Special Session 33: Nonlinear Elliptic and Parabolic Problems in Mathematical Sciences

Yoshihisa Morita, Ryukoku University, Japan
Junping Shi, College of William and Mary, USA

Recent developments of mathematical study for nonlinear PDEs (partial differential equations) provide new ideas and various techniques based on calculus of variations, dynamical systems, asymptotic analysis, qualitative theory etc. In this session we bring together researchers in this research area to present new results for nonlinear parabolic and elliptic equations arising from mathematical science and related problems. Various lectures will be delivered by both senior and junior experts in the field.

Uniqueness of solutions in a gravitational gauge field theory with coexistence of vortices and antivortices

Jann-Long Chern
Central University, Taiwan
chern@math.ncu.edu.tw
Sze-Guang Yang

We consider an elliptic equation arising from the study of static solutions with prescribed zeros and poles of the Einstein equations coupled with the classical sigma model and an Abelian gauge field. In the radially symmetric cases, the solutions can be completely described. We also establish the uniqueness of solutions with the presence of vortices and antivortices.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Reduction of parabolic PDEs

Hayato Chiba
Kyushu university, Japan
chiba@imi.kyushu-u.ac.jp

It is known that the Swift-Hohenberg equation can be reduced to the Ginzburg-Landau equation (amplitude equation) by means of the singular perturbation method. This means that a solution of the latter equation provides an approximate solution of the former one on a certain time interval. In this talk, a reduction of a certain class of (a system of) nonlinear parabolic equations is proposed. An amplitude equation of the system is defined and an error estimate of solutions is given. Further, it is proved under certain assumptions that if the amplitude equation has a stable steady state, then a given equation has a stable periodic solution . In particular, near the periodic solution, the error estimate of solutions holds uniformly in $t > 0$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Evolution and long-time behavior of the free boundary in nonlinear Stefan problems

Yihong Du
University of New England, Australia, Australia
ydu@turing.une.edu.au

We consider the following free boundary problem

$$\begin{cases} u_t - d\Delta u = f(u) & \text{for } \ x \in \Omega(t), t > 0, \\ u = 0 \text{ and } u_t = \mu|\nabla_x u|^2 & \text{for } \ x \in \Gamma(t), t > 0, \\ u(0,x) = u_0(x) & \text{for } \ x \in \Omega_0, \end{cases}$$

where $\Omega(t) \subset \mathbb{R}^n$ $(n \geq 2)$ is bounded by the free boundary $\Gamma(t)$, with $\Omega(0) = \Omega_0$, μ and d are given positive constants. Our assumptions on $f(u)$ include monostable, bistable and combustion type nonlinearities.

We show that the free boundary $\Gamma(t)$ is C^1 outside the convex hull of Ω_0, and as $t \to \infty$, either $\Gamma(t)$ remains bounded and $u(t, .) \to 0$ in the L^∞ norm, or $\Gamma(t)$ goes to infinity in the sense that it is contained in an annulus of the form $\{R(t)-C_0 \leq |x| \leq R(t)\}$, with $R(t) \to \infty$ as $t \to \infty$. Moreover, $R(t)/t \to k_0 > 0$ as $t \to \infty$.

This is joint work with Hiroshi Matano (Tokyo) and Kelei Wang (Sydney).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple positive solutions for p-Laplacian equation with allee effect growth rate

Chan-gyun Kim
College of William and Mary, USA
cgkim75@gmail.com
Junping Shi

A p-Laplacian equation with Allee effect growth rate and Dirichlet boundary condition is considered. The existence, multiplicity and bifurcation of positive solutions are proved with comparison and variational techniques. The existence of multiple positive solutions implies that the related ecological system may exhibit bistable dynamics.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Bifurcation from a degenerate simple eigenvalue

Ping Liu
College of William and Mary, USA
liuping506@gmail.com
Junping Shi, Yuwen Wang

