

## Special Session 19: Waves and Convection

Sam Stechmann, University of Wisconsin-Madison, USA  
 Leslie Smith, University of Wisconsin-Madison, USA

This special session is focused on waves, convection, and their intertwined dynamics. Mathematical aspects include interesting new PDEs, multiscale asymptotics, numerical modeling, and data mining, among other topics. Applications include rotating and stratified fluid dynamics, atmospheric and oceanic science, turbulence, and biology, to name a few.

### Multi-modal dynamics in parallel and wave-induced stratified shear layers

**Roberto Camassa**  
 UNC, USA  
 camassa@amath.unc.edu  
**Claudio Viotti**

In this talk we discuss the evolution of perturbations containing multiple non-normal eigenmodes in stratified shear flows. We perform detailed analysis of the Taylor–Goldstein equation, the stability operator in parallel stratified shear flows, and we present numerical observations of multi-modal dynamics arising in both parallel and non-parallel wave-induced shear layers. We show how the Taylor–Goldstein spectrum can be used to explain subtle aspects of perturbations evolution that affect the instability of internal waves beside other mechanisms, like Kelvin–Helmholtz instability, that so far have received most of the attention in such context.

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### Modulation of shallow water equatorial waves due to a varying equivalent height background

**Juliana Dias**  
 PSD - ESRL/NOAA, USA  
 juliana.dias@noaa.gov  
**P. L. Silva Dias, G. N. Kiladis**

The dynamics of equatorial convectively coupled Kelvin waves (CCKWs) is analyzed in an idealized model for the large-scale equatorial circulation. The model is composed of a linear rotating shallow water system with a variable equivalent depth, which is assumed to vary in space and time. This model is based on the hypothesis that moist convection acts to reduce the equivalent depth of a shallow water system. Here, asymptotic solutions are derived in the case of a small perturbation around a constant equivalent depth. The first order solutions correspond to the free normal modes of the shallow water system. The second order flow satisfies a forced shallow water system, where the forcing (representing the convection) is proportional to the divergence of the first order flow solution. We first demonstrate how solutions vary depending on the space-time behavior of the equivalent depth perturbation. In particular, propagating solutions exist in both limits where the equivalent depth oscillates fast

or slow in comparison to the first order wave scale. However, the second order wave amplitude decays as the period of the variable coefficient decreases. This result implies that the overall flow is less affected by high frequency equivalent depth oscillations. This analytical framework is applied to the study of a synoptic scale (1000 km) Kelvin wave propagating through a background where the equivalent depth oscillates at planetary scales (10000 km). The modeled flow share some remarkable similarities with observed CCKWs. First, as in observed CCKWs, the modeled wave develops a weak secondary meridional circulation. Second, not surprisingly, the phase speed and meridional trapping scale are modulated by changes in the equivalent depth. This modulation is consistent with the fact that CCKWs tend to propagate more slowly when they are embedded in a larger scale convective envelope such as the Madden Julian Oscillation. Based on space-time spectral analysis of tropical convection data, CCKWs propagate at speeds between 7-25 m/s. While highly idealized, the model used here provides a mechanism for both the variability in the observed phase speed of CCKWs, and for changes in the longitude-latitude structure of the observed waves in comparison to the dry modes.

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### Capturing intermittent and low-frequency variability in high-dimensional data through nonlinear Laplacian spectral analysis

**Dimitrios Giannakis**  
 New York University, USA  
 dimitris@cims.nyu.edu  
**Andrew J. Majda**

Nonlinear Laplacian spectral analysis (NLSA) is a recently developed technique for spatiotemporal analysis of high-dimensional data, which represents temporal patterns via natural orthonormal basis functions on the nonlinear data manifold. Through the use of such basis functions, determined efficiently via graph-theoretic algorithms, NLSA captures intermittency, rare events, and other nonlinear dynamical features which are not accessible via linear approaches [e.g., singular spectrum analysis (SSA)]. Here, we apply NLSA in a comparative study of North Pacific SST data from extended control integrations of the CCSM3 and ECHAM5/MPI-OM models. Without performing spatial coarse graining (i.e., operating in ambient space dimensions up to  $1.6 \times 10^5$  after

lagged embedding), or seasonal cycle subtraction, the method reveals families of periodic, low-frequency, and intermittent spatiotemporal modes. The intermittent modes, which describe variability in the Western and Eastern boundary currents, as well as variability in the subtropical gyre with year-to-year reemergence, are not captured by SSA, yet are likely to have high significance in a predictive context and utility in cross-model comparisons.

