Special Session 34: Multi-phase Flows in Porous Media and Related Systems

David Ambrose, Drexel University, USA
Xiaoming Wang, Florida State University, USA
Steven Wise, University of Tennessee, USA

Multiphase fluid flow in porous media is of great importance in many areas of science and engineering applications. Two well-known examples are the flow of groundwater and oil recovery in petroleum engineering. There are also classical applications in materials science (such as Hele-Shaw cells), as well as emerging applications in fuel cell technology (water management in PEM fuel cells), and biomedical science (tumor growth modeled as flow in porous media). The associated mathematical problems are intriguing and challenging, involving free boundaries as well as singular perturbations and stiff problems.

Much effort has been devoted to the investigation of multi-phase flows in porous media, with much recent progress on the mathematical theories for the physical models. Nevertheless there are still many unresolved mathematical and physical issues. Hence it seems a good time to have a special session devoted to the problem of multi-phase flows in porous media and related problems. There are two principal approaches to multi-phase flow. The first approach treats the interface as a sharp one with zero width. The second approach recognizes the micro-scale mixing and hence treats the interface as a transition layer with finite (small) width (the so-called diffuse interface model or phase field model). Our plan is to have some of the leading experts from both camps, including people working on modeling, analysis and simulation, to report on recent progress and future challenges associated with multi-phase flow in porous media and closely related systems.

Removing the stiffness from 3D interfacial flow with surface tension

David Ambrose
Drexel University, USA
ambrose@math.drexel.edu
Michael Siegel, Svetlana Tlupova

We present a boundary integral method for the efficient computation of three-dimensional irrotational free surface flows in the presence of surface tension. Specifically, we will consider a model problem for porous media flow. By considering a model problem, we are able to focus on the removal of stiffness, rather than other issues such as computing the Birkhoff-Rott integral.

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Numerical methods for some phase-field models

Franck Boyer
Aix-Marseille Universite, France
fboyer@cmi.univ-mrs.fr
C. Lapuerta, S. Minjeaud, B. Piar

In this talk, I will describe some numerical methods that we propose for the computation of multiphase flows through a phase-field model consisting in the coupling of a Cahn-Hilliard system and the Navier-Stokes equation.

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Solvability of a generalized Buckley-Leverett model

Nikolai Chemetov
CMAF / University of Lisbon, Portugal
chemetov@ptmat.fc.ul.pt
W. Neves

We propose a new mathematical modelling of the Buckley-Leverett system, which describes the two-phase flows in porous media.

We prove the solvability of the initial-boundary value problem for a deduced model, being a coupled system of hyperbolic/elliptic type equations. In order to show the solvability result, we consider an approximated parabolic-elliptic system. Since the approximated solutions do not have ANY type compactness property, the limit transition is justified by the kinetic method.

The main issue is to study a linear (kinetic) transport equation, instead of the nonlinear original system.

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Functionalized Cahn-Hilliard equation: competitive evolution of bilayers and pores

Shibin Dai
Michigan State University, USA
sdai@math.msu.edu
Keith Promislow

The functionalized Cahn-Hilliard equation is introduced as a phase field model to describe the evolution of complex nanoscale structures similar to those observed in Polymer Electrolyte Membrane (PEM) fuel cells. Such complex structures include single lay-

ers, bi-layers, pore networks and micelles, etc. We concentrate on the motion of closed bi-layers and pores. Using asymptotic analysis, we analyze their inner structures and derive the leading order normal velocity in different time scales. Also we will show the mechanism under which they compete with each other.

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Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions

Maurizio Grasselli
Politecnico di Milano, Italy
maurizio.grasselli@polimi.it
Sergio Frigeri

A well-known diffuse interface model consists of the Navier-Stokes equations nonlinearly coupled with a convective Cahn-Hilliard equation. This system describes the evolution of an incompressible isothermal mixture of fluids and it has been investigated by many authors. Here we discuss a variant of this model where the standard Cahn-Hilliard equation is replaced by its nonlocal version. More precisely, the gradient term in the free energy functional is replaced by a spatial convolution operator acting on the order parameter. We intend to present some recent results on the global longtime behavior of the (weak) solutions.