It is proved that a symmetry-breaking bifurcation occurs at a simple eigenvalue despite the usual transversality condition fails, and this bifurcation from a degenerate simple eigenvalue result complements the classical one with the transversality condition. The new result is applied to an imperfect pitchfork bifurcation, in which a forward transcritical bifurcation changes to a backward one. Several applications in ecological and genetics models are shown.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Bifurcation analysis for the Lugiato-Lefever equation in two space dimensions

Tomoyuki Miyaji
Kyoto University, Japan
tmiyaji@kurims.kyoto-u.ac.jp
I. Ohnishi, Y. Tsutsumi

We study the steady-state bifurcation of a spatially homogeneous stationary solution for cubic nonlinear Schrödinger equation (CNLS) with damping, detuning, and driving force. It is a model equation derived by L.A. Lugiato and R. Lefever for describing the evolution of the envelope of electric field in an optical cavity with a Kerr medium. It is known by numerical simulations that the Lugiato-Lefever equation (LL equation) in one or two dimensional space has spatially localized solutions in a certain range of parameters. In contrast to CNLS, LL equation does not satisfy any conservation law of CNLS. A localized structure of LL equation appears as a stationary solution, and lots of localized patterns coexist at the same parameter value. It is also known by simulation that, for 2D LL equation, it undergoes a Hopf bifurcation, and a spatially localized and temporary periodic pattern occurs. We analyze the steady-state mode interactions near the pattern-forming instability. Especially, we focus on the solutions which are periodic with respect to a planar square or hexagonal lattice.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stable patterns and Morse index one solutions

Yasuhito Miyamoto
Tokyo Institute of Technology, Japan
miyayan@sepia.ocn.ne.jp

In this talk we study the shapes of the stable patterns of two problems. One is the stationary problem of a shadow reaction-diffusion system and the other is the minimization problem of a functional with constraint. We will show the following: If a steady state of a shadow system is stable, then the Morse index of the solution is zero or one, and if a critical point of the minimization problem with constraint is a local minimizer, then the Morse index of the solution of the associated Euler-Lagrange equation is zero or one. It is well-known that the solution with Morse index zero is constant provided that the domain is convex. Therefore we are interested in the shape of the Morse index one solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Gradient-like property of a reaction-diffusion system with mass conservation

Yoshihisa Morita
Ryukoku University, Japan
morita@rins.ryukoku.ac.jp

We consider the following two-component system of reaction-diffusion equations

$$u_t = d\Delta u - g(u+v) + v,\ v_t = \Delta v + g(u+v) - v,$$

in a bounded domain Ω with the homogeneous Neumann boundary conditions. We first prove that the system possesses a Lyapunov function if d is less than 1. Then we establish a comparison theorem for the spectrum of the linearized operators around an equilibrium solution of the system and a related scalar elliptic equation with a nonlocal term. This talk is based on the recent joint work with Shuichi Jimbo (Hokkaido University).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Diffusion-induced blowup and bifurcation from infinity of reaction-diffusion systems

Hirokazu Ninomiya
Meiji University, Japan
ninomiya@math.meiji.ac.jp
Chihiro Aida, Chao-Nien Chen

The diffusion process is usually thought as a trivializing one. However, for some reaction-diffusion systems, the blowup of solutions may occur, though the corresponding ODE possesses a globally attractor. This is called diffusion-induced blowup. To study this phenomenon, in this talk, we consider the bifurcation from infinity. In some class of reaction-diffusion systems, this bifurcation takes place by adding the diffusion.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Long time existence of shortening-straightening flow for non-closed planar curves with infinite length

Shinya Okabe
Mathematical Institute, Tohoku University, Japan
okabe@math.tohoku.ac.jp