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### A model of convective Taylor columns in rotating Rayleigh-Benard convection

**Ian Grooms**

New York University, USA  
grooms@cims.nyu.edu

**Keith Julien, Jeffrey B. Weiss, Edgar Knobloch, Antonio M. Rubio**

Equations derived by asymptotic reduction of the rotating Navier-Stokes equations in the limit of strong rotation, weak stratification, and tall aspect ratio are presented, their relation to the hydrostatic equations and quasigeostrophic equations is explored, and a new multiscale interpretation of the Taylor-Proudman theorem is given. The discussion is then specialized to the problem of rotating Rayleigh-Bénard convection, and the results of numerical simulations in this setting are briefly summarized. In certain parameter regimes the simulations exhibit localized vortical structures termed ‘convective Taylor columns’; similar structures are also observed in laboratory experiments and in direct simulations of the unreduced Navier-Stokes equations. The ubiquity and dominance of the net heat transport by these structures motivates the derivation of a nonlinear, non-separable model which is an approximate solution of the reduced equations. The model specifies the horizontal structure in terms of the real part of a complex Hankel function, also known as a Bessel function of the third kind, while the vertical structure is specified by the numerical solution of a two-point boundary value problem.

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### Minimal models for precipitating organized convection

**Gerardo Hernandez-Duenas**

University of Wisconsin, USA  
hernande@math.wisc.edu

**Andrew J. Majda, Leslie M. Smith, Samuel N. Stechmann**

Simulations of precipitating convection are usually carried out with cloud resolving models, which typically represent all the different phases of water: water vapor, cloud water, rain water and ice. Here

we investigate the question: what is the minimal possible representation of water processes that is sufficient for these models? The simplified models that we present assume fast auto conversion and neglect ice. To test the simplified models, we present simulations of squall lines and scattered convection and show that they qualitatively capture observations made in nature and also seen in more comprehensive cloud resolving models, such as propagation of squall lines with tilted profiles, cold pools, and scattered convection.

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### Low rossby number heat transport in rotating Rayleigh-Benard convection

**Keith Julien**

University of Colorado at Boulder, USA  
keith.julien@colorado.edu

**Antonio Rubio, Ian Grooms, Geoff Vasil, Edgar Knobloch**

Recent laboratory experiments of turbulent rotating Rayleigh-Benard convection, performed *entirely within* the regime of strong rotational constraint, have revealed a sharp transition in the scaling of the heat transport as a function of the thermal forcing. This is embodied by the nondimensional Nusselt-Rayleigh scaling law,  $Nu \propto Ra^\alpha$ , where a steep scaling regime ( $\alpha > 1$ ) gives way to a comparatively shallower regime ( $\alpha < 1/2$ ) typical of non-rotating turbulent convection. A crossover between the thermal and viscous boundary layers has been proposed as the root-cause of this remarkable result, yet a similar transition is found in the presence of stress-free boundary conditions where viscous layer boundary layers are absent. Unfortunately, the dynamics within the thermal boundary layer remain poorly understood due to resolution challenges at low Rossby number. Utilizing numerical simulations of the asymptotically exact nonhydrostatic balanced geostrophic equations we present an alternative explanation, not reliant on the form of the mechanical boundary conditions, but based on loss of geostrophic balance within the thermal boundary layers as a result of vigorous vortical motions. Furthermore, in contrast to nonrotating convection, we show prior to loss of balance that the bottleneck to heat transport is the turbulent interior not the boundary layers.

