Special Session 7: Recent Progress in the Mathematical Theory of Compressible and Incompressible Fluid Flows

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The session is devoted to the recent development of the mathematical theory of compressible and incompressible fluids in motion. The main topics include the qualitative properties of weak and strong solutions, long-time behavior, fluids confined to special geometries, rotating fluids, among others

Decay estimates of the Oseen flow in the plane

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We consider the initial value problem for the Oseen system in 2D exterior domains and study the local energy decay of solutions. For 3D case the theory was well developed by Kobayashi and Shibata, while 2D case has remained open. The result is applied to deduction of L^p-L^q estimates of the Oseen semigroup. The dependence of estimates on the Oseen parameter is also discussed.

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Fluid mechanical models of self-organized dynamics

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In nature one can find a wide range of species exhibiting self-organizing behavior. For instance, in a school of fish, or in a flock of birds, there are no external forces to coordinate the group, there is no leader to guide them. Starting from individual based models at the microscopic scale one can derive corresponding kinetic models for the mesoscopic scale. In this talk, we will rigorously study the hydrodynamic limit of a class of such kinetic modelsand obtain appropriate macroscopic models. The limiting system will be the compressible isentropic Euler equations with additional non-local terms modeling flock behavior such as attraction and alignment. We will also discuss several open problems related to the macroscopic model such as existence, regularity, and, in particular, large time behavior. The latter will be illustrated with numerical experiments.

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Long time behavior of solutions to nonlinear flow-structure interactions

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We shall consider a model of flow-structure interaction which consists of perturbed wave equation coupled with a nonlinear plate. The interaction between two media takes place on the edge of the plate with the dissipation occurring in a small layer near the edge of the plate. We shall consider both subsonic and supersonic case. It is known that in the latter case the static problem looses ellipticity. Questions such as existence and uniqueness of finite energy solutions will be addressed first . The final goal is to determine geometric conditions for the configuration of the plate which would lead to existence of global attractors capturing solutions near the structure (wing of the airplane). Of particular interest are dissipation mechanisms which are geometrically constrained. It turns out that nonlinear effects in the model are critical in proving ultimate boundedness of solutions. The proofs rely on weighted energy methods with suitably constructed geometric multipliers combined with microlocal analysis and nonlinear elliptic theory which is degenerate.

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Analysis and simulation of shear-dependent non-Newtonian fluids in moving domains

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We present our recent results on mathematical modelling and numerical simulation of non-Newtonian flows in compliant two-dimensional domains having applications in hemodynamics. Two models of the shear-thinning non-Newtonian fluids, the power law Carreau model and the logarithmic Yeleswarapu model, will be considered. For the structural model the generalized string equation for radially symmetric tubes will be generalized to stenosed vessels and vessel bifurcations. The arbitrary Lagrangian-Eulerian approach is used in order to take into account moving computational domains. The analytical result for the existence of a weak solution for the shear-thickening power-law fluid is based on the global iteration with respect to the domain deformation, energy estimates, compactness arguments using the semi-continuity in time and the theory of monotone operators. We will also present several numerical experiments for the Carreau and the Yeleswarapu model, comparisons of the non-Newtonian and Newtonian models and the results for hemodynamical wall parameters; the wall shear stress and the oscillatory shear index. Numerical experiments confirm high order accuracy and stability of new fluid-structure interaction methods. The results have been obtained in a cooperation with S. Necasova (Academy of Sciences, Prague) and A. Hundertmark, G. Rusnakova (University of Mainz).

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Modeling effective pressure interface law between a free fluid and a porous medium

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In this talk we present rigorous justification of the interface law describing contact between the flow in an unconfined fluid and a porous bed. The velocity of the free fluid dominates the filtration velocity, but the pressures are of the same order. Main results are the following:
1. We confirm Saffman's form of the Beavers and Joseph law in a new, more general, setting.
2. We show that a perturbation of the interface position, which is an artificial mathematical boundary, of the order $O(\varepsilon)$ implies a perturbation in the solution of order $O(\varepsilon^2)$. Consequently, there is a freedom in fixing position of the interface. It influences the result only at the next order of the asymptotic expansion.
3. We obtain a uniform bound on the pressure approximation. Furthermore, we prove that there is a jump of the effective pressure on the interface and that it is proportional to the free fluid shear at the interface.

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Some new regularity criteria for weak solutions of the Navier-Stokes equations

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We present two local criteria which imply that a chosen point (x,t) in the space-time cylinder is a regular point of solution v. One of the criteria imposes conditions on function v only in the exterior of a certain space-time paraboloid with the vertex at point (x,t). Furthermore, we present a global criterion which imposes conditions on a certain spectral projection of the associated vorticity or only on its one component.

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Evolution of non-isothermal nematic liquid crystals flows

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We discuss two models describing the time evolution of nematic liquid crystals: one is the non-isothermal version of a variant of the celebrated Leslie-Ericksen model of liquid crystals, the other in the framework of DeGennes-Landau theory, where the natural physical constraints are enforced by a singular free energy potential proposed by J.M. Ball and A. Majumdar. Here the thermal effects are present through the component of the free energy that accounts for intermolecular interactions. The models are consistent with the general principle of thermodynamics and mathematically tractable. We identify the a priori estimates for the associated system of evolutionary partial differential equations and construct global-in-time weak solutions for arbitrary physically relevant initial data.

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Fokker-Planck equations and neuroscience

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Jose Carrillo, Maria del Mar Gonzalez, Maria Gualdani.

I will discuss the global existence of classical solutions to the initial boundary value problem for a nonlinear parabolic equation (Fokker-Planck type) describing the collective behavior of an ensemble of neurons. After some transformations the Fokker-Planck system will be handled by methods used for a classical Stefan problem.

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Regularity problems related to Navier-Stokes equations with mass diffusion

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We discuss global in time well-posedness for the initial-boundary value problems of some coupled Navier-Stokes models with mass diffusion, for compressible or incompressible fluids. In the case of two dimensional space, we establish the global-in-time existence(and uniqueness) of strong solution with initial data of arbitrary size. In 3D case we give some regularity criteria.

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