Special Session 67: Applied Analysis and Dynamics in Engineering and Sciences

Thomas C. Hagen, University of Memphis, USA
Janos Turi, University of Texas at Dallas, USA

The goal of this session is to bring together mathematicians who work in different areas of applied mathematics and might thus not meet and exchange ideas and points of view. Consequently, the session program addresses a cross section of theoretical, numerical and computational developments and their applications to fluid dynamics, solid mechanics and life sciences. Areas of analytical interest include the theory of linear/nonlinear partial differential equations, the qualitative behavior of solutions, stability and asymptotics, control theoretic issues, and related aspects. The areas of application range from viscoelasticity and water waves to industrial flows and applications in mathematical biology and theoretical physics.

Minimal norm control asymptotics and numerical approximations for the null controllability of non-standard parabolic-like PDE dynamics

George Avalos
University of Nebraska-Lincoln, USA
gavalos@math.unl.edu

Semidiscrete finite difference approximation schemes are presented for the null controllability of structurally damped plate equations. The key feature here is that the null controllers being approximated exhibit the asymptotics of the associated minimal energy function. We focus here upon the "nonspectral case"; i.e., the fourth-order elastic component of the dynamics does not necessarily obey (beneficient) hinged boundary conditions.

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Models for growth of heterogeneous sandpiles via Mosco convergence

Marian Bocea
Loyola University Chicago, USA
mbocea@luc.edu

The asymptotic behavior of power-law functionals involving variable exponents is studied in the framework of Mosco convergence, leading to the analysis of a class of quasilinear parabolic problems which in the fast/slow diffusion limit models the growth of sandpiles whose critical slopes depend explicitly on the position in the sample. This is joint work with M. Mihailescu, M. Perez-Llanos, and J.D. Rossi.

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Linear models for fluid-elasticity interactions

Lorena Bociu
NC State University, USA
lvbociu@ncsu.edu

We consider the non-steady, nonlinear coupling of Navier-Stokes and elasticity. We provide a new linearized model for the system, that takes into account the curvature of the common interface and shows that the free boundary plays a key role in the analysis of the coupled system and its influence can not be neglected. The clear advantage of our approach is that it highlights the geometrical aspects of the problem, like curvature and boundary acceleration, which are unaccounted for in current linear models.

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On the stability of closed vortex filaments

Annalisa Calini
National Science Foundation and College of Charleston, USA
acalini@nsf.gov
Thomas Ivey, Stephane Lafortune

In its simplest form, the self-induced dynamics of a vortex filament in an ideal fluid is described by the Vortex Filament Equation, a completely integrable PDE related to the cubic focusing Nonlinear Schrödinger (NLS) equation by the well-know Hasimoto map. I will describe a general framework for studying the linear stability of solutions of the VFE, based on the correspondence between the VFE and the NLS equation, and on the construction of solutions of its linearization in terms of NLS squared eigenfunctions. Focusing on closed vortex filaments, I will discuss: the linear stability analysis of small-amplitude torus knot solutions of the VFE; criteria for instability of vortex filaments associated with periodic traveling wave solutions of the NLS; and a precise characterization of the linear instabilities of filaments associated with cnoidal NLS potentials, concluding that all knotted filaments in this class are linearly unstable. This work is part of on-going collaborations with Tom Ivey and Stephane Lafortune.

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The Navier-Stokes flow of a liquid jet with moving free surface

Shaun Ceci
Le Moyne College, USA
cecisj@lemoyne.edu
Thomas C. Hagen

We analyze a Stokes problem arising in the study of the Navier-Stokes flow of a liquid jet. The analysis is accomplished by showing that the relevant Stokes operator accounting for a free surface gives rise to a sectorial operator which generates an analytic semigroup of contractions. Estimates on solutions are established using Fourier methods. The result presented is the key ingredient in a local existence and uniqueness proof for solutions of the full nonlinear problem.

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Analysis on stability of solitary-wave solutions for a system of nonlinear dispersive equations

Hongqiu Chen
University of Memphis, USA
hchen1@memphis.edu
Jerry L. Bona

Considered here is the system

$$\begin{cases} u_t + u_{xxx} + P(u,v)_x = 0, \\ v_t + v_{xxx} + Q(u,v)_x = 0 \end{cases}$$

of coupled KdV-equations introduced by Bona, Cohen and Wang, where $u = u(x,t), v = v(x,t)$ are functions defined on $\mathbb{R} \times \mathbb{R}^+$,

$$P(u,v) = Au^2 + Buv + Cv^2$$

and

$$Q(u,v) = Du^2 + Euv + Fv^2$$

in which $A, B, \cdots, F$ are real number constants. There are up to three explicit solitary-wave solutions given in terms of hyperbolic secants. Moreover, if the system of linear equations

$$\begin{cases} 2Ba + (E-2A)b - 4Dc = 0, \\ 4Ca + (2F-B)b - 2Ec = 0 \end{cases}$$

has solutions (a,b,c) such that $4ac > b^2$, then, the stability and instability of the solitary-wave solutions can be checked by simple and straightforward algebra.

