

## Special Session 43: Stochastic Networks with Applications to Neuroscience

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Dynamical networks feature a rich variety of spatio-temporal phenomena including synchrony, bursting and avalanches, formation of clusters and waves, intermittency, and chaos. Understanding the mechanisms underlying network dynamics requires a combination of tools from several mathematical disciplines: dynamical systems, stochastic processes, and discrete mathematics. The talks in this special session explore different aspects of network dynamics emphasizing the role of stochasticity and its applications to neuroscience.

**Modeling collective neural activity: when are pairwise maximum entropy methods good enough?**

**Andrea Barreiro**

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**Julijana Gjorgjieva, Fred Rieke, Eric Shea-Brown**

Recent experimental studies find that the activity patterns of many neural circuits are well described by pairwise maximum entropy (PME) models — which require only the activity of single neurons and neuron pairs — even in cases where circuit architecture and input signals seem likely to create a richer set of outputs. Why is this the case? We study spike patterns in a general class of circuits, and attempt to draw general principles about the effects of network architecture and input statistics on the complexity of output spiking patterns. Two significant findings have emerged: in feedforward circuits, responses to unimodal inputs were well described by PME models, while bimodal input signals drove significant departures. Secondly, biophysically motivated recurrence can drive significant departures when added to feedforward circuits.

As a particular application, we investigate the performance of PME models in retinal ganglion cell (RGC) circuits with different architectures and inputs. We find that the distinct filtering properties of parasol cells suppress higher-order interactions by suppressing bimodality in light stimuli, offering a possible mechanism for the remarkable success of PME models in this experimental setting.

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**Dynamics of a stochastic neuronal network model with inhibitory neurons**

**Stephen Berning**

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We will consider extensions and generalizations of the stochastic neuronal network model developed by DeVille et al.; their model corresponded to an all-to-all network of discretized integrate-and-fire excitatory neurons where synapses are failure-prone. It was shown that this model exhibits different metastable phases of asynchronous and synchronous behavior, since the model limits on a mean-field deterministic system with multiple attractors. Our work investigates the effect of adding inhibition and heterogeneity into the model and considers several statistical measures of the dynamics. We will also show that there exists a family of network-supported continuous-time Markov chains that converge to this neuronal network model in a singular limit.

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**Emergence of computation in random networks**

**Stefano Boccaletti**

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**M. Zanin, D. Papo, R. Gutierrez**

One of the open problems in neuroscience is the understanding of how neurons can self-organize to create structures able to perform some meaningful computation. While several theoretical models have been proposed (from Hebbian learning, up to Hopfield networks), they are usually based on regular, or fully connected, initial configurations, which are not found in the early stages of the development of a brain. In this contribution, we will show how a non-trivial computation, i.e., including memory on past states, can be performed by random Erdős-Renyi networks; furthermore, we will show that the probability of the emergence of such feature depends on simple characteristics of the graph, such as the number of nodes and the density of links. These results may represent a new paradigm explaining how natural neural networks, without the help of any external intervention or of any pre-defined architectures, are able to perform the complex computations that we continuously experience.

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**Stochasticity and phase-locking in small neuronal networks**

**Amitabha Bose**

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**Andrea Barreiro, Katka Bodova, Victoria Booth, Badal Joshi, Jung Eun Kim, Jasmin Nirody, Erin Munro**

We investigate stochastic effects in feed-forward neuronal networks. We study a simple network consisting of an oscillator (O) that forces a follower (F) cell to determine the phase-locked solutions of the network. The case of having a deterministic synapse from O to F has been previously studied. We add to the results by explicitly deriving a one-dimensional map involving the PRC of F that reproduces the devil's staircase of solutions. We then show how adding stochasticity to the synapse allows us to continue to use the map formalism to show which portions of the staircase persist. The analysis allows us to give meaning to the term "phase-locking" even in a stochastic setting.

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**Spatially structured networks from sequences****Carina Curto**University of Nebraska-Lincoln, USA  
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Spatially structured networks, such as bump attractor networks, have enjoyed considerable success in modeling a wide range of phenomena in cortical and hippocampal networks. A key question that arises in the case of hippocampal models, however, is how such a spatial organization of the synaptic connectivity matrix can arise in the absence of any a priori topographic structure of the network. Here we demonstrate a simple mechanism by which robust sequences of neuronal activation, such as those observed during hippocampal sharp waves, can lead to the formation of spatially structured networks that exhibit robust bump attractor dynamics.

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**Communicability distance in networks****Ernesto Estrada**University of Strathclyde, Scotland  
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We introduce the concept of communicability distance on the basis of a transformation of the exponential of the adjacency matrix of the network of  $n$  nodes. We prove that the communicability distance is a Euclidean distance. In addition we also prove that the points given rise to the communicability distance can be embedded into a hyper-sphere of dimension  $n-1$  and certain radius determined by the inverse of the communicability distance matrix. We show a few mathematical properties of the communicability distance and compare it with the shortest path distance and the resistance distance. We show that the communicability distance is very useful in identifying the best routes for information traffic in crowded networks. We show some illustrative examples for the case of brain networks.