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A rescaling scheme and its applications to free boundary problems

Shuwang Li
Illinois Instaitute of Technology, USA
sli@math.iit.edu

In this talk, I present a time and space rescaling scheme for the computation of moving interface problems. The idea is to map time-space such that the interfaces can evolve exponentially fast in the new time scale while the area/volume enclosed by the interface remains unchanged. The rescaling scheme significantly reduces the computation time (especially for slow growth), and enables one to accurately simulate the very long-time dynamics of moving interfaces. We then implement this scheme in a Hele-Shaw problem, examine the dynamics for a number of different injection fluxes, and present the largest and most pronounced viscous fingering simulations to date. I then generalize the idea to be able to compute multiple interfaces. I will also give examples of the curvature weakening model of the Hele-Shaw problem.

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Stability of 2D incompressible flows under 3D perturbations

Milton Lopes Filho
IMECC-UNICAMP, Brazil
clavyus@gmail.com
C. Bardos, D. Niu, H. Nussenzveig Lopes, E. Titi

We discuss results on preservation of 2D symmetry under 3D viscous flows with low regularity and corresponding results for inviscid flow.

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Liquid drops sliding down an inclined plane

Antoine Mellet
University of Maryland, USA
mellet@math.umd.edu
Inwon Kim

We investigate the properties of a model describing the motion of liquid drops sliding down an inclined plane (the so-called quasi-static approximation model). We prove existence and uniqueness of a solution and investigate its long time behavior for both homogeneous and inhomogeneous medium (i.e. constant and non-constant contact angle).

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A Cahn-Hilliard model with dynamic boundary conditions

Alain Miranville
Universite de Poitiers, France
miranv@math.univ-poitiers.fr

Our aim in this talk is to discuss the dynamical system associated with the Cahn-Hilliard equation with dynamic boundary conditions. Such boundary conditions take into account the interactions with the walls for confined systems. We are in particular interested in a model which accounts for the conservation of mass, both in the bulk and on the walls.

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Capillary driven viscous fingering in Buckley-Leverett models

Iain Moyles
University of British Columbia, Canada
imoyles@math.ubc.ca

The modelling of two fluid displacement in porous media is of great industrial interest in soil remediation and environmental engineering. One of the phenomena of interest is the formation of fingering patterns that emerge at the fluid interface. There

has been a large focus from the academic community to accurately model the formation of fingers under various physical scenarios. In this talk, we look at an oil-water in situ flushing model based on Buckley-Leverett formulations. We start with an overview of the Taylor-Saffman instability applied to lateral flow when we negelect surface tension between fluids. We then consider the effect of introducing a small local capillary effect at the interface between fluids, motivated by a physical scaling based on industrially relevant parameters. With the introduction of the capillary effect we present some singular perturbation analysis as well as numerical computations to analyze and compare the finger growth to that of the case with no surface interaction. Depending on the capillary-saturation dependence, the steady-state solutions that form can either be C^∞ smooth or C^0 with compact support; this is due to the appearance of degenerate terms in the differential equations. We present results under degenerate and non-degenerate conditions.

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Thermal compositional model based on the diffuse interface assumption

Paul Papatzacos
University of Stavanger, Norway
paul.papatzacos@uis.no

A thermal compositional model for three phase flow must face a number of challenges. (1) The interrnal degrees of freedom of the atoms must be taken into account in the thermodynamic description of the fluid. (2) The transport coefficients must be known functions of the temperature and the densities, even for unstable states. (3) Numerical instabilities due to small thermal conductivities must be overcome. These challenges are discussed and solutions presented. A numerical example involving injection of cold fluid in a warmer reservoir is presented.