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### The vertical structure of baroclinic turbulence in the ocean

**Shane Keating**

Courant Institute of Mathematical Science, USA  
skeating@cims.nyu.edu

**K. Shafer Smith**

Mesoscale eddies in the ocean, which are primarily driven by baroclinic instability of the mean shear

and stratification profiles, dominate the ocean kinetic energy and play a crucial role in the transport of heat, salinity and biogeochemical tracers. However, no well-accepted scaling theory for baroclinic eddies exists, in part because of the complexity of the full nonlinear instability problem for realistic mean states, and this remains a critical limitation on efforts to develop mesoscale eddy parameterizations. In this study, we exploit the observation that the potential vorticity (PV) inversion problem for a general mean state can be decomposed into a part forced by surface boundary conditions and a part forced by the interior PV distribution. This allows for a convenient categorization of the mean state into Charney-type — in which surface-intensified modes interact with a background meridional PV gradient — or Phillips-type, which is unstable when there exists an inflection point in the mean PV profile. Within this framework, we examine idealized Charney, Phillips and mixed mean states in high-resolution quasigeostrophic simulations with the goal of elucidating the equilibration, vertical structure, and transport properties of the resulting baroclinic eddies.

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#### **A multiscale framework for analysis and simulation of the stratified wind-driven ocean surface boundary layer**

**Ziemowit Malecha**

University of New Hampshire, USA  
ziemowitzima@gmail.com

**Gregory Chini, Keith Julien**

A primary challenge in physical oceanography is to understand the interaction between small-scale turbulent convective flows in the upper ocean, particularly wind- and surface-wave-driven Langmuir circulation (LC), and submesoscale eddies, fronts, internal waves (IWs), and their associated instabilities. This problem is challenging because LC is strongly non-hydrostatic, is only indirectly affected by density stratification and the Earth's rotation, and has O(50) m length scales. In contrast, submesoscale flows are approximately hydrostatic, are strongly affected by density stratification and Coriolis accelerations, and have O(10) km lateral scales. In this investigation, we use multiscale asymptotic analysis to develop a physically consistent and computationally efficient model of the dynamics of the ocean surface boundary layer. Numerical experiments with this new model reveal novel dynamical phenomena induced by the two-way inter-scale coupling between submesoscale IWs and fine-scale LC: the IWs modulate the phase and intensity of the LC, while the rectified effect of the resulting non-uniformly distributed small-scale convective structures modifies the IW dynamics.

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#### **Internal solitary waves in two-layer flows with shear.**

**Paul Milewski**

University of Bath, England  
p.a.milewski@bath.ac.uk

**Anakewit Boonkasame**

Internal solitary waves are ubiquitous in the ocean and atmosphere. They often coexist with a background shear and also generate shear at their crests or troughs. Most realistic models for these waves are ill posed, and this ill posedness is often misinterpreted as a consequence of the shear. Often ad-hoc filtering must then be used to stabilize the computations. We consider the weakly dispersive long wave limit of two layer flow in the presence of background (and also induced) shear and present a model that is stable as long as a depth dependent Richardson parameter is above a threshold. Comparisons of solitary waves of this model with those of other models will be made.

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#### **Rossby waves in rotating shallow water on the sphere**

**David Muraki**

Simon Fraser University, Canada  
muraki@sfu.ca

**Kevin Mitchell**

Linear waves in the tropics are well-understood by the equatorial beta-plane theory of Matsuno (1966). When linearized about a quiescent background flow, the Rossby wave dispersion relation allows for waves at all length scales near the equator. However, it is also well known that for climatologically-typical zonal shear, midlatitude Rossby waves are prevented from crossing the tropics due to the presence of critical latitudes where the phase speed matches that of the background wind. These seemingly contradictory behaviors are typically found in disjoint chapters in the textbooks on atmospheric dynamics and the consistency of their coexistence is not addressed. Based upon the Rotating Shallow Water (RSW) equations on the sphere, we present a unified understanding of how both phenomena fit into a consistent picture of the atmosphere in both the tropics and midlatitudes. This perspective also offers clarification on the RSW wave modes of Kasahara (1980), and observations of global Rossby waves in the recent review by Madden (2007).

