Special Session 57: Nonlinear and Dispersive Partial Differential Equations

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Regularity of ∞ for elliptic equations with measurable coefficients and its consequences

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This lecture introduces a notion of regularity (or irregularity) of the point at infinity (∞) for the unbounded open set $\Omega \in R^N$ concerning second order uniformly elliptic equations with bounded and measurable coefficients, according as whether the A-harmonic measure of ∞ is zero (or positive). A necessary and sufficient condition for the existence of a unique bounded solution to the Dirichlet problem in an arbitrary open set of R^N , $N \geq 3$ is established in terms of the Wiener test for the regularity of ∞. It coincides with the Wiener test for the regularity of ∞ in the case of Laplace equation. From the topological point of view, the Wiener test at ∞ presents thinness criteria of sets near ∞ in fine topology. Precisely, the open set is a deleted neigborhood of ∞ in fine topology if and only if ∞ is irregular.

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Dispersive blow-up phenomena

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The lecture will discuss singularity formation due to dispersion. Interestingly, such singularities persist even when nonlinearity is present. Possible applications to rogue-wave formation will also be touched upon.

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Long-wave limit of periodic solutions of nonlinear wave equations

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In this lecture, we consider some non-linear, dispersive equations posed on the entire real line and the interest here is the relationship between two different types of solutions. One type is in Sobolev spaces defined on $\mathbb{R}$ and the other is periodic, say of period $2l$. For the same equation, if the two types of solutions are close in some sense at some time as $l \to \infty$, say initially, then we show that as $l \to \infty$, the periodic solutions converge to the solution in the Sobolev space.

Similarly, solitary-wave solution are often approximated by using periodic data. Our theory is a nice justification.

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Exploiting bifurcations in waveguide arrays for light detectors

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An array of finite number waveguides driven laterally by injecting light at the outer waveguides is considered. The array is modelled by a discrete nonlinear Schrödinger equation. It has been shown that when the injected light is in the proximity of a bifurcation point, such system can be sensitive to small disturbances, making it possible to act as a light detector. Here, the optimum intensity of the injected light is discussed and an analytical approximation is presented.

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Complex-valued Burgers and KdV Burgers equations

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Spatially periodic complex-valued solutions of the Burgers equation and KdV-Burgers equations will be discussed. It is shown that for any sufficiently large time T, there exists an explicit initial datum such that its corresponding solution of the Burgers equation blows up at finite time T.

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NLS with quantum potential, the related reaction-diffusion systems and Kaup-Broer system

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This talk will be presented by two parts.We will review some old results and plan to present some new parts.

At first, we will review some old results about 2 by 2 AKNS system with linear and quadratic spectral parameter and the related evolution equations and the inverse scattering method under the set-up of Beals-Coifman.

Then we will make a brief report on the nonlinear Schrödinger equations with quantum potential, derivative nonlinear Schrödinger equations and the related reaction-diffusion systems. We will discuss some possible applications.

$$i\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+\frac{\Lambda}{4}|\psi|^2\psi=s\frac{1}{|\psi|}\frac{\partial^2|\psi|}{\partial x^2}\psi.$$

A novel integrable version of the NLS equation with quantum potential has been termed the resonant nonlinear Schrödinger equation (RNLS). It can be regarded as a third version of the NLS, intermediate between the defocusing and focusing cases. The critical value $s=1$ separates two distinct regions of behaviour. Thus, for $s<1$ the model is reducible to the conventional NLS, (focusing for $\Lambda>0$ and defocusing for $\Lambda<0$). However, under the condition, for $s>1$, it is reducible to a reaction-diffusion system, which can be transformed into Kaup-Broer system. In this case, the model exhibits novel solitonic phenomena. The RNLS can be interpreted as an NLS-type equation with an additional quantum potential $U_Q=|\psi|_{xx}/|\psi|$. Recently it was shown that RNLS naturally appears in a reduced equation in the plasma physics. Via a Hirota bilinear representation of the Reation-Diffusion system, here some exact solutions are obtained by this bilinear method. We will also consider a non-Madelung type hydrodynamic representation for resonant nonlinear Schrödinger type equations. New Broer-Kaup type systems of hydrodynamic equations are also derived from the derivative reaction-diffusion systems arising in SL(2,R) Kaup-Newell hierarchy, represented in the non-Madelung hydrodynamic form. The relation with Kaup-Broer system will be mentioned here. (This is a joint work with O.K.Pashaev.)

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Global regularity results for the 2D MHD equations with horizontal dissipation and horizontal magnetic diffusion

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We study the global regularity issue concerning the 2D incompressible magnetohydrodynamic (MHD) equations with horizontal dissipation and horizontal magnetic diffusion. We establish a global bound for the L^{2r}-norm of the horizontal component of the velocity and of the magnetic field for any $1\le r<\infty$ and the bound grows no faster than the order of $\sqrt{r\log r}$ as r increases. A global L^q-bound, $1<q\le 3$, for pressure is also obtained. In addition, we establish a conditional global regularity in terms of the $L^2_tL^\infty_x$-norm of the horizontal component and the global regularity of a slightly regularized version of the aforementioned MHD equations. This is a joint work with C. Cao and J. Wu

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An initial boundary value problem for one-dimensional shallow water magnetohydrodynamics in the solar tachocline

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In this talk we investigate the shallow water magnetohydrodynamics equations in space dimension one with Dirichlet boundary conditions only for the velocity. This model has been proposed to study the phenomena in the solar tachocline. In this talk, the local well-posedness in time of the model is presented by constructing the approximate solutions and showing the strong convergence of the approximate solutions.

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Recent results on the Serrin-type regularity criteria for Navier-Stokes and related equations

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We study the Serrin-type regularity criteria for Navier-Stokes and related equations such as magneto-hydrodynamics and others. Recently a significant amount of effort has been devoted to reduce this criteria from a classical Lp bound on the gradient of the solution to only its components or partial derivatives. Regularity criteria in the mixed Lp-space will also be discussed.

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Fifth-order complex Korteweg-de Vries type equations

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We discuss spatially periodic complex-valued solutions of the fifth-order Korteweg-de Vries (KdV) type equations. The aim is at several fundamental issues including the existence, uniqueness and finite-time blowup problems. Special attention is paid to the Kawahara equation, a fifth-order KdV type equation. When a Burgers dissipation is attached to the Kawahara equation, we establish the existence and uniqueness of the Fourier series solution with the Fourier modes decaying algebraically in terms of the wave numbers. We also examine a special series solution to the Kawahara equation and prove the convergence and global regularity of such solutions associated with a single mode initial data. In addition, finite time blow-up results are discussed for the special series solution of the Kawahara equation.

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