Special Session 44: Applications of Chaotic and Stochastic Multiscale Dynamics

Rafail Abramov, Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, USA
Gregor Kovacic, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, USA
Ilya Timofeyev, Department of Mathematics, University of Houston, USA

Multiscale, chaotic and stochastically parameterized processes are common in many areas of contemporary science, such as molecular dynamics, genetics, neuroscience, nonlinear optics, and geosciences, among others. Key topics on applications of chaotic and stochastic dynamics for complex multiscale systems will be brought together in this interdisciplinary session, ranging from fluid dynamics to geophysical sciences, nonlinear optics and computational biology. Examples of applications will include low frequency variability and climate change, parameter estimation in complex unresolved systems, prediction of statistical behavior under external perturbations, applications of multiscale high-dimensional coagulation processes, random dynamics of neuronal networks, stochastic soliton dynamics, and more. Common to all these topics is the mathematical approach used to address them, which will be highlighted in the session.

A simple linear response closure approximation for slow dynamics of a multiscale system

Rafail Abramov
University of Illinois at Chicago, USA
abramov@math.uic.edu

We propose a method of determining the closed model for slow variables of a multiscale system, which requires only a single computation of appropriate statistics for the fast dynamics with a certain fixed state of the slow variables. The method is suitable for situations with linear, quadratically nonlinear, and multiplicative coupling, and is based on the first-order Taylor expansion of the mean state and covariance matrix of the fast variables with respect to changes in the slow variables, which can be computed using the linear fluctuation-dissipation theorem. We show that the method produces quite comparable statistics to what is exhibited by a complete two-scale model. The main advantage of the method is that it applies even when the statistics of the full multiscale model cannot be simulated due to computational complexity, which makes it practical for real-world large scale applications.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Compressed Sensing in Retinal Image Processing

Victor Barranca
Rensselaer Polytechnic Institute, USA
barrav@rpi.edu

Retinal image processing transforms photons into membrane potentials via several nonlinear transformations. This process begins in a large network of photoreceptors and ends in a relatively small ganglion cell network. We posit the loss of visual information despite the decrease in network size is minimized via compressed sensing. Using an idealized mathematical model of the retina and a mean-field analytical reduction, we demonstrate firing patterns among ganglion cells can be used to reconstruct input images.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Low-dimensional descriptions of neural networks

Andrea Barreiro
Southern Methodist University, USA
abarreiro@smu.edu
Nathan Kutz, Eric Shea-Brown, Eli Shlizerman

Biological neural circuits display both spontaneous asynchronous activity, and complex, yet ordered activity while actively responding to input. When can model neural networks demonstrate both regimes? Recently, researchers have demonstrated this capability in large, recurrently connected neural networks, or "liquid state machines", with chaotic activity. We study the transition to chaos in a family of such networks, and use principal orthogonal decomposition techniques to provide a lower-dimensional description of network activity. Finally, we connect our findings to the ability of these networks to perform as liquid state machines.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A general method for parametric estimation of stochastic volatility models

Arjun Beri
Mathematical Biosciences Institute, USA
beri.3@mbi.osu.edu
Robert Azencott, Ilya Timofeyev

Modeling dynamics of a physical phenomenon by parametric stochastic differential equation is widespread. In many situations classical techniques of parametric inference given discrete observations do not provide consistent estimates for the drift and diffusion parameters. I will present asymptotic results for a test case of parametric estimation of a stochastic differential equation. In particular, I consider a stochastic volatility model specified as a system of two-dimensional stochastic differential equation (X, Y), where Y represents the stochastic dynamics of the diffusion coefficient of process X. Assuming unobservability (or, latency) of the volatility process Y, parametric estimation of Y is based only on the available discrete observations from X. Estimation of parameters characterizing Y involves an intermediate step of constructing a dataset of empirical estimates for quadratic variation of X, denoted by $V_t^\varepsilon = [X]_{t-\varepsilon}^t$, such that V_t^ε converges to Y_t as $\varepsilon \to 0$.I will present optimal subsampling conditions, verified numerically, for asymptotic consistency of estimators for parameters in Y based on observations from approximate process V_t^ε, and also highlight the general context of estimation under indirect observability, estimation of reduced order model given multiscale (fast-slow) system, and the role of path properties, such as modulus of continuity, in parameter estimation of a limiting model.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Finding quasipotential for nongradient SDE's

