Special Session 3: Mathematics of Social Systems

Andrea Bertozzi, UCLA, USA

This session concerns mathematical modeling of social systems - including problems in biology involving social interaction and problems in social science modeling. Is it the intention of the organizer to form a broad based session that will be useful for young mathematical scientists interested in broadening their research. The session will include work on agent based models, methods in linear and nonlinear partial differential equations, variational methods for inverse problems, and statistical and stochastic models. Examples of applications range from crime modeling including gang interactions and patterns in property crime to models for social interactions of swarms in biology.

Stationary states for the aggregation equation with power law attractive-repulsive potentials

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We analyze the stability of a uniform distribution on a sphere as stationary solution ofthe aggregation equation with power law attractive-repulsive potentials. We give a sharp condition that establishes its stability under radial perturbations.

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Modeling social dynamics

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In recent years, agent-based models have become very popular for modeling social dynamics. This technique has been used to model organisms as diverse as fish, insects, birds, and even people. Studying these models at a kinetic level opens new mathematical perspectives into the dynamics of such systems, raising new and interesting mathematical questions. Here, we will present some agent-based models for social systems, and examine the PDEs arising from these models.

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An adversarial evolutionary game for criminal behavior

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Traditional models of human cooperation are usually cast in the form of a prisoner's dilemma, where although cooperation may be beneficial, players may choose to "defect" and pursue selfish goals. In this talk we consider an adversarial evolutionary game developed for criminal activity where players not only choose whether or not to cooperate for the common good but also whether or not to actively harm others by committing crimes. This new choice gives rise to four possible strategies among which that of the so called "informant", a player who cooperates while still committing crimes. We find two possible equilibration regimes, a defection-dominated and an ideal, cooperation-dominated one and show that the number of informants is crucial in determining which of these two regimes is achieved. Since large numbers of informants lead to the ideal cooperative society we also study their active recruitment from the overall society, by considering differential recruitment costs and benefits, via an optimal control problem where finite resources are included. We discuss our results in the context of extreme adversarial societies, such as those marred by wars, insurgencies and organized crime.

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A mathematical model for flight guidance in honeybee swarms

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When a colony of honeybees relocates to a new nest site, less than 5% of the bees (the scout bees) know the location of the new nest. Nevertheless, the small minority of informed bees manages to provide guidance to the rest and the entire swarm is able to fly to the new nest intact. The streaker bee hypothesis, one of the several theories proposed to explain the guidance mechanism in bee swarms, seems to be supported by recent experimental observations. Originally proposed by Lindauer in 1955, the theory suggests that the informed bees make high-speed flights through the swarm in the direction of the new nest, hence conspicuously pointing to the desired direction of travel. Once they reach the front of the swarm, they return at low speeds to the back, by flying along the edges of the swarm, where they are less visible to the rest of the bees. This work presents a mathematical model of flight guidance in bee swarms based on the streaker bee hypothesis. Numerical experiments, parameter studies and comparison with experimental data will be presented.

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Swarming on random graphs

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We study a standard aggregation model where instead of the usual assumption of all-to-all interaction, we assume that two given particles interact with probability p. We are particularly interested in the following question: How does the connectivity of the underlying graph affect the confinement property of the swarm? Under certain generic assumptions on the interaction kernel, we show that the confinement preserved up to p=O((log n)/n), where n is the number of particles. In other words, very few edges are needed to preserve the confinement. The results rely on basic probability arguments and eigenvalues of random matrices.

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Analysis of a new PDE based model for ant foraging

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The dynamics of ant foraging is of interest in understanding how local interactions lead to global emergent behaviors in complex systems. We present a new mathematical model, inspired by the Keller Segel model of bacterial chemotaxis, for ant foraging that accounts for different behaviors exhibited by foragers in search of food and food carrying ants. The model distinguishes between the dynamics of food searching (foraging) ants and food carrying ants. Numerical simulations based on the model show trail formation in foraging ant colonies to be an emergent phenomenon and also account for adaptive behavior observed in recent experiments. The model has interesting characteristics related to the absence of finite time singularities that emerge in the Keller-Segel model of chemotaxis. We also discuss preliminary steady state stability results.

