Special Session 4: Nonlinear PDEs and Control Theory with Applications

Barbara Kaltenbacher, University of Alpen-Adria, Klagenfurt, Austria
Irena Lasiecka, University of Virginia, USA
Petronela Radu, University of Nebraska-Lincoln, USA
Lorena Bociu, NC State University, United States

In the large context of nonlinear evolution equations we will focus on systems of PDEs which exhibit a hyperbolic or parabolic-hyperbolic structure. The topics of this special session will revolve around qualitative and quantitative properties of solutions to these equations, such as existence and uniqueness, regularity of solutions, and long time asymptotic behavior. Associated control theoretic questions such as stabilization, controllability and optimal control will be addressed as well. Both bounded and unbounded domain problems will be under considerations. Of special interest in our discussions are interactions involving nonlinearity and geometry . Several methods available for investigation of such problems will be presented. We anticipate also to discuss specific problems that arise in applications such as nonlinear acoustics, traveling waves in elasticity and viscoelasticity, plasma dynamics, and semiconductors.

Rational decay of structural acoustic dynamics

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A rate of rational decay is obtained for solutions of a PDE model which has been used in the literature to describe structural acoustic flows. This structural acoustics PDE consists partly of a wave equation which is invoked to model the interior acoustic flow within a given cavity. Moreover, a structurally damped elastic equation is invoked to describe time-evolving displacements along the flexible portion of the cavity walls. The coupling between these two distinct dynamics will occur across a boundary interface. We obtain the uniform decay rate of this structural acoustic PDE without incorporating any additional boundary dissipative feedback mechanisms. By way of deriving this stability result, necessary a priori inequalities for a certain static structural acoustics PDE model are generated, thereby allowing for an application of a recently derived resolvent criterion for rational decay.

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Weak and regular solutions for nonlinear waves with super-critical sources and nonlinear dissipations

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We consider nonlinear wave equations characterized by energy building, super-critical sources and nonlinear dissipation terms. We provide sharp results on the range of parameters for damping terms and sources that determine the exact regions for existence and uniqueness for both weak and regular solutions, and finite time blow-up.

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Algebraic Riccati equations with unbounded coefficients lacking analyticity of the free dynamics semigroup

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The talk will report on the recent advances in the study of the Linear-Quadratic problem on an infinite time horizon for composite systems of evolutionary Partial Differential Equations which comprise both parabolic and hyperbolic components, such as models for thermoelastic, acoustic-structure and fluid-solid interactions. Emphasis will be placed on the far-reaching role of functional analytic methods at a theoretical level, as well as of parabolic regularity and microlocal analysis in order to establish the appropriate regularity of boundary traces which ultimately allows to ensure well-posedness of the corresponding algebraic Riccati equations.Most results presented in the talk are obtained jointly with Irena Lasiecka (Virginia) and Paolo Acquistapace (Pisa).

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Uniform decay rates for the wave equation with locally distributed nonlinear damping in unbounded domains with finite measure

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Valeria Domingos Cavalcanti, Flavio R. Dias Silva

This talk is concerned with the study of the uniform decay rates of the energy associated with the wave equation with locally distributed nonlinear damping

$$u_{tt} - \Delta u + a(x)\, g(u_t) = 0 \quad \text{in } \Omega \times (0, \infty)$$

subject to Dirichlet boundary conditions, where $\Omega \subset \mathbb{R}^n$ $n \geq 2$, is an *unbounded* open set with *finite measure* and *unbounded* smooth boundary $\partial\Omega = \Gamma$. The

function $a(x)$, responsible for the localized effect of the dissipative mechanism, is assumed to be nonnegative, essentially bounded and, in addition,

$$a(x) \geq a_0 > 0 \ \text{ a.e. in } \omega,$$

where $\omega = \omega' \cup \{x \in \Omega; ||x|| > R\}$ $(R > 0)$ and ω' is *a neighbourhood* in Ω *of the closure of* $\partial\Omega \cap B_R$, where $B_R = \{x \in \Omega; ||x|| < R\}$.