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Numerical optimal unbounded control with a singular integro-differential equation as a constraint

Shihchung Chiang
Chung Hua University, Taiwan
Chiang@chu.edu.tw

We consider numerical methods for optimal controls with an integro-differential equation of singular kernel as a constraint. We set the objective to minimize gap between optimal state and some target function for certain period of time. By assuming control to be unbounded, we propose a method of feedback correction which makes correction for optimal control proportionally to the corresponding error until certain criterion is satisfied. There are several advantages about this method such as user-decided accuracy, user-decided number of iterations and time saving. We compare the numerical results with the results by other methods from previous work.

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Pathogen evolution in switching environments: a hybrid dynamical system approach

Peter Hinow
University of Wisconsin - Milwaukee, USA
hinow@uwm.edu
Jozsef Farkas, Jan Engelstaedter

We propose a hybrid dynamical system approach to model the evolution of a pathogen that experiences different selective pressures according to a stochastic process. In every environment, the evolution of the pathogen is described by a version of the Fisher-Haldane-Wright equation while the switching between environments follows a Markov process with a given generator matrix. We investigate how the qualitative behavior of a simple single-host deterministic system changes when the stochastic switching process is added. In particular, we study the exchange of stability between monomorphic equilibria. Our results are consistent with the view that in a fluctuating environment, the genotype with the highest mean fitness will eventually become fixed. However, if the probability of host switching depends on the genotype composition of the population, polymorphism can be stably maintained.

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Global stability lobes of turning processes with state-dependent delay

Qingwen Hu
The University of Texas at Dallas, USA
qingwen@utdallas.edu
Wieslaw Krawcewicz, Janos Turi

We obtain global stability lobes of two models of turning processes with inherit non-smoothness due to the presence of state-dependent delays. In the process, we transform the models with state-dependent delays into systems of differential equations with both discrete and distributed delays and develop a procedure to determine analytically the global stability regions with respect to parameters. We find that a spindle speed control strategy can provide essential improvement on the stability of turning processes with state-dependent delay and furthermore we show the existence of a proper subset of the stability region which is independent of system damping. Numerical simulations are presented to illustrate the general results.

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Non-linear discrete-time singularly-disturbed control dynamical systems and their steady state sets

Byungik Kahng
University of North Texas at Dallas, USA
byungik.kahng@unt.edu

It is known that Multiple Valued Iterative Dynamical Systems (MVIDS) can be used to model non-linear discrete-time control dynamical systems with disturbance. MVIDS turns out to be a useful tool particularly when the disturbance contains singularity, on which the traditional calculus-based method fails to apply. Under the MVID-modeling, it turns out that the steady state set of the disturbed control dynamical system corresponds to a set called the locally maximal strongly full-invariant set. In this talk, we discuss how to reach the afore-mentioned set in countable steps so that the finite-step approximate control of the steady state set is well-posed. Time permitting, we also discuss the invariant fractal structure that often arises as a byproduct of the singular disturbance.

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Stability analysis of the delayed oscillator subjected to digital feedback control

David Lehotzky
Budapest University of Technology and Economics, Hungary
lehotzkydavid@gmail.com
Tamas Insperger

The continuous-time delayed oscillator is considered subjected to digital feedback controllers. The motivation for the analysis is the digital control of regenerative machine tool vibrations. While the governing equation of the open loop system inherently involves a term with continuous-time point delay, the sampling effect of the digital feedback controller presents delayed terms of another type: terms with piecewise constant arguments. Since these latter terms can be represented as terms with a piecewise linearly varying time delay, the resultant system is time-periodic at the sampling period. The semi-discretization method is used to construct approximate stability diagrams in the plane of the system parameters. The combination of the two types of delays results in an intricate stability picture.

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Conservative time discretization of large, stiff Hamiltonian systems, applied to models of molecular chains and nonlinear optics

Brenton Lemesurier
College of Charleston, USA
lemesurierb@cofc.edu

A variety of problems in biophysics and nonlinear optics lead to the need to solve large, stiff, mildly nonlinear systems of ODEs that have Hamiltonian form, or perturbations of such by slight dissipation or noise. This talk presents a discrete calculus approach to constructing unconditionally stable, time-reversal symmetric discrete gradient conservative schemes for Hamiltonian systems, akin to the methods developed by Simó, Gonzales, et al, together with an iterative scheme for the solution of the resulting nonlinear systems which preserves unconditional stability and exact conservation of quadratic first integrals. Methods for increasing the order of accuracy are also discussed, along with handling of perturbations like noise. Several applications are considered, as time permits. In particular, some new coherent phenomena in molecular chain models of forms related to the discrete nonlinear Schrödinger equation, leading to a novel continuum limit approximation, and solving coupled systems of NLS equations.