Maria Cameron
University of Maryland, USA
cameron@math.umd.edu

Nongradient Stochastic Differential Equations describe overdamped systems without detailed balance that arise e.g., in chemical reactions far from equilibrium, evolutionary biology, and stochastically modeled computer networks. Nongradient systems allow more complex attractors than stable equilibrium points. The behavior of such a system can be quantified in terms of the quasipotential. It allows to estimate the probability density and transition rates between different attractors, and to find the most likely transition paths between them. I will discuss some theoretical properties of the quasipotential and present a numerical algorithm for computing it on a regular mesh.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear stochastic inverse models with memory, and prediction of climatic phenomena

Mickael Chekroun
UCLA, USA
mchekroun@atmos.ucla.edu
Dmitri Kondrashov, Michael Ghil, Andrew Robertson

Recent works on stochastic Navier-Stokes equations and previous approaches coming from Statistical Mechanics such as the Mori-Zwanzig (MZ) formalism, have advocated the idea of representing the higher (unresolved) modes as functionals of the time history of the low (resolved) modes to deal with the closure problem. In the MZ formalism, these functional dependence arise typically in complicated integral terms obtained by repeated convolutions between decaying memory kernels and the resolved variables. In the case of a lack of scale separation, these memory kernels roughly decay at the same rate than the decorrelation rate of the solution itself; which renders challenging the numerical computations of these integral terms and thus the obtention of an efficient solution to the closure problem via this approach.

By considering a certain class of memory kernels within the MZ formalism, it will be presented a numerically tractable data-driven approach to deal with this problem while allowing the cases where the separation of scales is not necessarily pronounced. The approach will be illustrated for the inverse modeling of two major tropical climatic phenomena: the El Niño-Southern Oscillation (ENSO) and the Madden-Julian Oscillation (MJO). Prediction capabilities of the resulting nonlinear stochastic inverse models will be discussed in each case.

The past noise forecasting method recently developed by Chekroun, Kondrashov and Ghil (PNAS, 108 (29), 2011) will be then presented. It will be shown how this method allows to extend the prediction skill of our MJO and ENSO models by using one hand, information from the estimated path on which the inverse stochastic model lives; and on the other, dynamical features associated with the low-frequency variability captured by these models.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Strong collapse turbulence in quintic nonlinear Schroedinger equation.

Yeojin Chung
Southern Methodist University, USA
ychung@smu.edu
P.M. Lushnikov

We consider the quintic one dimensional nonlinear Schrödinger equation with forcing and both linear and nonlinear dissipation. Quintic nonlinearity results in multiple collapse events randomly distributed in space and time forming forced turbulence. Without dissipation each of these collapses produces finite time singularity but dissipative terms prevents actual formation of singularity. In statistical steady state of the developed turbulence the spatial correlation function has a universal form with the correlation length determined by the modulational instability scale. The amplitude fluctuations at that scale are nearly-Gaussian while the large amplitude tail of probability density function (PDF) is strongly non-Gaussian with power-like behavior. The small amplitude nearly-Gaussian fluctuations seed formation of large collapse events. The universal spatio-temporal form of these events together with the PDF for their maximum amplitudes define the power-like tail of

PDF for large amplitude fluctuations, i.e., the intermittency of strong turbulence.

⟶ ∞ ◇ ∞ ⟵

Stochastic dynamics on networks

Lee Deville
University of Illinois, USA
rdeville@illinois.edu

Dynamical systems defined on networks have applications in many fields, including neuroscience, ecology, biophysics, condensed matter theory, etc. In particular, it is interesting to understand if and when networked dynamical systems exhibit synchronous or coherent collective behaviors, but this question has proved surprisingly difficult to answer in many contexts. Yet another question is how stable coherent behavior is to random perturbation, or, conversely, what types of network-level structures can be induced by noise alone. We will examine several applications that fall into this framework and present a few theorems that make this analysis possible.

