Special Session 46: Discrete/Continuous and Nonstandard Analysis

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Our session "discrete/continuous and nonstandard analysis" contains variational problem and linear/nonlinear differential equations and logics itself. Taking intervals for state/time very small in each system, there exists a problem on discrete/continuous and nonstandard analysis. In some cases, we have a solution of these relations, but generally not yet enough. In quantum field, Prof S.Nagamachi gave "minimum length". We need such an example. There are many problems when discretizing differential equations, for example, wavelet but not only one thing. Even taking a usual limit, we met many singular states, especially for parameters. We are going to discuss such a singular state in our session.

Smooth models of discontinuous systems

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In this talk we present smooth models of discontinuous systems and make a comparative study of their dynamics and bifurcations using Nonstandard Analysis, once the difference between continuity and S continuity allows the existence of smooth models of quick changing situations, where usually a discontinuous approach is used.

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The Osgood integral: an extraordinary tool

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Initially, Osgood [4] used the integral of Osgood for an unicity criterion to the differential equation $dy/dx = f(y), f(0) = 0$. The trivial solution is unique iff this integral goes to the infinite at the origin. Then he can prove the unicity of the trivial solution of $dy/dx = yLn|y|$, although the second member is not lipschitzian. Later, Bernfeld [1] shows that all the solutions of $dy/dx = f(y)$ do not explose iff the same integral goes to the infinite at the infinite and Ceballos-Lira, Macias-Diaz and Villa [3] generalize this Osgood's test. Finally, we can adapt a result from the Cauchy works [2] as follows: the trivial solution is a singular solution iff the same integral vanishes at the origin. Using non standard analysis, we extend the different criterions to non autonomous differential equations and generalize them.

References:

[1] Bernfeld S.R. : the extendability and uniqueness of solutions of ordinary differential equations, Pacific J.Math 69 (1977) No.2 pp 307-315.

[2] Cauchy A.L. : Equations differentielles ordinaires, cours inedit , co-publication CNRS, coll. Academic Press(1981).

[3] Ceballos-Lira M.J., Macias-Diaz J.E., Villa J. : a generalization of Osgood's test on a comparison criterion for integral equations with noise, EJDE Vol 2011 (2011) No.5 pp 1-8.

[4] Osgood W.F.: Beweis der Existenz einer Lösung der differentialgleichungen dy/dx=f(x,y) ohne Hinzunahme der Cauchy-Lipschitzschen Bedingung, Monatsch.Math.Phys. 9 (1898) pp 331-345.

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How to find infinitesimals in a big genetic-metabolic model?

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In the study of metabolism and expression of genes, it appears commonly differential systems of large dimension (dozens of unknowns). To make the most of these models, it is necessary to take account of the orders of magnitude. In fact, we want to modelize the differential system, with non standard (infinitely small or infinitely large parameters) coefficients. The aim of the talk is to discuss the different methods to introduce these infinitesimals in a real system.

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Error estimation for approximate solutions of SDE

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The numerical solution of Ito's stochastic differential equation (SDE) is realized by pseudo-random numbers in terms of an approximate solution on computers. Pseudo-random numbers are defined by some algebric algorithms such as the linear congruential method, M-sequence, etc. For more details, see e.g. Knuth (1981) and Fushimi (1989). Since any algorithm has an essential defect for independence and distribution, as Knuth (1981) pointed out, pseudo-random numbers do not behave completely as a sequence of independent random variables with the uniform distribution. Furthermore pseudo-normal random numbers, which are generated from pseudo-random numbers by the Box-Muller method or the Marsaglia method, can not obey the normal distribution exactly. In this note we focus on the distribution of pseudo-random numbers and consider the error estimation of the Euler-Maruyama approximation when the distribution of underlying random variables is different from the normal distribution.

