Special Session 30: Recent Developments on Turbulence

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Turbulence is often referred to as the last open problem of classical mechanics. It is a problem of both fundamental importance, posing unique mathematical challenges, and with a wide range of applications, in earth and atmospheric science, plasma physics, aerospace engineering, and many other areas. Despite nearly a century of efforts by the best minds, many questions remain open, and considerable progress has only been achieved for some idealized situations, such as homogeneous and isotropic turbulence. The goal of the session is to bring together specialists from various areas of turbulence research.

Non-linear cascades in rotating stratified Boussinesq flows

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We use high-resolution simulations of Boussinesq flows, forced in the large-scales, with fixed rotation and stable stratification along the vertical axis, to study the downscale cascades of energy and potential enstrophy in three different regimes of stratification and rotation. In addition, we analyze the spatial distribution of the cascades using a coarse-graining approach which allows simultaneous resolution of the dynamics in scale and in space.(1) For strongly stratified flows with moderate rotation, we observe anisotropic fluxes of energy and potential enstrophy into Fourier modes with large vertical component k_z, predominantly due to a highly non-local transfer from the large-scales directly to the smallest scales. The energy cascade is predominantly due to three vortical-mode interactions.(2) For strongly rotating flow with moderate stratification, there are anisotropic fluxes to modes with large k_h, due to a "diffusely" local transfer much like in isotropic Navier-Stokes turbulence. The energy cascade is primarily due to three vortical-mode interactions, as in the strongly stratified case, although wave-vortical-wave and vortical-wave-vortical interactions also make a noticeable contribution.(3) In the third case of equally strong rotation and stratification, there are only slightly anisotropic fluxes, mostly to modes with large k_h, due to an ultra-local transfer in which the energy gained by an inertial scale comes almost exclusively from the adjacent larger scales. We confirm that the cascades in the third regime are primarily due to wave-vortical-wave interactions, in agreement with previous work.

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Maximal spatial analyticity radius for the Navier-Stokes equations

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The regular solutions of the Navier-Stokes equations (NSE) are well-known to be analytic in both space and time variables. Moreover, the space analyticity radius has an important physical interpretation. It demarcates the length scale below which the viscous effect dominates the (nonlinear) inertial effect. Foias and Temam introduced an effective approach to estimate space analyticity radius via the use of Gevrey norms which, since then, has becomea standard tool for studying analyticity properties for dissipative equations. Using this approach, we study the maximal space analyticity radius of solutions of the 2D and 3D NSE as a function of time. Our main tool is an ODE associated to the NSE which can be solved on a maximal domain. The upper boundary of this domain is the maximal analyticity radius of the solution to the NSE. The main objective is to study the connection between (topological) features of this domain and physical phenomena such as intermittency, energy and estrophy cascades. This is a jointwork with Professors C. Foias and M. Jolly.

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Analytical approach to intermittency in turbulence

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We introduce in precise mathematical terms some of the empirical concepts used to describe intermittency in fully developed turbulence. We give definitions of the active turbulent region, volume, eddies, energy dissipation set, and derive rigorously some power laws of turbulence. In particular, the formula for the Hausdorff dimension of the energy dissipation set will be justified, and upper/lower bounds on the energy spectrum will be obtained.

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Geodesic detection of coherent vortices in two-dimensional turbulence

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We study the evolution of vortices in two-dimensional turbulence through the recently developed geodesic theory of transport barriers [Haller & Beron-Vera (2012)]. First, we review the main results of the theory and its numerical implementation. We then use the theory to locate hyperbolic and elliptic Lagrangian Coherent Structures (LCSs) in two-dimensional turbulence. The hyperbolic LCSs correspond to generalized stable and unstable manifolds while the elliptic LCSs correspond to the cores of Lagrangian vortices. In addition, we use these structures to explore the Lagrangian signature of vortex merger.

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Energy and potential enstrophy flux constraints in the two-layer quasi-geostrophic model

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We investigate an inequality constraining the energy and potential enstrophy flux in the two-layer quasi-geostrophic model. This flux inequality is unconditionally satisfied for the case of two-dimensional Navier-stokes turbulence. However, it is not obvious that it remains valid under the multi-layer quasi-geostrophic model. We derive the general form of the energy and potential enstrophy dissipation rate spectra for a generalized multi-layer model. We then specialize these results for the case of the two-layer quasi-geostrophic model under dissipation configurations in which the dissipation terms for each layer are dependent only on the streamfunction or potential vorticity of that layer. We derive sufficient conditions for satisfying the flux inequality and discuss the possibility of violating it under different conditions.

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Bounds on energy, enstrophy for the 2D NSE with single mode forcing

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We examine how the global attractor of the 2-D periodic Navier-Stokes equations projects in the energy-enstrophy-plane when the force is an eigenvector of the Stokes operator. We prove the existence semi-integral curves, which form the upper boundary. Along the way we find regions of the plane in which the energy and enstrophy must decrease, and calculate the curvature of the projected solution at certain initial data. Finally, we obtain restrictions on solutions that project onto a single point in the plane.

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Recent numerical results for the 3D MHD-Voigt model and related models

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Recently, the Voigt-regularization, which is related to the alpha-models of turbulent flows, has been investigated as a regularization of various fluid models. It overcomes many of the problems present in other alpha-models; for example, it is well-posed in bounded domains, and its global bounds in the relevant spaces are independent of viscosity. Moreover, in studying the limit as the regularization parameter tends to zero, a new criterion for the finite-time blow-up of the original equations arises. I will discuss recent analytical and numerical work on the Voigt-regularization in the context of the 3D MHD equations and, given time, the 2D Boussinesq equations.