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Dissipative acoustic solitons

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Lagrangian-averaged models for compressible flow have recently been proposed [Bhat & Fetecau, DCDS-B 6 (2006) 979–1000]. Like their counterparts in turbulence, these models introduce a minimal ("cut-off") length scale beyond which energy dissipation cannot occur. When applied to weakly-nonlinear acoustic phenomena in inviscid, lossless single-phase fluids, the Lagrangian-averaged model represents a higher-order dispersive regularization of the governing equation, which exhibits nonlinear dissipation as well [Keiffer et al., Wave Motion 48 (2011) 782–790]. Kink-type solitary wave solutions are derived analytically, and an implicit finite-difference scheme with internal iterations is constructed in order to study their collisions. It is shown that two kinks can interact and retain their identity after a collision, meaning that these waves represent dissipative acoustic solitons. For a different choice of parameters, finite-time blow-up can be observed numerically. Finally, while the classical equations of nonlinear acoustics can be reduced to Burgers' equation, we show that the present model reduces to the Korteweg–de Vries equation.

This work is, in part, a collaboration with R.S. Keiffer, R. McNorton and P.M. Jordan from the Naval Research Laboratory, Stennis Space Center.

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A unified theory for damped evolutionary equations

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Marcelo Cavalcanti, Vilmos Komornik, Jose Henrique Rodrigues

The main goal of this talk is to establish an abstract theory which allows us to determine the exponential decay in a certain Hilbert space H for locally damped evolution equations. Our approach can be used for a wide assortment of equations as, for instance, the heat equation, Airy-Burgers equation, Schrödinger equation, among others.

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Partial continuity for parabolic systems

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Consider the parabolic system

$$u_t - \operatorname{div}[a(x,t,u,Du)] = 0 \quad \text{ in } \Omega_T := \Omega \times (-T, 0),$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain and $T > 0$. The vector field $a : \Omega_T \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to \mathbb{R}^N$ satisfies natural p-growth and ellipticity assumptions with $p > 2n/(n+2)$. I will present partial continuity results for weak solutions to the above problem. More precisely, the results establish that there is an open set of full measure in Ω_T in which the solution is Hölder continuous. The key assumption for the problem being considered is that the vector field a is continuous with respect to the arguments x, t and u. This distinguishes this result from others which assume Hölder continuity of a with respect to x, t and u and provide partial continuity for the spatial gradient of the solution. The talk will focus mostly on the subquadratic setting, where $2n/(n+2)$.

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Some elliptic and parabolic problems with boundary conditions of diffusive type

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We will mainly focus on parabolic and elliptic equations with boundary conditions of diffusive type, which are sometimes dubbed as dynamic(al) or Wentzell boundary-type conditions. We wish to discuss issues involving global well-posedness and regularity of solutions, the asymptotic behavior as time goes to infinity in terms of global/exponential attractors. Finally, some explicit dimension estimates are provided to show a different degree of structural complexity of these systems when compared to the same equations subject to the usual Dirichlet and Neumann-Robin type of boundary conditions.

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The polarization in a ferroelectric thin film: local and nonlocal limit problems

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In this joint work with K. Hamdache (Ecole Polytechnique, Palaiseau, France), starting from classical non-convex and nonlocal 3D-variational model of the electric polarization in a ferroelectric material, via an asymptotic process we obtain a rigorous 2D-variational model for the polarization in a ferroelectric thin film. Depending on the initial boundary conditions, the limit problem can be either nonlocal or local. Thin films of ferroelectric material are used for the realization of “Ram” for computers and “RFID cards ”.