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Relativistic quantum field theory with a fundamental length

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The relativistic equation of quantum mechanics called Dirac equation

$$i\frac{\hbar}{c}\gamma_\mu\frac{\partial}{\partial x_\mu}\psi(x) - M\psi(x) = 0,$$

$x_0 = ct, x_1 = x, x_2 = y, x_3 = z$, contains two constants c (velocity of light), the fundamental constant in the relativity theory and $h = 2\pi\hbar$ (Planck constant), the fundamental constant in quantum mechanics. Dimension of c is $[\mathrm{LT}^{-1}]$ and that of h is $[\mathrm{ML}^2\mathrm{T}^{-1}]$. W. Heisenberg thought that the equation must also contain aconstant l with dimension [L]. Then arbitrary dimensions are expressed by the combination of c, h and l, e.g., $[\mathrm{T}] = [\mathrm{L}]/[\mathrm{LT}^{-1}]$, $[\mathrm{M}] = [\mathrm{ML}^2\mathrm{T}^{-1}]/([\mathrm{LT}^{-1}][\mathrm{L}])$
In 1958, Heisenberg with Pauli introduced the equation

$$\frac{\hbar}{c}\gamma_\mu\frac{\partial}{\partial x_\mu}\psi(x) \pm l^2\gamma_\mu\gamma_5\psi(x)\bar{\psi}(x)\gamma^\mu\gamma_5\psi(x) = 0, \quad (1)$$

which is later called the equation of universe. The constant l has the dimension [L] and is called the fundamental length. But equation (1) is difficult to solve. So, we consider the following soluble equation having the constant l with the dimension [L]:

$$\begin{cases} \Box\phi(x) + \left(\frac{cm}{\hbar}\right)^2\phi(x) = 0 \\ \left(i\frac{\hbar}{c}\gamma_\mu\frac{\partial}{\partial x_\mu} - M\right)\psi(x) \\ +2\gamma_\mu l^2\psi(x)\phi(x)\frac{\partial\phi(x)}{\partial x_\mu} = 0 \end{cases} \quad (2).$$

This equation has no solutions in the axiomatic framework of of Wightman, that is, the field $\psi(x)$ is not an operator-valued tempered distribution. But $\psi(x)$ is an operator-valued tempered ultrahyperfunction.
The equation (2) has a solution in the framework of ultrahyper function quantum field theory which is a generalization of Wightman's tempered field theory. We present the axioms based on tempered ultrahyper functions which have no localization property and discuss their relations to the fundamental length l: The two events occurred within the distance l can not be distinguished. Only if the distance between two events is greater than l, they are distinguished.

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On relative stability in 4-dimensional canard

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This talk gives the existence of a relatively stable duck solution ina slow-fast system in R^{2+2} with an invariant manifold. It has a 4-dimensional canard having a relatively stable region when there exists the invariant manifold near the pseudo singular node point. It is a revised version of AIMS 2010 and added further results.

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Transitions between discreteness and continuity of all orders of regularity.

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We study discrete functions on regular and irregular infinitesimal grids in one or two dimensions. We consider its difference quotients of higher order and give conditions for them to be infinitely close to the corresponding (partial) derivatives. Important tools are the formula of Faa di Bruno for higher order derivatives and a discrete version of it. Applications include a general method for transitions from difference equations to differential equations, a DeMoivre-Laplace Theorem of higher order and regularity properties of discrete free-boundary problems.

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A proof-theoretic approach for nonstandard analysis

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Significant tools for nonstandard analysis are the transfer principle, the existence of the standard part, some saturation/enlargement principles, and so on.

Formalizing these, we can construct a formal system for nonstandard analysis. One formal system based on ZFC is given by Edward Nelson in 1977. In fact, most part of nonstandard analysis can be done within a very small fragment of this formal system. In this talk, I will introduce several weak formal systems for nonstandard analysis, and discuss which axioms are essentially needed for nonstandard analysis. This is a proof-theoretic study of nonstandard analysis from the standpoint of so called "reverse mathematics". By this, we can see that several basic theorems of nonstandard analysis are actually equivalent to important axioms of nonstandard analysis. Moreover, I will give a canonical interpretation of nonstandard proofs into elementary proofs without nonstandard methods. This means that, in theory, there is a canonical way to remove nonstandard techniques from a nonstandard proof. By this, we can also compare the length of a proof using nonstandard techniques with a proof without nonstandard analysis.

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