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A nonstiff boundary integral method for 3D interfacial flow with surface tension

Michael Siegel
NJIT, USA
misieg@oak.njit.edu
David Ambrose, Svetlana Tlupova

A nonstiff boundary integral method for moving boundaries in 3D flow is presented, with an application to porous media flow. The velocity of the interface is given in terms of the Birkhoff-Rott (B-R) integral. Numerical stiffness due to surface tension is removed by using a generalized isothermal parameterization of the surface and approximating the B-R integral at high wavenumbers by a small scale decomposition. Results are presented for a doubly-periodic interface and a closed interface, the latter of which is discretized using moving coordinate patches and partitions of unity.

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3-D pore-scale resolved model for coupled species/charge/fluid transport in a vanadium redox flow battery

Ying Sun
Drexel University, USA
ysun@coe.drexel.edu
Gang Qiu, Abhijit S. Joshi, C.R. Dennison, K.W. Knehr, E.C. Kumbur

The vanadium redox flow battery (VRFB) has emerged as a viable grid-scale energy storage technology that offers cost-effective energy storage solutions. In this paper, a novel methodology is introduced for modeling of the transport mechanisms of electrolyte flow, species and charge in the VRFB at the pore scale of the electrodes; that is, at the level where individual carbon fiber geometry and electrolyte flow are directly resolved. The detailed geometry of the electrode is obtained using X-ray computed tomography (XCT) and calibrated against experimentally determined pore-scale characteristics. The processed XCT data is then used as geometry input for modeling of the electrochemical processes in the VRFB. The flow of electrolyte through the pore space is modeled using the lattice Boltzmann method (LBM) while the finite volume method (FVM) is used to solve the coupled species and charge transport and predict the performance of the VRFB under various conditions. An electrochemical model using the Butler-Volmer equations is used to provide species and charge coupling at the surfaces of the carbon fibers. Results are obtained for the cell potential distribution, as well as local concentration, overpotential and current density profiles undergalvanostatic discharge conditions. The cell performance is investigated as a function of the electrolyteflow rate and external drawing current. The model developed here provides a useful tool for building the structure-property-performance relationship of VRFB electrodes.

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A coupled level set-moment of fluid method for incompressible two-phase flows

Mark Sussman
Florida State University, USA
sussman@math.fsu.edu
M. Jemison, E. Loch, M. Shashkov, M. Arienti, M. Ohta, Y. Wang

A coupled level set and moment of fluid method (CLSMOF)is described for computing solutions to incompressible two-phase flows. The local piecewise linear interface reconstruction (the CLSMOF reconstruction)uses information from the level set function, volume of fluid function, and reference centroid, in order to produce a slope and an intercept for the local reconstruction. The level set function is coupled to the volume-of-fluid function and reference centroid by being maintained as the signed distance to the CLSMOF piecewise linear reconstructed interface. The nonlinear terms in the momentum equations are solved using the sharp interface approach recently developed by Raessi and Pitsch (2009). We have modified the algorithm of Raessi and Pitsch from a staggered grid method to a collocated grid method and we combine their treatment for the nonlinear terms with the variable density, collocated, pressure projection algorithm developed by Kwatra et al (2009). A collocated grid method makes it convenient forusing block structured adaptive mesh refinement (AMR) grids. Many 2D and 3D numerical simulations of bubbles, jets, drops, and waves on a block structured adaptive grid are presented in order to demonstrate the capabilities of our new method.

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Two phase fluid flows with biological microstructures

Xiaoqiang Wang
Florida State University, USA
wwang3@fsu.edu

Phase field method has become a widely used tool to characterize the evolution of surfaces in material science, biology, imaging processing, etc. In this talk the speaker presents how to apply phase field method to the evolution of biological microstructures in fluid. The content includes the study of the shape of lipid vesicles and its transformation and oscillations in fluid systems.

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