We consider a steepest descent flow for the modified total squared curvature defined on curves. We call the flow the shortening-straightening flow. First it has been proved by A. Polden (1996) that the flow admits smooth solutions globally defined in time, when the initial curve is smooth, closed, and has finite length. In 2002, G. Dziuk, E. Kuwert, and R. Schätzle extended Polden's result to closed curves with finite length in n-dimensional Euclidean space. We are interested in the following problem: "What is a dynamics of non-closed planar curves with infinite length governed by shortening-straightening flow?" In this talk, we will talk about a long time existence of solution of shortening-straightening flow starting from non-closed planar curve with infinite length. This work is a joint research with M. Novaga (Padova Univ.).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics for an evolution equation describing micro phase separation

Yoshihito Oshita
Okayama University, Japan
oshita@math.okayama-u.ac.jp

We study the free boundary problem describing the micro phase separation of diblock copolymer melts in the regime that one component has small volume fraction such that micro phase separation results in an ensemble of small spheres of one component. On some time scale, the evolution is dominated by coarsening and subsequent stabilization of the radii of the spheres. Starting from the free boundary problem restricted to spheres (particles) we derive the effective equations describing the dynamics in this time regime called mean-field models. We first consider the spatially uniform mean-field models in the dilute case, where the self-interaction of particles is dominated. We identify all the steady states and their stabilities, and show the convergence of solutions. Furthermore we see that the steady states are of the form of the sum of at most two Dirac deltas except for disappeared particles. We then consider the models for the joint distribution of particle radii and centers in the screened case, which is an inhomogeneous extension of the dilute case, where a small migration term is remained. We obtain the form of steady states for this model.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Connection graphs for Sturm attractors of S^1-equivariant parabolic equations

Carlos Rocha
Instituto Superior Tecnico, Portugal
crocha@math.ist.utl.pt
Bernold Fiedler, Mathias Wolfrum

We consider a semilinear parabolic equation of the form $u_t = u_{xx} + f(u, u_x)$ defined on the circle $x \in S^1 = \mathbb{R}/2\pi\mathbb{Z}$. For a dissipative nonlinearity f this equation generates a dissipative semiflow in the appropriate function space, and the corresponding global attractor $\mathcal{A}_f$ is called a Sturm attractor. We use the Sturm permutation σ_f introduced for the characterization of Neumann flows to obtain a purely combinatorial characterization of the Sturm attractors $\mathcal{A}_f$ on the circle. With this Sturm permutation σ_f we show how to construct a connection graph $\mathcal{G}_f$ representing the Sturm attractor $\mathcal{A}_f$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Turing type instabilities in diffusion systems

Kunimochi Sakamoto
Hiroshima University, Japan
kuni@math.sci.hiroshima-u.ac.jp

The purpose of this talk is to display various situations in which Turing type instabilities occur. The essential ingredients are: (1) diffusion and interaction; (2) existence of unstable subsystems in a stable full system. The diffusion and interaction may take place in the interior or on the boundary of domains for interaction-diffusion systems. In particular, we will show that linear diffusion in the interior and nonlinear interaction on the boundary tend to produce temporally oscillating patterns with nontrivial spatial modes.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Wavenumber selection in closed reaction-diffusion systems

Arnd Scheel
University of Minnesota, USA
scheel@umn.edu

Motivated by the plethora of patterns observed in precipitation experiments starting with Liesegang's 1896 study, we investigate pattern formation in the wake of fronts in closed reaction-diffusion systems. We will briefly describe some models and the relation to phase separation models such as the Cahn-Hilliard equation and the Phase-Field System. We will then present results that characterize patterns formed in the wake of fronts.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Time delay induced instabilities and Hopf bifurcations in general reaction-diffusion systems

Junping Shi
College of William and Mary, USA
jxshix@wm.edu
Shanshan Chen, Junping Shi, Junjie Wei

The distribution of the roots of a second order transcendental polynomial is analyzed, and it is used for solving the purely imaginary eigenvalue of a transcendental characteristic equation with two transcendental terms. The results are applied to the stability and associated Hopf bifurcation of a constant equilibrium of a general reaction-diffusion system or ordinary differential equation with delay effects. Examples from chemical reaction and predator-prey models are analyzed using the new techniques. In particular the stability and associate Hopf bifurcation of a Gierer-Meinhardt system with the gene expression time delays is analyzed by using our method.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Spatially inhomogeneous time-periodic solutions in delayed Nicholson's blowflies model

