Special Session 6: Dispersal in Heterogeneous Landscape

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This special session aims to address some recent developments in mathematical modeling and analysis of dispersal in heterogeneous landscape. Dispersal of organisms is a key component of ecological and evolutionary processes. Incorporating heterogeneity and dispersal into models for biological processes leads to new mathematical models and brings new mathematical challenges. More generally, understanding dispersal in spatially and/or temporally varying environments can significantly enhance our understanding of how diversity is maintained in complex foodwebs and how organisms can respond to global change.

A new inequality on the spectral bound of resolvent positive operators that unifies results on the evolution of dispersal

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The spectral bound, $s(aA+bV)$, of a combination of a resolvent positive linear operator A and an operator of multiplication V, was shown by Kato (1982) to be convex in $b \in R$. Kato's result is shown here to imply, through an elementary "dual convexity" lemma, that $s(aA+bV)$ is also convex in $a > 0$, and notably, $\partial s(aA+bV)/\partial a < s(A)$. Diffusions typically have $s(A) < 0$, so that for diffusions with spatially heterogeneous growth or decay rates, greater mixing reduces growth. Models of the evolution of dispersal in particular have found this result when A is a Laplacian or second-order elliptic operator, or a nonlocal diffusion operator, implying selection for reduced dispersal. These cases are shown here to be part of a single, broadly general, "reduction" phenomenon. A key open problem is to characterize operators B for which $s(aA+bB)$ decreases in a. By dual convexity, a sufficient conditions is that $(aA+bB)$ be convex in b.

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Regularity for solutions of non local parabolic equations

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We study the regularity of solutions of parabolic fully nonlinear nonlocal equations. We prove C^a regularity in space and time and, under different assumptions on the kernels, $C^{1,a}$ in space for translation invariant equations. The proofs rely on a weak parabolic ABP in order to prove a Point Estimate which allows to decrease the oscillation of the solution geometrically in the interior. Our results remain uniform as $s \to 2$ allowing us to recover most of the classical regularity results for fully non linear parabolic equations.

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Evolutionary stability of ideal free dispersal strategies: a nonlocal dispersal model

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An important problem in the study of the evolution of dispersal is determining what kinds of dispersal strategies are evolutionarily stable. This talk will present results on the evolutionary stability of ideal free dispersal strategies in the context of continuous time nonlocal dispersal models. This work extends recent results by Cosner et al. [1], where a class of ideal free dispersal kernels was introduced and results were obtained that suggested the dispersal strategies they define should be evolutionarily stable. This talk will describe a more general class of ideal free dispersal kernels and give results showing that the ideal free dispersal strategies they define are indeed evolutionarily stable. These results also partially extend some recent work on the evolutionary stability of ideal free dispersal for reaction-diffusion equations and patch models to nonlocal dispersal models.

[1] C. Cosner, J. Davila and S. Martinez, Evolutionary stability of ideal free nonlocal dispersal, J. Biological Dynamics, v.6 (2012), 395-405

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Recent advance in heterogeneous nonlocal models for population dynamics

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to be announced

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A Lane-Emden equation with the fractional Laplacian

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We consider the Lane-Emden equation

$$(-\Delta)^s u = u^p, u > 0$$

either in entire space or a bounded domain and where $p > 1$. One of the contributions is the nondegeneracy of the bubble solution for the critical exponent, and its application for concentration phenomena for slightly subcritical equations. We also study the supercritical equation in entire space.

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Fish biomass production and dispersal across a seasonally flooded marsh

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Small fishes in seasonally flooded environments such as the Everglades are capable of spreading into newly flooded areas and building up substantial biomass. We tested three hypothesized mechanisms for movement of fish into flooding areas; (1) spread through reaction-diffusion, (2) the refuge mechanism that remnant populations of small fishes survive the dry season in small permanent bodies of water, and (3) the dynamic ideal free distribution mechanism that a combination of expansion of habitat by flooding and consumption of prey by the fish creates a prey density gradient and that fish actively follow this. We concluded that although refugia may play an important role in recolonization by the fish population during reflooding, active movement up the prey gradient also seems capable of matching empirical observations. Further studies of the fish biomass dynamics on a seasonally varying, spatially heterogeneous wetland required simulations. We simulated population dynamics (growth, mortality, predation, movement patterns, and resource competition) of three small fish functional groups across a grid-based spatially-explicit, heterogeneous, two-dimensional marsh slough landscape using hydrologic variability as the primary driver for macro-spatial scale movement. The model predicts the spatio-temporal pattern of fish biomass stranded (thus easily available for wading birds) as water levels decline.

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Linking individual movements and population patterns in dynamic landscapes

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Thomas Mueller, Justin Calabrese, Chris Fleming, Chris Cosner, Steve Cantrell

Real landscapes are dynamic in space and time, and the scale over which such variation occurs can determine the success of different strategies of population growth and movement. Real species rely on a variety of individual-level behaviors to move in and navigate through their landscapes. Such behaviors include (1) non-oriented movements based on diffusion and kinesis in response to proximate stimuli, (2) oriented movements utilizing perceptual cues, and (3) memory mechanisms that assume prior knowledge of movement targets. Species use of these mechanisms depends on life-history traits and resource dynamics, which together shape population-level patterns such as range residency, migration, and nomadism. This talk will draw upon empirical data, remote sensing imagery, and a variety of mathematical models to demonstrate the connections among individual movements, landscape dynamics, and population-level patterns. Preliminary work on our current project using integrodifferential equations to represent non-local information gathering will be discussed, focusing on results for bounded domains.

