Special Session 27: Transport Barriers in Dynamical Systems

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Transport barriers are ubiquitous generators of coherent patterns in complex dynamical systems. In recent years, significant advances have been made on the mathematics and computations of such barriers in temporally aperiodic flows. This special session surveys recent analytic and computational techniques for the identification of generalized transport barriers in dynamical systems, and reviews applications to geophysical and engineering flows.

Transport barrier detection via braid theory

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The definition of transport barriers as minimally stretching material lines has recently been established. A detection method was developed for regions where the full velocity field is known; however this is not always the case. Our work attempts to detect similar boundaries in situation where only sparse trajectory information is known. We do this through the use of tools from topology, in particular braid theory. We will briefly review our detection method and a proof of concept in a mixing example. Additionally, we will analyze trajectories in the double-gyre system to try to establish the relationship between transport barriers detected using the geodesic method and our braid based method.

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Explicit stable and unstable manifolds in a class of unsteady 2D and 3D flows

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Stable and unstable manifolds strongly govern fluid transport in unsteady flows, yet are often difficult to determine even numerically. This talk introduces a class of unsteady Eulerian velocity fields in both two- and three-dimensions, in which the time-varying invariant manifolds can be expressed explicitly. It is shown that exponential dichotomy conditions – which ensure that fluid particles on these manifolds decay exponentially to hyperbolic trajectories – are satisfied for these flows. Methods for generating finite-time analogues of these manifolds are also discussed; in particular, it is shown how the violation of "infinite-timeness" can be quantified and tuned using a parameter. While there is in reality no chaotic motion associated with this class of examples, Lagrangian trajectories which are seemingly arbitrarily complicated can be generated. As such, these will serve as useful testbeds for researchers developing and improving diagnostic methods for extracting flow barriers in genuinely time-dependent two- and three-dimensional flows.

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Uncovering the Lagrangian skeletons of oceanic and atmospheric flows

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Haller and Beron-Vera (2012) have recently developed a geodesic theory for the objective (i.e., frame-independent) identification of key material curves (transport barriers) which shape global transport patterns in unsteady 2-d flows defined over a finite-time interval. In the geodesic theory transport barriers emerge as material curves closely shadowed by least-stretching geodesics for the Cauchy–Green tensor. In the incompressible case, the geodesic theory identifies hyperbolic (generalized stable and unstable manifolds), elliptic (generalized eddy boundaries), and parabolic (generalized shear jets) transport barriers. Such transport barriers, which can be regarded as generalized Lagrangian Coherent Structures or LCSs, compose the Lagrangian skeleton of an incompressible 2-d flow. Here we apply the geodesic theory to oceanic and atmospheric velocity fields in an attempt to uncover their Lagrangian skeletons.

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Lagrangian coherent structures, biological invasions, and limits of forecasting

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The language of Lagrangian coherent structures (LCSs) provides a new means for discussion of transport and mixing of pathogens that are transported through the atmosphere, paving the way for new modeling and management strategies for the spread of infectious diseases affecting plants, domestic animals, and humans. Our research group is investigating the influence of atmospheric dynamical structures on the aeroecology of pathogen populations collected with autonomous unmanned aerial vehicles (UAVs). Recent work analyzing collections of pathogens across multiple UAV sampling missions has demonstrated a statistical correlation between punctuated changes in the population structure and passages of LCSs over the sampling location, based on data from archived meteorological models which constitute our best estimate of the atmospheric flow. If LCSs prove to be important for understanding the atmospheric transport of invasive microbial species into previously unexposed regions, accurate forecasting of LCSs could be incorporated into early warning systems for plant pathogens. We will consider the consequences of wind forecast errors on the finite-time Lyapunov exponent (FTLE) field and the results from proper orthogonal decomposition for comparing forecast and pastcast LCS features. Finally, we describe stochastic FTLE fields which take into account the uncertainty of forecasts.

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KAM-like Lagrangian coherent structures in geophysical flows

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Transport barriers in the vicinity of zonal (east-west) jets in planetary atmospheres are investigated. Recent results relating to KAM (Kolmogorov-Arnold-Moser) theory predict that when such a flow is perturbed, invariant tori in the vicinity of the jet core should be present and will serve as impenetrable barriers to meridional (north-south) transport. Numerical estimates of invariant tori of this type are referred to as KAM-like Lagrangian Coherent Structures (LCSs). Relevant theory and numerical methods will be reviewed, and applications, including the annually recurring Antarctic ozone hole and Jupiter's belts and zones, will be discussed.