⟶ ∞ ◇ ∞ ⟵

Stochastic models for organized tropical convection

Boualem Khouider
University of Victoria, Canada
khouider@uvic.ca
Andrew Majda, Yevgeniy Frenkel

Physical processes associated with clouds and convection have a big impact on large scale circulation and climate variability in the tropics. Current operational climate models capture very poorly the radiative-green house effects and latent heat release due to the formation of clouds and the inherent large scale organization into large (synoptic to planetary) scale convectively coupled waves. To help improve the variability in climate models due to unresolved organized convection, we present and analyze a hierarchy of stochastic and deterministic multicloud cloud models.

⟶ ∞ ◇ ∞ ⟵

Dynamics of light interacting resonantly with an active optical medium

Gregor Kovacic
Rensselaer Polytechnic Institute, USA
kovacg@rpi.edu
E. P. Atkins, G. Biondini, I. R. Gabitov, P. R. Kramer

Resonant interaction of light pulses with a randomly-prepared, lambda-configuration active optical medium is described by exact solutions of a completely-integrable, random partial differential equation, thus combining the opposing concepts of integrability and disorder. An optical pulse passing through such a material will switch randomly between left- and right-hand circular polarizations. Exact probability distributions of the electric-field envelope variables describing the light polarization and their switching times will be presented. The dynamics of pulses on top of continuous waves will also be discussed.

⟶ ∞ ◇ ∞ ⟵

Faster dynamic Monte Carlo via Markov couplings

Kevin Lin
University of Arizona, USA
klin@math.arizona.edu
Jonathan Goodman, Jonathan Mattingly

Dynamic Monte Carlo methods are widely used in scientific and engineering computations, and are frequently applied to stochastic dynamical systems exhibiting chaotic behavior. In this talk, I will report on recent efforts to accelerate dynamic Monte Carlo calculations using a tool from probability theory, namely Markov couplings. Specifically, I will discuss coupling-based algorithms for two distinct but related problems: sensitivity analysis for stochastic differential equations and variance reduction for nonequilibrium steady-state calculations in statistical physics.

⟶ ∞ ◇ ∞ ⟵

A traffic model for pedestrian and its comparison with experimental data

Sebastien Motsch
CSCAMM, University of Maryland, USA
smotsch@cscamm.umd.edu
Pierre Degond, Cecile Appert-Rolland, Mehdi Moussaid, Guy Theraulaz

Understanding the various factors which determine pedestrian flow efficiency is an important societal issue (e.g. traffic safety, optimization of traffic flow). With this aim, a simple model has been constructed and calibrated on the basis of experimental recordings. The novelty of the model (2-way Lighthill-Whitham-Richards model) relies on the introduction of a two direction flow depending on co and counter-moving pedestrians. At high density, the model becomes "non-hyperbolic" which leads to the formation of clusters similar to traffic jam observed in high density crowd. A comparison between data and model is performed and we observe good agreement for the apparition of clusters and traveling bands. However, some discrepancies are observed, which are due to the inhomogeneities of the pedestrians comfort walking speeds. To account for these inhomogeneities, an extension of the model is presented (2-Way Aw-Rascle model). The comfort speed is assimilated from the data by observing the pedestrian velocity at low densities. The resulting model and assimilated initial data are compared to the experimental results and agree quite well.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Magnetization reversal in thin film magnetic elements

Katherine Newhall
Courant Institute, NYU, USA
newhall@cims.nyu.edu
Eric Vanden-Eijnden

There is considerable interest in understanding thermally induced magnetization reversals in thin film magnetic elements, with application to random access memory storage. We investigate the rare event of thermally induced magnetization reversal in thin film magnetic elements. We include the effect of spin transfer torques, and their influence on the average switching times. To determine the average time for these rare events in the stochastic multi-scaled system, we consider the evolution of the averaged slow variable, the energy, and use a combination of computer simulations and analytical techniques.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The neurochemical dynamics of the mammalian sleep-wake regulatory network

Lisa Rogers
NYU, USA
lrogers@cims.nyu.edu

The mammalian sleep/wake system is governed by several interacting populations of neurons in and around the hypothalamus. We present here a model of the sleep and wake promoting populations in the VLPO (ventrolateral preoptic nucleus), BF (basal forebrain) and PB/PC (parabrachial nucleus). The model is formed using Morris-Lecar firing dynamics and the chemical kinetics of receptor-neurotransmitter interaction to quantify synaptic input. We also present a novel but simple way of relating firing rates of neuron populations to the corresponding concentrations of neurotransmitter, allowing us to track both electrical and chemical output. Rate and equilibrium constants are obtained using appropriate mammalian data from the BRENDA enzyme database.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A multiscale method for epitaxial growth