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The wave equation with abstract nonlinear acoustic boundary conditions

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In this talk we introduce a wave equation with “abstract” nonlinear acoustic boundary conditions, which consists of a wave equation coupled with an abstract second-order PDE where the coupling occurs at the boundary interface. This system resembles the original model proposed by Morse and Ingard but is sufficiently general to cover a wide range of problems studied in the literature. We show that under quite general assumptions, the system generates a nonlinear semigroup, and, in the presence of appropriately selected nonlinear boundary damping, the semigroup is uniformly stable. These results serve to unify a number of the available studies in the literature on PDE models of acoustic/structure interactions. In addition, we study the effects of a ”nonlinear coupling” which arises in the case of a porous structure.

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Asymptotic behavior of solutions to nematic liquid crystal models

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Wu Hao

We present some recent results on the longtime behavior of solutions to some simplified versions of the Ericksen-Leslie model for the nematic liquid crystal flow. More precisely, we consider first the one proposed by F.-H. Lin et al. In this case the evolution system consists of the Navier-Stokes equations coupled with a convective Ginzburg-Landau type equation for the (vector-valued) averaged molecular orientations. We discuss the convergence of given trajectories to single equilibria when the Dirichlet boundary conditions for the order parameter and the external force acting on the flow are time dependent. Then we introduce a more refined model proposed by C. Liu et al. and we show that the corresponding dynamical system in two spatial dimensions has a finite dimensional global attractor.

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Global attractors for von Karman plate model with a localized damping

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In this study dynamic von Karman equations with localized interior damping supported in a boundary collar are considered. Hadamard well-posedness for von Karman plates with various types of nonlinear damping are well-known, and the long-time behavior of nonlinear plates has been a topic of recentinterest. Since the von Karman plate system is of ”hyperbolictype” with critical nonlinearity (noncompact with respect to the phase space), this latter topic is particularly challenging in the case of geometrically constrained and nonlinear damping. In this paper we first show the existence of a compact global attractor for finite-energy solutions, and we then prove that the attractoris both smooth and finite dimensional. Thus, the hyperbolic-like flow is stabilized asymptotically to a smooth and finite dimensional set.

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Free liquid fibers and films

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The mathematical description of free liquid fibers and films poses serious analytical challenges. In the simplest case of a highly viscous material, this description, based on a long-wave approximation of the Navier-Stokes equations with free surface, is essentially due to Matovich, Pearson and Yeow. It takes the form of a nonlinear transport equation coupled to an elliptic momentum equation or system of such equations.In this presentation I will address recent results on stability and global existence in the absence of surface tension for equations of this type. Some reduced models will also be addressed.

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Non-linear plate coupled with Darcy flows for slightly compressible fluid

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In this work, we consider the dynamical response of a non-linear plate with viscous damping, perturbed in both vertical and axial directions interacting with a Darcy flow. We first study the problem for the non-linear elastic body with damping coefficient. We prove existence and uniqueness of the solution for the steady state plate problem. We investigate the stability of the dynamical non-linear plate problem under some condition on the applied loads. Then we explore the fluid structure interaction problem with a Darcy flow in porous media. In an appropriate Sobolev norm, we build an energy functional for the displacement field of the plate and the gradient pressure of the fluid flow. We show that for a class of boundary conditions the energy functional is bounded by the flux of mass through the inlet boundary.

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Nonlinear poroacoustic flow in rigid porous media

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An acoustic acceleration wave is defined as a propagating singular surface (i.e., wavefront) across which the first derivatives of the velocity, pressure, or density exhibit jumps. In this talk, the temporal evolution of the amplitude and speed of such waves are investigated in the context of nonlinear, fluid-acoustic propagation in rigid porous media, where the fluid-solid interaction is described by the resistance law of Darcy. It is shown that there exists a critical value, the constant $\alpha^*(> 0)$, of the initial jump amplitude. It is then established that the acceleration wave magnitude either goes to zero, as $t \to \infty$, or blows-up, in finite time, depending on whether the initial jump amplitude is less than or greater than α^*.