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Escape and diffusion through small holes

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A dynamical system may be "opened" by allowing trajectories to leak out through one or more holes, subsets of phase space. Given a distribution of initial conditions, We can then pose questions about the probability of surviving within the system, as a function of time, the size and position of the hole(s). Open billiard dynamics can be related to a number of physical experiments and applications involving escape of particles from a cavity. In several geometries the leading coefficient of the survival probability can be determined, including connections with the Riemann Hypothesis and phenomena such as asymmetric transport. A chain of systems linked by their holes can also model deterministic diffusion. Very recent results for escape and diffusion in one-dimensional maps will be discussed, including an expansion for the escape rate beyond linear order in hole size, and an exact additivity formula for diffusion coefficients.

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Transport in time-dependent dynamical systems: finite-time coherent sets

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We study the transport properties of nonautonomous chaotic dynamical systems over a finite time interval. We are particularly interested in those regions that remain coherent and relatively nondispersive, despite the chaotic nature of the system. We detect maximally coherent transport pathways using singular vectors of a matrix of transitions induced by the dynamics. The methodology is illustrated on an idealized stratospheric flow and two and three-dimensional analyses of European Centre for Medium Range Weather Forecasting reanalysis data.

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Using LCS to study the transition vortex shedding on a cylinder in cross-flow

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The transition from steady separation to unsteady vortex shedding downstream of a circular cylinder in cross-flow is examined using a Lagrangian coherent structure analysis. Velocity data is gathered from both 2D and 3D simulations at Reynolds numbers close to transition ($Re = 47$). The LCS is compared against common Eulerian criteria, such as vorticity, the Q criterion, and the acceleration minima. At transition, when flow begins to entrain into and detrain from the cylinder wake, the wake as described using LCS undergoes a distinct qualitative change. This event in the evolution of the LCS will offer new information about possible timing and location at which to implement effective flow control to mitigate the shedding and unsteady forces on the cylinder body.

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Geodesic theory of transport barriers

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I describe a unified approach to locating key material transport barriers in unsteady flows induced by two-dimensional, non-autonomous dynamical systems. Seeking transport barriers as minimally stretching material lines, one obtains that such barriers must be shadowed by minimal geodesics under the metric induced by the Cauchy-Green strain tensor field associated with the flow map. As a result, snapshots of transport barriers can be explicitly computed as trajectories of ordinary differential equations. Using this approach, hyperbolic barriers (generalized stable and unstable manifolds), elliptic barriers (generalized KAM curves) and parabolic barriers (generalized shear jets) can be found with high precision in temporally aperiodic flows defined over a finite time interval. I illustrate these results on unsteady flows arising in mechanics and fluid dynamics. Further applications to geophysical flows will be discussed by Francisco Beron-Vera in another talk within this special session.

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Out of flatland: 3D aspects of Lagrangian coherent structures in oceanography

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Since the seminal work of Haller (Chaos, 99-108, 2000) and Haller and Yuan (Physica D, 352-370, 2000) finite time Lyapunov exponents (FTLE) have been used to identify Lagrangian coherent structures (LCS). Until very recently applications of this methodology in oceanography have been restricted to analyses along a few selected surfaces even though the theory is readily extended to fully three-dimensional flows. In rotating stratified geophysical scale fluids LCS are 2D surfaces embedded within a finite fluid volume so the intersections of these surfaces with a few level surfaces clearly are inadequate descriptors of true LCS. The only study that reports on the 2D structure of LCS in oceanography that we are aware of is Branicki and Kirwan (Int. J. Engr. Sci., 1027-1042, 2010). In their investigation of a large anticyclonic ring in the Gulf of Mexico they reported that the 2D LCS were nearly vertical and that there was entrainment into the ring near its base and detrainment near the surface. However, for technical reasons peculiar to most data assimilating ocean and atmospheric general circulation models they elected not to use the vertical velocity in their analysis. Thus the question addressed in this talk: how representative of true 2D LCS are reduced FTLE representations? Specifically we discuss two strategies for computing 2D LCS from reduced representations of the Cauchy Green tensors. Our analysis indicates that under typical oceanographic conditions it is important to account for the vertical shear of the horizontal currents in constructing a reduced representation of this tensor. We apply this approach to a high-resolution data assimilating general circulation model to report on the vertical structure of 2D LCS in the Gulf of Mexico.