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### Tropical cyclogenesis and vertical shear in a moist Boussinesq model

**Leslie Smith**

University of Wisconsin, Madison, USA  
lsmith@math.wisc.edu

**Leslie Smith, Qiang Deng, Andy Majda**

Tropical cyclogenesis is studied in the context of idealized three-dimensional Boussinesq dynamics with a simple model for bulk cloud physics. With low-altitude input of water vapor, numerical simulations capture the formation of vortical hot towers. From measurements of water vapor, vertical velocity, vertical vorticity and rain, it is demonstrated that the structure, strength and lifetime of the hot towers is similar to results from models including more detailed cloud microphysics. Furthermore, the idealized model captures merger of vortical hot towers into a larger-scale, cyclonic moist vortex. The effects of low-altitude vertical shear are investigated by varying the initial zonal velocity profile. In the presence of weak low-level vertical shear, the hot towers retain the low-altitude monopole vorticity structure characteristic of the zero-shear case (starting from zero velocity). For stronger vertical shear, the individual hot towers develop a vorticity dipole rather than a cyclonic monopole. Linear analysis helps to explain the transition from monopole to dipole vorticity structure as the shear increases. The dipoles are not as conducive to merger, and thus strong enough low-level shear prevents the vortical-hot-tower route to cyclogenesis.

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### Nonlinear dynamics and regional variations in the MJO skeleton

**Samuel Stechmann**

University of Wisconsin-Madison, USA  
stechmann@wisc.edu

**Andrew J. Majda**

The Madden-Julian Oscillation (MJO) is a propagating envelope of complex multi-scale convection/storms in the tropics. With characteristic scales of 30-60 days and 20,000 km, it significantly affects El Nino, monsoons, and midlatitude predictability. Despite its importance, no theory for the MJO has yet been generally accepted, and climate models typically have inadequate representations of it.

In this talk, a minimal, nonlinear oscillator model is analyzed for the MJO "skeleton," i.e., its fundamental features on intraseasonal/planetary scales: (i) slow eastward phase speed of roughly 5 m/s, (ii) peculiar dispersion relation with group velocity of roughly 0, and (iii) horizontal quadrupole vortex structure. Originally proposed in recent work by the authors, the fundamental mechanism involves neutrally stable interactions between (i) planetary-scale, lower-tropospheric moisture anomalies, and (ii)

the envelope of sub-planetary-scale, convection/wave activity. Here, the model's nonlinear dynamics are analyzed in a series of numerical experiments, using either a uniform sea surface temperature (SST) or a warm-pool SST. The results show both standing oscillations and eastward propagation, and there are significant variations in the number, strength, and/or locations of MJO events. Besides these numerical experiments, it is also shown that the nonlinear model conserves a total energy that includes a contribution from the convective activity.

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### The diurnal cycle and the meridional extent of the tropics

**Esteban Tabak**

New York University, Courant Institute, USA  
tabak@cims.nyu.edu

**Paul A. Milewski**

This talk proposes an explanation for the sharp transition between tropics and extra-tropics at a latitude of 30 degrees. This transition, at the outer edges of the Hadley cells, is marked by a steep jump in the height of the troposphere, from sixteen kilometers in the tropics to nine in the mid and high latitudes. The tropics, equatorwards of 30 degrees, are characterized by easterly surface winds -the Trades- and a strong diurnal signal in the wind, pressure and temperature. Polewards of 30 degrees, the winds are westerly, and the weather systems have longer spatio-temporal scales. This change of behavior can be explained in terms of diurnal baroclinic waves due to solar forcing and trapped equatorwards of 30 degrees by the Coriolis effect. Their effect can be illustrated in simple two-layer models for the meridional circulation, where both convection and the entrainment of stratospheric air into the troposphere are represented by energy-preserving shock waves.

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### Bioconvection revisited

**Jean-Luc Thiffeault**

University of Wisconsin, Madison, USA  
jeanluc@mailaps.org

Bioconvection is convection driven by density differences due to concentration of swimming organisms. It is often driven by phototaxis or other biased motion. Here I will discuss a related phenomenon, where the swimming action of large swarms of microorganisms, such as some types of plankton, can actually drive large scale flows. I will also discuss the likelihood of observing such flows in the laboratory or the environment.

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