Yi Sun
University of South Carolina, USA
yisun@math.sc.edu
Russel Caflisch, Bjorn Engquist

We investigate a heterogeneous multiscale method (HMM) for interface tracking and apply the technique to the simulation of epitaxial growth. HMM relies on an efficient coupling between macroscale and microscale models. When the macroscale model is not fully known explicitly or not accurate enough, HMM provides a procedure for supplementing the missing data from a microscale model. Here we design a multiscale method that couples kinetic Monte-Carlo (KMC) simulations on the microscale with the island dynamics model based on the level set method and a diffusion equation. We perform the numerical simulations for submonolayer island growth and step edge evolutions on the macroscale domain while keeping the KMC modeling of the internal boundary conditions. Our goal is to get comparably accurate solutions at potentially lower computational cost than for the full KMC simulations, especially for the step flow problem without nucleation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

From stochastic to coarse-grained models of pedestrian traffic

Ilya Timofeyev
University of Houston, USA
ilya@math.uh.edu
A. Chertock, A. Kurganov, A. Polizzi

Microscopic rules for pedestrian traffic in a narrow street or corridor are discussed and the corresponding stochastic system modeling the pedestrian bi-directional flow is introduced. Mesoscopic and macroscopic PDE models for the pedestrian density are derived. The macroscopic PDE model is a system of conservation laws which can change type depending on the strength of interaction between the pedestrian flows and initial conditions. Behavior of the stochastic and coarse-grained models is compared numerically for several different regimes and initial conditions. Finally, nonlinear diffusive corrections to the PDE model are derived systematically. Numerical simulations show that the diffusive terms can play a crucial role when the conservative coarse-grain PDE model becomes non-hyperbolic.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Irreversible Monte Carlo algorithms for efficient sampling

Marija Vucelja
Courant Institute of Mathematical Sciences, USA
vucelja@cims.nyu.edu
Konstantin S. Turitsyn, Michael Chertkov

Equilibrium systems evolve according to Detailed Balance (DB). This principle guided the development of Monte Carlo sampling techniques, of which the Metropolis-Hastings (MH) algorithm is the famous representative. It is also known that DB is sufficient but not necessary. We construct irreversible deformation of a given reversible algorithm capable of dramatic improvement of sampling from known distribution. Our transformation modifies transition rates keeping the structure of transitions intact. To illustrate the general scheme we design an Irreversible version of Metropolis-Hastings (IMH) and test it on an example of a spin cluster. Standard MH for the model suffers from critical slowdown, while IMH is free from critical slowdown.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The effects of time iteration schemes on the climate of the Lorenz 96 model

Celestine Woodruff
Florida State University, USA
cwoodruf@math.fsu.edu
Xiaoming Wang

We consider the effects of using several different time iteration schemes on the chaotic Lorenz 96 model to predict statistics, which give us the "climate". This is a simple model, but it exhibits energy-preserving advection, damping, and forcing (some of the same properties that can be found in geophysical models), shares statistical features of weather wave packets of the atmosphere, and is an analogue of Rossby waves. The schemes that we consider are Forward Euler, Backward Euler, 4th order Runge-Kutta, and four new schemes which we call Semi-Implicit 1 (SI1) and Semi-Implicit 2 (SI2), 2nd order Semi-Implicit 1 (SSI1) and 2nd order Semi-Implicit 2 (SSI2). We give theoretical results for stability, with unconditional stability for Backward Euler, SI2, and SSI2. Using a numerical truth, we find that for a small enough time step, all of the methods perform very well. We also see grossly inaccurate solutions and blowup when using Forward Euler for the time step $h = 10^{-2}$, which should be a reasonable time step. We show that SI2, which also has energy-preserving properties, is a much better choice for this model and that care needs to be taken when choosing time iteration schemes for climate models in general.

$\longrightarrow \infty \diamond \infty \longleftarrow$