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Optimal stirring strategies with fixed energy, fixed power or fixed palenstrophy constraint

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We consider passive scalar mixing by a prescribed divergence-free velocity vector field in a periodic box and address the following question: Starting from a given initial inhomogeneous distribution of passive tracers, and given a certain energy budget, power budget or finite palenstrophy budget, what incompressible flow field best mixes the scalar quantity? We adopt the optimal stirring strategy recently proposed by Lin, Thiffeault and Doering which subsequently determine the flow field that instantaneously optimizes the depletion of the H^{-1} mix-norm. In this work we bridge some of the gap in the best available *a priori* analysis and the simulation results. We recall some previously derived rigorous analysis and then present a new explicit analytical example establishing finite-time perfect mixing with finite energy constraint. On the other hand, using techniques pioneered by Yudovich in proving uniqueness of solutions to the 2-d Euler equations, we establish that

if the flow is constrained to have constant palenstrophy $\mathcal{P} := \|\Delta u\|_{L^2}^2$, then the H^{-1} mix-norm decays at most $\sim e^{-c\mathcal{P}t^2}$. That is, finite-time perfect mixing is certainly ruled out when too much cost is incurred by small scale structures in the stirring. As observed from direct numerical simulations, we discuss the impossibility of finite-time perfect mixing for flows with fixed power constraint and conjecture an exponential lower bound on the H^{-1} mix-norm. Interestingly, we discuss results about some related problems from other areas of analysis which are quantitatively equivalent or similarly suggestive of an exponential lower bound for the H^{-1} mix-norm.

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Dissipative length scales of the Navier-Stokes equations

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The radius of analyticity of the Navier-Stokes equations indicates a length scale below which viscous effects dominate the inertial ones, and in the context of turbulence can be couched in terms of the so-called Kolmogorov length-scale, the unique length scale determined by the viscosity and energy dissipation rate alone. This talk will address a semigroup method initiated by [Biswas-Swanson '07] for obtaining a lower bound on this radius in terms of the Gevrey norm of the initial data. While this approach does not improve the best known estimate obtained by [Kukavica '98] it does recover and generalize the estimate made by [Doering-Titi '95], as well as identify a Kolomogorov-type length scale based on a quantity formally analogous to the energy dissipation rate.

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A new instability that breaks the spatial homogeneity symmetry in wave turbulence

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Spatial homogeneity, the symmetry property that all statistical moments are functions only of the relative geometry of any configuration of points, can be spontaneously broken by the instability of the finite flux Kolmogorov-Zakharov spectrum in certain (usually one dimensional) situations. As a result, the nature of the statistical attractor changes dramatically, from a sea of resonantly interacting dispersive waves to an ensemble of coherent radiating pulses.

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Characterizing the layer thickness in unit- and small-aspect-ratio rotating Boussinesq turbulence

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High-resolution simulations are used to quantify how small domain aspect ratio conspires with strong stratification to set the dynamical vertical length scale of 'pancake' structures in strongly stratified Boussinesq flow. All simulations are in an asymptotic parameter regime defined by quadratic potential enstrophy. The relevant parameters of the problem are the buoyancy frequency N, the Coriolis parameter f, the domain height H, the domain length and width L, and the Burger number NH/(fL). There are two sets of calculations with (i) Burger number fixed at unity and decreasing domain aspect ratio H/L = 1, 1/4, 1/8 and 1/16, and (ii) increasing N/f = 4, 8, 16 and aspect ratio H/L fixed at unity. The first set of calculations is relevant for mid-latitude atmosphere/ocean dynamics with Burger number close to unity.

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Phase transitions in optical turbulence

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We consider turbulence in the Gross-Pitaevsky model and study the creation of a coherent condensate via an inverse cascade originated at small scales. The growth of the condensate leads to a spontaneous breakdown of symmetries of small-scale overcondensate fluctuations: first, statistical isotropy is broken, then series of phase transitions mark the change of symmetry from the two-fold to three-fold to four-fold. At the highest condensate level reached, we observe a short-range positional and long-range orientational order (similar to a hexatic phase in the condensed matter physics). In other words, the longer one pumps the system the more ordered it becomes. We show that these phase transitions happen when the driving term corresponds to an instability but not when the system is pumped by a random force. Thus we demonstrate non-universality of the inverse-cascade turbulence.

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Fractal contours of scalar in smooth flows

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A passive scalar field was studied under the action of pumping, diffusion and advection by a smooth flow with a Lagrangian chaos. We present theoretical arguments showing that scalar statistics is not conformal invariant and formulate a new effective semi-analytic algorithm to model scalar turbulence. We then carry out massive numerics of scalar turbulence focusing on nodal lines. The distribution of contours over sizes and perimeters is shown to depend neither on the flow realization nor on the resolution (diffusion) scale for scales exceeding this scale. The scalar isolines are found fractal/smooth at the scales larger/smaller than the pumping scale. We characterize the statistics of bending of a long isoline by the driving function of the Loewner map, show that it behaves like diffusion with diffusivity independent of resolution yet, most surprisingly, dependent on the velocity realization and time (beyond the time on which the statistics of the scalar is stabilized).

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