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Evolutionary ecology of habitat selection by predators and prey

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An essential key to explaining the mechanistic basis of ecological patterns lies in understanding the consequences of adaptive behavior for distributions and abundances of organisms. We developed a model that simultaneously incorporates (i) ecological dynamics across three trophic levels, (ii) evolution of behaviors via the processes of mutation, selection, and drift in populations of variable, unique individuals, and (iii) coevolution of traits mediating interactions between predators and prey. Results from the model yield empirically testable predictions about dispersal strategies, spatial distributions, and ecological dynamics of predator-prey-resource systems.

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Accelerating solutions in integro-differential equations

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I present the spreading properties of the solutions of an integro-differential equation of the form $u_t = J * u - u + f(u)$. I focus on equations with slowly decaying dispersal kernels $J(x)$ which correspond to models of population dynamics with long-distance dispersal events. I show that for kernels J which decrease to 0 slower than any exponentially decaying function, the level sets of the solution u propagate with an infinite asymptotic speed. Moreover, I obtain lower and upper bounds for the position of any level set of u. These bounds allow me to estimate how the solution accelerates, depending on the kernel J: the slower the kernel decays, the faster the level sets propagate. My results are in sharp contrast with most results on this type of equation, where the dispersal kernels are generally assumed to decrease exponentially fast, leading to finite propagation speeds.

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On pointwise estimates for non-local elliptic equations

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Russell Schwab

Convexity has played an important role in fully nonlinear equations, for instance in obtaining pointwise bounds for viscosity solutions, as in the celebrated Aleksandrov-Bakelman-Pucci theorem (ABP), or in understanding the regularity of the free boundary in the obstacle problem. In this talk I will describe the shortcomings of convexity when dealing with nonlocal problems and will discuss alternatives as well as their applications. In particular, I will describe a result obtained with Russell Schwab regarding pointwise bounds (analogous tothe ABP) for weak solutions of non-local elliptic equations which is new even for non-local linear operators.

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PDE vs ODE dynamics

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Dynamics, or behavior of solutions of a given nonlinear reaction-diffusion system is deeply related to that of its kinetic problem. In this lecture, we will use the classical Lotka-Volterra competition model as an example to illustrate some connections between the two. Also, the asymptotic behavior of the principal eigenvalue of linear cooperative elliptic systems, as the diffusion rates approach zero, will be motivated and studied.

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Optimization of the persistence threshold in spatial envirnonments with localized patches.

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Determining whether a habitat with fragmented or concentrated resources best supports a contained population is a natural question to ask in Ecology. Such fragmentation may occur naturally or as a consequence of human activities related to development or conservation. In certain mathematical formulations of this problem, a critical value known as the persistence threshold indicates the boundary in parameter space for which the species either persists or becomes extinct. By assuming simple diffusive logistic dynamics for the population and accommodating the heterogeneous nature of the landscape with a spatially varying growth rate, a simple formulation for the persistence threshold is afforded in terms of an indefinite weight eigenvalue problem. In this talk I will show that for a growth rate with strongly localized patches of favorable habitat, the persistence threshold can be calculated implicitly and minimized with respect to the location and fragmentation of patches. This reveals an optimal strategy for minimizing the persistence threshold, thereby allowing the species to persist for the largest range of physical parameters. The techniques developed can be extended to study the effects of heterogeneity in a variety of ecological models.

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Asymptotic behavior of a nonlocal inhomogeneous equation

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In this talk we will study the asymptotic behavior of the solutions of

$$u_t(x,t) = \int_{\mathbb{R}} J\left(\frac{x-y}{g(y)}\right)\frac{u(y,t)}{g(y)}dy - u(x,t),$$

with $J : \mathbb{R} \to \mathbb{R}$ is a nonnegative even function with compact support such that

$$\int_{\mathbb{R}} J(y)dy = 1.$$

In this equation the dispersal is inhomogeneous in space and the step size, g(y), of the dispersal depends on the position y. We assume that $g(y) > 0$ in R. In particular, we will study the behavior of the steady state solutions of this equation.

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Dynamics of a three species competition model

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We investigate the dynamics of a three species competition model, in which all species have the same population dynamics but distinct dispersal strategies. Gejji et al. (*Evolutionary convergence to ideal free dispersal strategies and coexistence*, Bull. Math. Biol., 74 (2012), 257-299) introduced a general dispersal strategy for two species, termed as an ideal free pair in this talk, which can result in the ideal free distributions of two competing species at equilibrium. We show that if one of the three species adopts a dispersal strategy which produces the ideal free distribution, then none of the other two species can persist if they do not form an ideal free pair. We also show that if two species form an ideal free pair, then the third species in general can not invade. When none of the three species is adopting a dispersal strategy which can produce the ideal free distribution, we find some class of resource functions such that three species competing for the same resource can be ecologically permanent by using distinct dispersal strategies.

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The dynamics of resource theft in a spatially continuous habitat

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We describe the dynamics of a producer-scrounger system in a spatial habitat using a partial differential equation model. Both species are assumed to increase logistically and to move randomly in their environment. Although only producers can utilize the resource in its raw form, they are parasitized by nearby scroungers stealing resources from them. When possible, parameter combinations which allow producers and/or scoungers to persist either alone or together are distinguished from those in which they cannot.

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A cross-diffusion model for avoidance behavior in an intraguild predation community

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Intraguild predation (IGP) refers to a community module that blends competitive and predator-prey dynamics. Although IGP is widespread in nature, spatially homogeneous models for IGP communities predict that stable coexistence is only possible if restrictive conditions on resource productivity, competitive ability and predation susceptibility are satisfied. This talk will consider the population dynamics of an IGP module in a spatially heterogeneous landscape and examine how avoidance strategies deployed by the intraguild prey can lead to more robust coexistence states.

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