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Computational study of a dynamic contact problem

Jigarkumar Patel
The University of Texas at Dallas, USA
jigar_jcp1@yahoo.com
Janos Turi

In this paper we describe a computational framework to study the influence of a normal crack on the dynamics of a cantilever beam; i.e., changes in its natural frequency, amplitude, period etc. Due to the opening and closing of the crack during beam vibrations unilateral contact boundary conditions are assumed at the crack location. In the numerical implementation the contact conditions lead to the consideration of a linear complementarity problem. We have introduced a very effective method for solving linear complementarity problem. Numerical experiments are included.

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Stability analysis of human respiratory system with both central and peripheral control

Saroj Pradhan
University Of Central Oklahoma, USA
spradhan10@uco.edu
Janos Turi

Stability analysis of a physiologically correct model of respiratory control where the transport delay from the lung to the central controller (τ_b) is assumed to be always greater than transport delay from the lung to the peripheral controller (τ_p)is carried out. It is seen that the multiple switching between stable and unstable regions occur with respect to changes in transport delays. Effects of variation in other internal and external parameters such as atmospheric pressure, central control gain (G_c) and peripheral control gain (G_p) on the stability of the human respiratory control is also studied.

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On the jellyfish joyride: Mathematical analysis of catastrophes in maritime ecosystems

Florian Rupp
TU Munich, Germany
rupp@ma.tum.de
Jürgen Scheurle

Dramatic increases in jellyfish populations which lead to the collapse of formally healthy ecosystems are repeatedly reported. Due to their devastating effects on fish populations the understanding of the causes for such boomings are of major ecological as well as economical importance. Here, we will study the reduced dynnamics of a fish-jellyfish ecosystem by assuming fish as the dominant predator species. By totally analytic means we completely classify all equilibria in terms of existence and non-linear stability, and give a complete description of this system's non-linear global dynamics. This ansatz complements the study given in [**?**], where an analogous system is studied which is derived under the assumption of jellyfish being the dominant predator species.

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Stability of homoclinic orbits of the nonlinear Schrödinger equation

Constance Schober
University of Central Florida, USA
drschober@gmail.com
Annalisa Calini

We study the linearization of the Nonlinear Schrödinger (NLS) equation about homoclinic orbits of unstable plane wave solutions with two unstable modes. The family of homoclinic orbits as well as a complete set of solutions of the associated linearized NLS equation can be constructed using Bäcklund transformations. We show that iterating Bäcklund transformations saturates instabilities of the seed solution, making the largest dimensional homoclinic orbits the most stable in the sense of linear stability.

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Stochastic wave and heat equations with cubic nonlinearity and additive space-time noise in 2D

Henri Schurz
Southern Illinois University, USA
hschurz@math.siu.edu

Stochastic wave and heat equations perturbed by cubic nonlinearities and additive Q-regular space-time white noise are studied. Existence and uniqueness of approximate strong solutions, stability and energy estimates are presented under Dirichlet-type boundary conditions and with L^2-integrable initial data on bounded rectangular domains D in R^2.

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Critical excitation and control of variational inequalities

Janos Turi
University of Texas at Dallas, USA
turi@utdallas.edu
Alain Bensoussan

We consider control problems for the variational inequality describing a single degree of freedom elastoplastic oscillator. We are particularly interested in finding the "critical excitation", i.e., the lowest energy input excitation that drives the system between

the prescribed initial and final states within a given time span. The response of the oscillator to an excitation typically can be described as a sequence of alternating elastic and plastic phases where the state variables have to satisfy size and sign constraints. Using the appropriate governing equations in the elastic and plastic regimes, respectively, and continuity conditions between connecting segments we are able to construct a forward process, which combined with an iterative procedure provides approximations of the critical excitation.

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Analyzing viscous effects on wave motion

Jin Wang
Old Dominion University, USA
j3wang@odu.edu

We study wave motion between two immiscible, incompressible, and slightly viscous fluids. A normal mode analysis is first presented for the linear viscous motion, and results on linear stability, dissipation, and dispersion are discussed. A formal asymptotic analysis is then conducted for the nonlinear progressive waves with slight viscosity. The analytical predictions are verified using numerical simulation, based on a recently developed numerical algorithm for two-phase Navier-Stokes flows, and the wave motion is followed sufficiently in time to quantify the effects of small viscosity.

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