is modifed and utilized in the reduction of pdes to ordinary differential equations(ODEs). We present an analysis of the reduced system of ODEs and show exact solutions for certain boundary-value problems with specifed rate constants. A brief discussion of the application of our results in the biological context will be also provided.

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A reaction-diffusion model of Bovine Viral Diarrhea Virus (BVDV) infection

Sami Shahin
Southwest Minnesota State Univesity, USA
sami.shahin@smsu.edu
Mufid Abudiab

BVDV (Bovine Viral Diarrhea Virus Infection) is an infection that affects many populations of cattle. A severe infection can be linked to many different causes such as the state of immune system, physiological status, and other diseases. Most BVDV infections are caused by non-cytopathic viruses. Severely infected animals can go through a brief shift in the number of circulating leukocytes. Due to virus multiplication, contaminated macrophages change its functions and cause the depletion of B and T type lymphocytes. In this paper, a PDE model of four reaction-diffusion equations that represents the basics of the contact between immune cells and the virus will be introduced. The system will be solved numerically and analytical via the upper-lower solution method.

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Polynomial ODEs for conservation laws

James Sochacki
James Madison University, USA
sochacjs@jmu.edu

Burger's equation without diffusion is the prototype for conservation laws and can be derived from Euler's equation for gas dynamics. Euler's equation for gas dynamics can be obtained from the Navier-Stokes equation which includes diffusion. Burger's equation with diffusion can be obtained from the Navier-Stokes equation. In this talk we discuss these relations through ordinary differential equations and show how these ordinary differential equations can be used to derive polynomial solutions to these partial differential equations and conservation laws in general. We demonstrate these ideas for the acoustic equations that can be derived from the Euler equations and highlight interesting consequences for these equations.

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Quasilinear differential equations in exterior domains and application

Nicolae Tarfulea
Purdue University Calumet, USA
tarfulea@purduecal.edu

In this talk we address the existence and uniqueness of weak solutions to a class of quasilinear elliptic equations with nonlinear Neumann boundary conditions in exterior domains. An important model case related to the initial data problem in general relativity is presented. Correlated to our results, we discuss the existence and uniqueness of the conformal factor for the Hamiltonian constraint in general relativity.

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Numerical solutions for weak scattering in a turbulent fluid

Christopher Trombley
Texas A&M Corpus Christi, USA
ctrombley@islander.tamucc.edu
D. Bogucki, D. Palaniappan

A mathematical model for optical scattering in a turbulent fluid is presented. Navier-Stokes equations are adapted for the turbulent fluid region while the scattered field is modeled using the Helmholtz equation. This set of nonlinear partial differential equations with the appropriate boundary conditions are solved numerically with the aid of the finite element method. The scattered amplitude is then given, which can be used to reduce errors in radio signals in a turbulent fluid.

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