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Transport in transitory dynamical systems

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A transitory dynamical system is nonautonomous only on a compact interval of time. With this particularly simple form, coherent structures can be identified with those of the "past" or "future" autonomous vector fields, and transport is quantified by a transition map. We study Liouville dynamics in which the incompressible vector field has a Lagrangian generating form. In this case the volume of regions bounded by past-unstable and future-stable surfaces can be efficiently computed using an action-like formula on heteroclinic orbits. Three-dimensional examples using the ABC flow and a model of droplet flow in a twisted pipe will be presented.

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Predicting instabilities in environmental pollution patterns using LCS-core analysis

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I present a methodology to predict major short-term changes in environmental contamination patterns, such as oil spills in the ocean and ash clouds in the atmosphere. The approach is based on new mathematical results on the objective (frame-independent) identification of key material surfaces that drive tracer mixing in unsteady, finite-time flow data. Some of these material surfaces, broadly known as Lagrangian Coherent Structures (LCSs), turn out to admit highly attracting cores that lead to inevitable material instabilities even under future uncertainties or unexpected perturbations to the observed flow. These LCS cores have the potential to forecast imminent shape changes in the contamination pattern, even before the instability builds up and brings large masses of water or air into motion. Exploiting this potential, the resulting LCS-core analysis provides a model-independent forecasting scheme that relies only on already observed or validated flow velocities at the time the prediction is made. The methodology is used to obtain high-precision forecasts of two major instabilities that occurred in the shape of the Deepwater Horizon oil spill in the Gulf of Mexico.

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Finite-time entropy: a probabilistic approach for measuring nonlinear stretching in dynamical systems

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We propose an entropy-based methodology for estimating nonlinear stretching in dynamical systems based on the evolution of probability densities. We present a very efficient computational approach that makes direct use of a discretized transfer operator. The novel methodoloy is illustrated by several example systems, highlighting the similarity to the frequently used concept of finite-time Lyapunov exponents. Thus the finite-time entropy approach is a first step towards formally linking geometric and probabilistic methods for the numerical analysis of transport.

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Inertial manifold dimensionality and finite-time instabilities in transient turbulent flows

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We examine the geometry of the inertial manifold associated with fluid flows described by Navier-Stokes equations and we relate its nonlinear dimensionality to energy exchanges between the mean flow and stochastic modes of the flow. Specifically, we employ a stochastic framework based on the dynamically orthogonal field equations to perform efficient order-reduction in terms of time-dependent modes and describe the inertial manifold in the reduced-order phase space in terms of the associated probability measure. We introduce the notion of local fractal dimensionality and we establish a connection with the finite-time Lyapunov exponents of the reduced-order dynamics. Based on this tool we illustrate in 2D Navier-Stokesequations that the underlying mechanism responsible for the finite dimensionality of the inertial manifold is, apart from the viscous dissipation, the reverse flow of energy from the stochastic fluctuations (containing in general smaller length-scales) back to the mean flow (which is characterized by larger spatial scales).

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Investigating fluid flows via individual trajectory complexity methods

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We consider the analysis of flows in terms of the complexity of the fluid particle trajectories. Two complexity measures - the correlation dimension and the ergodicity defect - are explored in the context of several examples. Both measures yield structures resembling Lagrangian coherent structures in the examples. The possible advantage of these individual trajectory complexity methods over more traditional approaches in the analysis of typical ocean float and drifter data sets is discussed.