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**Memory encoding via perturbations of spatially structured networks.****Vladimir Itskov**University of Nebraska-Lincoln, USA  
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Memories in the brain are believed to be encoded via patterns of synaptic strengths of recurrent neuronal networks. It is unclear how exactly this can work however. Given a prescribed list of memories, it is still an unsolved problem how the synaptic weights can be arranged so that exactly those memories are encoded, while avoiding unwanted "spurious" states. The problem is challenging because encoded patterns are usually heavily overlapping; this case is prevalent in many important areas of the brain such as hippocampus. Here we develop a perturbative approach to the memory encoding problem, where memory patterns are encoded via small perturbations of the synaptic weights around a spatially structured low-rank network. We find large classes of sets of overlapping memories that can be encoded exactly as steady states of threshold-linear networks.

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**The geometry of spontaneous spiking in neuronal networks****Georgi Medvedev**Drexel University, USA  
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We study electrically coupled networks of excitable neurons forced by small noise using techniques from dynamical systems and algebraic graph theories. The results are presented from two complementary perspectives of variational analysis of spontaneous network dynamics and slow-fast analysis of synchronization. The former approach yields geometric interpretation of various dynamical regimes including weakly correlated firing, formation of clusters, and complete synchronization; while the latter highlights the contribution of the network architecture to the stability of the synchronous state.

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**Synchrony in stochastic pulse-coupled neuronal network models****Katherine Newhall**Courant Institute, NYU, USA  
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We are interested in the synchronous dynamics of simple pulse-coupled models for neuron dynamics. These time correlations in firing times are seen experimentally. In our model, the size of synchronous firing events depends on the probabilistic dynamics between such events as well as the network structure representing the neuron connections. We presents both analytical results and numerical simulations of these global dynamics.

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**Network structure and synchronization of coupled oscillators****Takashi Nishikawa**Clarkson University, USA  
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Synchronization, in which individual dynamical units keep in pace with each other in a decentralized fashion, depends both on the dynamical units and on the properties of the interaction network. Yet, the role played by the network has resisted comprehensive characterization. In this talk, I will present results that illuminate the intricate relationship between structure and synchronization stability in coupled oscillator networks. Using networks with best complete synchronization, least coupling cost, and maximum dynamical robustness, I will show that "less can be more" in network synchronization: *negative* interactions as well as link *removals* can be used to systematically improve and optimize synchronization properties in both directed and undirected networks. I will also show that having local and/or global *directionality* structure in the network facilitates synchronization.

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**The influence of network structure on neuronal network dynamics****Duane Nykamp**University of Minnesota, USA  
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We investigate the influence of network structure on the dynamics of neuronal networks, with a focus on the emergence of synchronous oscillations. Network structure is specified using the framework of second order networks, a network model that captures second order statistics (correlations) among the connections between neurons. We demonstrate that the frequency of a chain motif in the network plays a crucial role in influencing network dynamics, not only modulating the emergence of synchrony but also possibly increasing the range of possible network behaviors.

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**Criticality and dynamic range in network cascading processes****Juan Restrepo**University of Colorado, USA  
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I will present recent work on the effect of network topology on cascading processes. The motivation for our work is a series of recent experiments on cascades of excitation in rat cortical tissue cultures, where it is found that these neural networks maximize their dynamic range (the range of stimulus intensities resulting in distinguishable network responses). We develop a theoretical framework to study the effect of network topology on the response to a stochastic stimulus, and find that the dynamic range is maximized when the largest eigenvalue of the transmission probability matrix is one. In the critical regime with maximum dynamic range, the response of the network is characterized by excitation avalanches with power-law distributions of size and duration. We find that these experimental signatures of criticality are robust to the underlying network structure. Using our theory, we characterize the network topologies that can achieve the largest dynamic range. I will discuss potential applications of these results to other networked systems.

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**Competing spatiotemporal neural codes in the olfaction of the *Manduca sexta* moth****Eli Shlizerman**University of Washington, USA  
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Experiments across different species have shown that perception of odors in the olfactory system is associated with neural encoding patterns. The neurobiological mechanisms responsible for such encoding patterns, their transient dynamics and interactions are yet to be fully understood. We show that a data-driven computational model reduction for the antennal (olfactory) lobe (AL) of the *Manduca sexta* moth reveals the nature of experimentally observed persistent spatial and temporal neural encoding patterns and its associated decision-making dynamics. Utilizing the experimental data we reduce a high-dimensional neural network model of the AL to a decision making model. Analyzing the model we conclude that the mechanism responsible for the robust and persistent appearance of neural codes is a stable fixed point. The model is used to explain, predict and direct experiments when odors are mixed or the structure of the network is altered.

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**Exact mean-field dynamics for a heterogeneous network of globally coupled theta neurons****Paul So**George Mason University, Krasnow Institute for Advanced Study, USA  
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The theta neuron is a canonical model for a spiking neuron with Type-I excitability. In this work, we consider a large heterogeneous network of such neurons coupled globally with a pulse-like synapse. Utilizing the recently developed Ott-Antonsen reduction method, we obtained an exact low dimensional mean field result for this network of theta neurons in the thermodynamic limit. Each individual theta neuron is chosen randomly with its parameter drawn from a given time-invariant distribution. By choosing the distribution function to straddle the SNIC (saddle-node on a limit cycle) bifurcation point, this network of theta neurons when uncoupled will naturally segregate into a group of resting but excitable neurons and another group of constantly-spiking neurons. Using the mean field reduction, we analyzed and categorized the range of possible collective behavior for this heterogeneous network of coupled theta neurons as the relative proportion of neurons belonging to the two fundamental groups was varied.

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