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Aggregation via Newtonian potential and aggregation patches

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Bertozzi, Leger

We consider the motion of a density of particles rho(x,t) by a velocity field v(x,t) obtained by convolving the density of particles with the gradient of the Newtonian potential, that is v = - grad N* rho. An important class of solutions are the ones where the particles are uniformly distributed on a time evolving domain. We refer to these solutions as aggregation patches, by analogy to the vortex patch solutions of the 2D incompressible Euler equations. Numerical simulations as well as some exact solutions show that the time evolving domain on which the patch is supported typically collapses on a complex skeleton of codimension one. We also show that going backward in time, any bounded compactly supported solution converges as t goes to minus infinity toward a spreading circular patch. We provide a rate of convergence which is sharp in 2D. This is a joint work with Bertozzi and Leger.

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Rethinking network analysis: topology, dynamics and network structure

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Many popular metrics used in social network analysis are based on the random walk. However, the random walk may not be appropriate for modeling and analyzing many social phenomena, including epidemics and information diffusion, in which one node may interact with many others at the same time, for example, by broadcasting the virus or information to its neighbors. To produce meaningful results, social network analysis algorithms have to take into account the nature of interactions between the nodes. We classify interactions as conservative (informally, one-to-one) and non-conservative (one-to-many) and show that while the former describes random walk dynamics, the latter is mathematically equivalent to epidemic dynamics. We then relate these to well-known metrics used in network analysis: PageRank and Alpha-Centrality. We demonstrate by ranking nodes in an online social network used for broadcasting information, that non-conservative Alpha-Centrality leads to a better agreement with an empirical ranking scheme than the conservative PageRank.Our work unifies two areas of network analysis — centrality and epidemic models — and leads to insights into the relationship between dynamic processes and network structure, specifically, the existence of an epidemic threshold for non-conservative processes.

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Moderation, as an escape from a persistent cycle of ideological revolutions

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In many interconnected social communities—for example a well unified political party, a network of credit-lending institutions, or a nationally bound community of academic linguists—we often observe a sort of punctuated equilibrium of ideas: The community as a whole focuses on a certain topic or methodological approach for a while, then rapidly converts to a new, seemingly more reasonable alternative under sustained pressure from a small group of committed adherents. In many cases, the new approach has as many shortcomings as the old, but this is not fully appreciated until it has been widely adopted. (Once it is appreciated however, the community is ripe for a new shift of thought, and the cycle continues.) As a way out of these swings between ideological extremes, we might seek to encourage a more balanced or qualified approach. Here, however, we show that (in the context of a simplified model) doing so successfully is surprisingly difficult—perhaps explaining the persistence of the dramatic ideological shifts in the first place. In particular, only one out of the seven strategies that we analyze succeeds in expanding the subcommunity of moderates without risking its extinction.

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Paladins as predators: invasive waves in a spatial evolutionary adversarial game

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Invasive waves are found in a novel variant of a reaction-diffusion system used to extend an evolutionary adversarial game into space wherein the influence of various strategies is allowed to diffuse. The waves are driven by a nonlinear instability that enables an otherwise unstable informant state to travel through an initially uncooperative state leaving a cooperative state behind. The wave speed's dependence on the various diffusion parameters is examined in one and two dimensions. Various other phenomena, such as pinning near a diffusive inhomogeneity, are also explored.

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Filtering and estimation of self-exciting Cox processes with applications to social systems

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Self-exciting point processes have shown promise for modeling social event patterns where the occurrence of an event increases the likelihood of subsequent events. However, standard models assume a Poisson background rate for spontaneous events, an unrealistic assumption in many social systems. We introduce a self-exciting Cox process model where the background rate is driven by an Ornstein-Uhlenbeck stochastic differential equation. We then develop a methodology for simultaneous filtering and estimation of the intensity. Application to crime and security data sets are investigated.