Additionally, numerical solutions of an idealized initial-boundary value problem involving sinusoidal signaling in a fluid-saturated porous slab are used to illustrate the finite-time transition from acceleration to shock wave, which occurs when the initial jump amplitude is greater than α^*, and comparisons with the linearized case (i.e., the damped wave equation) are presented whenever possible. Finally, the related phenomenon of weakly-nonlinear poroacoustic traveling waves, where an exact solution is possible in terms of the Lambert W-function, is briefly considered and connections to second-sound (i.e., thermal wave) phenomena are noted. (Work supported by ONR funding.)

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Remarks on the Fourier series method

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We review some recent applications of harmonic and nonharmonic Fourier series in control theory, including various results obtained in collaboration with C. Baiocchi, P. Loreti, M. Mehrenberger and A. Barhoumi. We discuss several possible generalizations of Parseval type inequalities due to Ingham and Beurling .

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On the exterior problem for nonlinear wave equations in 2D

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In this talk I wish to present a result on the exterior problem for the nonlinear wave equations in two space dimensions. Because the dispersive property for the solution to the wave equation is rather weak in 2D, it is not straightforward to get counterpart for the Cauchy problem. Nevertheless, u Under the geometric assumption on the obstacle, we are able to get an almost global solution for small initial data. Moreover, if we pose the null condition on the nonlinearity, then the solution actually exists globally.

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Recovering sound spend and initial data for the wave equation by a single measurement

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We consider a problem of recovering the sound speed and an initial condition for wave equations which is motivated from the photo/thermo-acoustic imaging model. We will also show the connection between such a problem and the classical inverse hyperbolic problem with a single measurement.

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Time optimal boundary controls for the heat equation

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The fact that the time optimal controls for parabolic equations have the bang-bang property has been recently proved for controls distributed inside the considered domain (interior control). The main result in this article asserts that the boundary controls for the heat equation have the same property, at least in rectangular domains. This result is proved by combining methods from traditionally distinct fields: the Lebeau-Robbiano strategy for null controllability and estimates of the controllability cost in small time for parabolic systems, on one side, and a Remez-type inequality for Müntz spaces and a generalization of Turán's inequality, on the other side.

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Inverse problem for the heat equation and the Schrödinger equation on a tree

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In this paper we establish global Carleman estimates for the heat and Schrödinger equations on a network. The heat equation is considered on a general tree and the Schrödinger equation on a star-shaped tree. The Carleman inequalities are used to prove the Lipschitz stability for an inverse problem consisting in retrieving a stationary potential in the heat (resp. Schrödinger) equation from boundary measurements.

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Instability for nonlinear evolution equations

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We consider steady solutions of semilinear parabolic and hyperbolic equations; the hyperbolic equations exhibit a sign changing damping term. We prove that linear instability with a positive eigenfunction gives rise to nonlinear instability by either exponential growth or finite-time blow-up. We then discuss a few examples to which our main theorems are immediately applicable, including equations with supercritical and exponential nonlinearities.

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Local and global well-posedness to a nonlinear model in viscoelasticity with m-Laplacian damping

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We consider a general viscoelasticity model in a bounded domain $\Omega \subset \mathbb{R}^3$ with a smooth boundary Γ:

$$u_{tt} - \Delta_p u - \Delta_m u_t = f(u) \quad \text{in} \quad \Omega \times (0,T),$$

subject to Dirichlét boundary condition: $u = 0$ on $\Gamma \times (0,T)$, or subject to p-generalized Robin boundary condition. We study the local and global solvability in the interesting case when p is between 2 and 3. In this case, if the interior source $f(u)$ is of order r (i.e., $|f(u)| \leq c|u|^r$ for all $|u| \geq 1$), then $f(u)$ is locally Lipschitz from $W^{1,p}(\Omega)$ into $L^2(\Omega)$, only for the values $1 \leq r \leq \frac{3p}{2(3-p)}$. Thus, our goal is to prove the local and global solvability of the problem when the interior source is of supercritical order, i.e., $\frac{3p}{2(3-p)} < r < \frac{3p}{3-p}$.

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Asymptotic behavior of the wave equation with dynamic boundary condition

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In this talk, we consider a semi-linear wave equation with a dynamic boundary conditions.