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Maximal stretching surfaces as potential platelet activation pathways

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As platelets are transported they are continuously stretched, compressed and sheared by local gradients in the flow. Exposure to elevated gradients can cause platelets to actively react with conformational, chemical and enzymatic responses, i.e. becoming activated. Once switched to the activated state, platelets perform multifaceted roles to orchestrate clotting. When one measures the activation potential for platelets in blood flow, the measure tends to be maximized along distinguished material surfaces that are otherwise referred to as "LCS." Furthermore, comparison of these structures with data on platelet deposition near arterial stenoses suggest that in fact, clot formation may be strongly influenced by these structures. Moreover, these results shed insight into the possible synergies between force-mediated and transport-mediated biomechanical processes, which have previously been considered independently.

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Mapping unstable manifolds using floats in a Southern Ocean field campaign

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Ideas from dynamical systems theory have been used in an observational field campaign in the Southern Ocean to provide information on the transport and mixing structure of the flow. Satellites can provide

information concerning surface currents at a scale of 10 km or so. This information was used in near-real time to provide an estimate of the location of stable and unstable manifolds in the vicinity of the Antarctic Circumpolar Current. As part of a large US/UK observational field campaign (DIMES: Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean) a number of floats were then released (at the surface and at a depth of approximately 1km) close to the estimated intersection of the two manifolds in several locations with apparently different dynamical characteristics. The subsequent trajectories of the floats has allowed the unstable manifolds to be tracked, and the relative separation of pairs of floats has allowed an estimation of Lyapunov exponents. The results of these deployments have given insight into the strengths and limitations of the satellite data which does not resolve small scales in the velocity field, and have elucidated the transport and mixing structure of the Southern Ocean at the surface and at depth.

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Topological chaos in systems ‘stirred’ by almost-cyclic sets

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In many dynamical systems, there are regions of phase space that remain coherent for an extended period of time. Identification of these coherent regions can be accomplished by, for example, computing the eigenvectors of the discretized Perron-Frobenius transfer operator via a set-oriented approach. The eigenvector(s) give the locations(s) of these regions, known as Almost-Invariant Sets (AIS), and the corresponding eigenvalue(s) quantify their ‘leakiness’. In some cases, there exist disconnected components of a single AIS that form Almost-Cyclic Sets (ACS). We extend this set-oriented approach by considering the relative interaction of trajectories from within the ACS. In certain (2+1)-dimensional systems, the braiding of these trajectories provides a framework for analyzing chaos in the system through application of the Thurston-Nielsen classification theorem. Sufficiently complex braiding patterns lead to what is known as ‘topological chaos’, enabling predictions of system behavior based solely on the spatio-temporal ACS structure. This approach makes it possible to investigate and quantify chaos in complex dynamical using limited information about the relative motion of a few coherent regions. We demonstrate this approach by considering stirring and mixing in a two-dimensional, time-dependent fluid system.

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Lagrangian coherent structures and eddy diffusion

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Lagrangian coherent structures (LCS) are known as the templates for chaotic mixing in nonlinear aperiodic dynamical systems, such as geophysical flows. In recent years, theoretical developments on the deterministic LCS have allowed the objective identification of mixing barriers and enhancers in geophysical flows. Stochastic transport associated with LCS, on the other hand, is less studied, partly due to the inherent scale separation between coherent structures and molecular diffusion. However, sub-grid scale uncertainty of geophysical flows cannot be neglected when one tries to quantify diffusive transport of substances. In this talk we will discuss some recent efforts on quantifying diffusive mixing associated with the LCS. In particular, eddy diffusivity tensors associated with advection-diffusion are constructed based on Lagrangian measures from LCS. Some archetypal examples will be discussed.

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Moving walls accelerate mixing

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Mixing in viscous fluids is challenging, but chaotic advection in principle allows efficient mixing. In the best possible scenario, the decay rate of the concentration profile of a passive scalar should be exponential in time. In practice, several authors have found that the no-slip boundary condition at the walls of a vessel can slow down mixing considerably, turning an exponential decay into a power law. This slowdown affects the whole mixing region, and not just the vicinity of the wall. The reason is that when the chaotic mixing region extends to the wall, a separatrix connects to it. The approach to the wall along that separatrix is polynomial in time and dominates the long-time decay. However, if the walls are moved or rotated, closed orbits appear, separated from the central mixing region by a hyperbolic fixed point with a homoclinic orbit. The long-time approach to the fixed point is exponential, so an overall exponential decay is recovered, albeit with a thin unmixed region near the wall.

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