

Special Session 61: PDE Models for Biological Pattern Formation

Thomas Hillen, University of Alberta, Canada
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The aim of this session is to discuss various state of the art approaches for the modeling of biological pattern formation, including chemotaxis, cell movement, tumor growth, and spatial ecology. The emphasis lies on the analysis and simulation of PDE-based models.

Mutations, competition and progression in cancer dynamics

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Tommaso Lorenzi

In this talk we present a mathematical model for the development of cancer at the cellular scale. Attention is focused on progression and heterogeneity aspects of carcinogenesis under an evolutionary perspective.

The model relies on a continuous structured formalism and consists of a set of integro-differential equations describing the dynamics of tumor cells under the effects of mutation and competition phenomena, cell proliferation as well as the action of therapeutic agents.

Asymptotic and computational analysis are developed with an exploratory aim and are devoted to study the role that the phenomena under consideration play in cancer evolution. The obtained results suggest that cancer progression selects for highly proliferative clones and point out how some therapeutic agents might act as an additional selective pressure leading to the selection for the most fitting, and then most resistant, cancer clones.

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Remarks on the global existence in super-critical cases for quasilinear degenerate Keller-Segel systems

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We consider the global existence in super-critical cases for quasilinear Keller-Segel systems with small initial data. In the non-degenerate case Winkler (2010) established the global existence and boundedness of solutions under the smallness of ∇v_0 . Recently, Ishida-Yokota (2012) proved the global existence under the smallness of Δv_0 . Note that there is a “gap” in differentiability imposed by Winkler and Ishida-Yokota. In this talk we try to fill this gap.

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Coupled chemotaxis-fluid models

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We consider coupled chemotaxis-fluid models aimed to describe swimming bacteria, which show bio-convective flow patterns on length scales much larger than the bacteria size. This behaviour can be modelled by a system consisting of chemotaxis equations coupled with viscous incompressible fluid equations through transport and external forcing. The global-in-time existence of solutions to the Cauchy problem in two and three space dimensions is established. Precisely, when the fluid motion is described by Stokes equations, we derive free energy functionals to prove global-in-time existence of weak solutions for cell density with finite mass, first-order spatial moment and entropy provided that the potential is weak or the substrate concentration is small. Moreover, with nonlinear diffusion for the bacteria, we give global-in-time existence of weak solutions in two space dimensions.

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Long time behaviour in some chemotaxis models arising in angiogenesis

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The aim of this talk is to study by different methods the long time behaviour of some models arising in angiogenesis. We will show linear as well as nonlinear stability of each semi-trivial steady-states for these models.

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Global existence of solutions to a parabolic-parabolic system for chemotaxis with logistic source in the higher-dimensional domain

Etsushi Nakaguchi

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We study the global existence of solutions to a parabolic-parabolic system for chemotaxis proposed by M.Mimura and T.Tsujikawa [Physica A 230 (1996)] in a three- or higher-dimensional domain. We will show the global existence of solutions under certain restriction on the degradation order of logistic source or on the smallness of initial data. We will also discuss the dynamical system and attractors for the system.

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Finite-time blowup and global-in-time unbounded solutions to a parabolic-parabolic quasilinear Keller-Segel system

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Tomasz Cieslak

We study radially symmetric solutions to a quasilinear parabolic-parabolic Keller-Segel system in a ball $B_R \subset \mathbb{R}^n$ with $n \geq 3$. Critical nonlinearities had been identified such that in the subcritical case the solution is global in time and bounded while in the supercritical case the solution blows up, but it was not known whether the blowup takes place in finite or infinite time. Assuming a non-decay for the nonlinear chemical sensitivity function, we prove that finite-time blowup occurs for any mass in the whole supercritical case. Moreover, we provide examples showing that in presence of a suitable decay of the sensitivity function some solutions blow up in infinite time. This shows that a non-decay assumption on the sensitivity function is necessary to have finite-time blowup in the whole supercritical case while for decaying sensitivity functions both finite-time and infinite-time blowup can occur. Our proof uses a detailed analysis of the Liapunov functional and generalizes a method which was introduced recently by M. Winkler.

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Global dynamics in a multi-dimensional chemotaxis-haptotaxis model

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Michael Winkler

This talk addresses a coupled chemotaxis-haptotaxis model of cancer invasion. We discuss the global existence and boundedness of solutions to this model. We

moreover analyze stability and attractivity properties of the nontrivial homogeneous equilibrium state and establish a quantitative result relating the domain of this steady state to the size of the parameter of logistic dampening of cell growth. This talk is based on a joint work with Michael Winkler.

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Competing effects of attraction vs repulsion in chemotaxis

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Most of past studies on chemotaxis models deal with attraction and repulsion separately. But in most biological processes (or experiments), the repulsive process usually follows the attractive process for balance in order to accomplish some biological objects. Hence an attraction and repulsion chemotaxis model will be more realistic than a sole attraction or repulsion chemotaxis model in this scenario. In this talk, we shall present the first mathematical results on an attraction-repulsion chemotaxis model and show the interplay of these two opposed biological processes. Some numerical simulations will be shown and various open questions will be presented.

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Reaching a maximal density threshold in some models of chemotaxis.

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We study a quasilinear parabolic system corresponding to models of chemo-taxis in which; 1) there is an impassable threshold for the density of aggregate cells. 2) the diffusion of cells becomes degenerate or singular (fast or super diffusion) when the density approaches the threshold. It is proved that for some range of parameters describing the relation between the diffusive and the chemotactic component of the cell flux there are global-in-time classical solutions which in some cases are separated from the threshold uniformly in time. In the case of fast diffusion existence and uniqueness of weak solutions are proved. For the case of non-degenerate diffusion the existence of solutions which attain the threshold in finite time is proved for the elliptic-parabolic version of the model. The results are contained in the following papers: 1. D. Wrzosek, Model of chemotaxis with threshold density and singular diffusion, *Nonlinear Analysis* 7 (2010) 338-349 .2. Zhi-An Wang, M. Winkler, D. Wrzosek, Singularity formation in chemotaxis systems with volume-filling effect, *Nonlinearity* 24, 3279-3297 (2011) 3. Zhi-An Wang, M. Winkler, D. Wrzosek, Global regularity

vs. infinite-time singularity formation in a chemotaxis model with volume filling effect and degenerate diffusion.

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On the solvability of generalized degenerate chemotaxis models

Tomomi Yokota

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Sachiko Ishida

We consider the solvability of generalized degenerate chemotaxis models. In particular, the solvability for the case of the porous medium-type diffusion has been studied by Sugiyama-Kunii (2006) and Ishida-Yokota (2012). In this talk we try to discuss the solvability for the case of “more generalized” degenerate diffusion by using the maximal regularity.

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