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Urban social dynamics: "Don't buy the house, buy the neighborhood"

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"Don't buy the house, buy the neighborhood" (Russian proverb). This talk is concerned with issues related to urban social segregation. Going beyond the simplest models introduced by T. C. Schelling in the 70's, we introduce a model of housing transactions between agents heterogeneous in their income, hence in their willingness to pay. The goal is to see how the spatial income segregation depends on both economic constraints and social interactions. A key feature of the model is that agents preferences for a place are expressed in terms of a local attractiveness, which depends on both an intrinsic attractiveness of the location and the social characteristics of its neighborhood. The demand for an asset thus depends on its attractiveness, which in turn evolves according to the social characteristics of the newcomers. The resulting stationary space distribution of income is analytically characterized and illustrated by agent-based simulations. The main results are that, (1), socio-spatial segregation occurs if, and only if, the social influence is strong enough, and, (2), even so, some social diversity is preserved at most locations. Comparing with the Parisian housing market, the results reproduce general trends concerning the price distribution and the income spatial segregation.

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Multi-Stage complex contagions

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The spread of ideas across a social network can be studied using complex contagion models, in which agents are activated by contact with multiple activated neighbors. The investigation of complex contagions can provide crucial insights into social influence and behavior-adoption cascades on networks. In this paper, we introduce a model of a multi-stage complex contagion on networks. Agents at different stages–which could, for example, represent differing levels of support for a social movement or differing levels of commitment to a certain product or idea–exert different amounts of influence on their neighbors. We demonstrate that the presence of even one additional stage introduces novel dynamical behavior, including interplay between multiple cascades, that cannot occur in single-stage contagion models. We find, for example, that cascades–and hence collective action–can be driven not only by higher-stage influencers but also by lower-stage influencers.

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Hotspot invasion: traveling wave solutions to a reaction-diffusion model for criminal behavior

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We study a class of reaction-diffusion models that can be taken as basic models for the dynamics of criminal activity. In this talk I will first discuss the existence of steady-state solutions and discuss the condition that determine whether there there are one, two, or three steady-states. The latter case corresponds to a bi-stable system and in this case we prove the existence of traveling wave solutions in one dimension. Physically, this correspond to the invasion of 'hotspots' into areas that have naturally low crime rates. I will also discuss some numerical results on obstruction of wave propagation.

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Social interactions on networks: self-excitation, third-party inhibition, and the link with game theory

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George Mohler, Jeff Brantingham, George Tita

We introduce a point process model for social interactions on a network, including self-excitation and third-party inhibition. Here, a coupled system of state-dependent jump stochastic differential equations is used to model the conditional intensities of the directed network of interactions. The model produces a wide variety of transient or stationary weighted network configurations and we investigate under what conditions each type of network forms in the continuum limit. We also explore the link between this model and recent work on repeated games.

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Locust dynamics: behavioral phase change, swarming, and nonlocal models

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Locusts exhibit two interconvertible phases, solitary and gregarious. Solitary (gregarious) individuals are repelled from (attracted to) others, and crowding biases conversion towards the gregarious form. We construct a nonlinear partial integrodifferential equation model of the interplay between phase change and spatial dynamics leading to the formation of locust swarms. Analysis of our model reveals conditions for the onset of a locust plague, characterized by a large scale transition to the gregarious phase. A model reduction to ordinary differential equations describing the bulk dynamics of the two phases enables quantification of the proportion of the population that will gregarize, and of the time scale for this to occur. Numerical simulations provide descriptions of the swarm structure and reveal transiently traveling clumps of gregarious insects.

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Pattern formation under nonlocal social interaction

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Pairwise particle interactions arise in diverse biological systems ranging from insect swarms and flocking, to self-assembly of nanoparticles. Particle systems which communicate through kernels with long-range attraction and short-range repulsion may exhibit rich patterns in their bound states. In this talk we present a theory to classify the morphology of various patterns in N dimensions from a given social interaction force. We also present a method to solve the inverse problem: Given an observed pattern, can we construct a social interaction potential which exhibits that pattern.

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Community detection among street gangs and gamma-convergence on graphs

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The goal of data clustering and community detection is to cluster nodes of a weighted graph into groups such that the edge weights between vertices in the same group is high and the outgroup weights are low. Linear methods based on graph Laplacians, such as spectral clustering, are popular tools to accomplish this goal. In this talk we investigate both linear methods and nonlinear methods based on the Ginzburg-Landau functional to detect social structures among street gangs. We use Gamma-convergence to connect the nonlinear method with the nonlocal means method (well known in image processing) as well as with classical continuum Ginzburg-Landau results.

This work has been done in collaboration with Andrea Bertozzi, Jeff Brantingham, Blake Hunter and others.

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