We prove the local existence and uniqueness of the solution of our problem. We use the Faedo-Galerkin approximation coupled with the fixed point theorem. Concerning the asymptotic behavior of the solution of this problems we prove the following :

When the initial data are large enough and the damping is nonlinear, then the energy solution is unbounded. In fact, we show that the Lp-norm of the solutions grows as an exponential function.

We prove that in the case of linear boundary damping, the solution blows up in finite time.

We showed also that if the initial data are in the stable set, the solution continues to live there forever.

In addition, we show that the presence of the strong damping forces the solution to go to zero uniformly and with an exponential decay rate. To obtain our results, we combine the potential well method with the energy method.

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Solutions to the Euler equation on a domain with a moving boundary

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Igor Kukavica

We present a new proof of local-in-time existence and regularity of solutions to the free boundary Euler Equation without surface tension under the Taylor-Rayleigh condition.

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Simultaneous controllability and stabilization of some uncoupled wave and plate equations

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First, we discuss the simultaneous controllability of uncoupled wave equations with different speed of propagation in a bounded domain. The control is locally distributed, and the control region satisfies the geometric control condition of Bardos-Lebeau-Rauch. Thanks to the Hilbert uniqueness method of Lions, the controllability problem is reduced to an observability one. To solve the observability problem, we employ a combination of the observability result of Bardos-Lebeau-Rauch for a single wave equation, and a new uniqueness result for the uncoupled system. Then, we address the related stabilization problem with the help of a theorem of Haraux and the new uniqueness result. Afterwards, we investigate similar problems for uncoupled plate equations with the help of the Holmgren uniqueness theorem. Finally, we discuss some generalizations, and some open problems.

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Carleman estimates and stabilization of hyperbolic systems in absence of geometric observability conditions

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The sharp sufficient conditions for observability in control systems are formulated via geometric optics and are linked to the structure of closed geodesics in the underlying physical domain. However, the necessary unique continuation property for PDEs is an intrinsically weaker requirement and does not impose such restrictions. It is, therefore, often possible to stabilize an evolution system by placing feedback controls on subsets of the domain that fail to satisfy the geometric conditions. The price to pay is the necessity to work with smoother solutions and the stabilization rates obtained thereby are no longer exponential.

In this work we present specialized Carleman estimates and a generalization of a pioneering strategy due to G. Lebeau and L. Robbiano to prove uniform stability (for strong solutions) of 1st-order hyperbolic systems without reliance on the geometric observability assumptions.

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An inverse problem for the ultrasound equation: uniqueness and stability

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We condsider a third order (in time) ultrasound equation arising in acoustics and we recover a critical coefficient by means of boundary measurement. More sopecifically, we obtain uniqueness and stability of the recovery

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Min-max game problem for coupled PDE system and an application to the fluid-structure interaction model

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We consider an established hyperbolic-parabolic fluid-structure interaction model with control and disturbance acting at the interface between the two media. Here, the structure (modeled by the system of dynamic elasticity) is immersed in a fluid (modeled by the linearized Navier-Stokes equations). A game theory problem between control and disturbance is studied, when both act at the interface. To this end, the main mathematical difficulty that one encounters is the fact that such model fails to satisfy the "singular estimate" from, say, control to state. This is a critical obstacle, as this is precisely the foundational property for a full theory to include solvability of the Differential Riccati equation in the study of the associated min-max game theory problem. Failure of the "singular estimate" property is due to a mismatch between the parabolic and hyperbolic component of the overall coupled dynamics. By introducing suitable observation or output operators, with incremental smoothness on the trajectory, it is shown that the resulting system satisfies a modified singular estimate, this time from the control to the observation space. This then allows one to adapt the complete min-max theory to the present

fluid-structure interaction model. More precisely, the approach followed is based on an abstract setting, of which the fluid-structure interaction model is a canonical illustration, which enjoys a desirable property not assumed for the abstract model.

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