Ying Su
University of Western Ontario, Canada
ysu62@uwo.ca
Xingfu Zou

In this talk, we focus on the spatially inhomogeneous and time periodic solutions of a delayed diffusive Nicholson's blowflies model subject to the no flux boundary condition. In particular, we investigate the existence and stability of the spatially inhomogeneous periodic solutions as well as their dependence on some model parameters, especially the diffusive rate of the mature population.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability of patterns in some reaction-diffusion systems with the diffusion-driven instability

Kanako Suzuki
College of Science, Ibaraki University, Japan
kasuzu@mx.ibaraki.ac.jp

We consider the following system of a single reaction-diffusion equation coupled with an ordinary differential equation:

$$u_t = f(u,v), \quad v_t = D\Delta v + g(u,v).$$

Here, $u = u(x,t)$, $v = (x,t)$ for $x \in \Omega$, $t > 0$, and Ω is a bounded domain with smooth boundary. We impose the Neumann boundary condition on v. This type of models exhibits the diffusion-driven instability. However, we show that, under some assumptions on non-linearities, all regular stationary solutions are unstable. This talk is devoted to understanding its mechanism. This work is a joint work with Anna Marciniak-Czochra (University of Heidelberg) and Grzegorz Karch (University of Wroclaw).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Isolated singularities of nonlinear polyharmonic inequalities

Steven Taliaferro
Texas A&M University, USA
stalia@math.tamu.edu

We discuss how the blow-up behavior at an isolated singularity of solutions of nonlinear polyharmonic inequalities depends on an exponent in the inequalities. In particular, we show that for certain exponents there exists an a priori bound on the rate of blow-up and for other exponents the solutions can blow up arbitrarily fast. Remarkably, the optimal bound for solutions may itself not be a solution.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Diffusion driven instabilities on evolving surfaces

Necibe Tuncer
University of Tulsa, USA
necibe-tuncer@utulsa.edu
Anotida Madzwamuse, A.J. Meir

Reaction diffusion systems defined on evolving surfaces has many application in mathematical biology. Examples of such applications include tumor growth, pattern formation on seashells, butterfly wing pigmentation patterns and animal coat markings. We develop and analyze a finite element method to approximate solutions of nonlinear reaction diffusion systems defined on evolving surfaces. The method we propose is based on radially projected finite elements.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On global bifurcation of bifurcation curves of some multiparameter problems

Shin-Hwa Wang
National Tsing Hua University, Taiwan
shwang@math.nthu.edu.tw
Kuo-Chih Hung

We study the global bifurcation of bifurcation curves of positive solutions for some nonlinear multiparameter problems of the form

$\longrightarrow \infty \diamond \infty \longleftarrow$

Qualitative analysis of a diffusive predator-prey model with modified Leslie-Gower and Holling-type II schemes

Jun Zhou
College of William and Mary, USA
jzhouwm@gmail.com
Junping Shi

We investigate the existence, multiplicity and stability of positive solutions to a prey-predator model with modified Leslie-Gower and Holling-Type II schemes

$$\begin{cases} -\Delta u = u\left(a_1 - bu - \dfrac{c_1 v}{u+k_1}\right) & \text{in } \Omega, \\ -\Delta v = v\left(a_2 - \dfrac{c_2 v}{u+k_2}\right) & \text{in } \Omega, \\ u \geq 0,\ v \geq 0 & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a bounded domain with a smooth boundary $\partial\Omega$, the parameters a_i, b, c_i, k_i $(i = 1, 2)$ are positive numbers, u and v are the respective populations of prey and predator. Here, we say (u, v) with $u|_{\partial\Omega} = v|_{\partial\Omega} = 0$ is a positive solution if (u, v) is a solution and $u, v > 0$ in Ω.

$\longrightarrow \infty \diamond \infty \longleftarrow$