Special Session 17: Singular Perturbations

Freddy Dumortier, Hasselt University, Belgium
Peter De Maesschalck, Hasselt University, Belgium
Martin Wechselberger, University of Sydney, Australia

The aim of this special session is to get informed about recent results on singular perturbations, both from a pure, applied and numerical point of view. Besides scheduling talks from established mathematicians, we will give opportunity to junior researchers to present their work. Topics include (non-exhaustive): (geometric) singular perturbation theory, mixed-mode oscillations, canards, singularly perturbed PDE, delay in bifurcations.

Delayed Hopf bifurcation with a focus node transition

Eric Benoît
Université de La Rochelle, France
ebenoit@univ-lr.fr

The delayed Hopf bifurcation is studied by Neishtadt and Callot with the *relief* (real part of the integral of an eigenvalue in the complex domain). A new lemma, using fine majorations on complex paths allows to improve the classical result. This lemma solves the problem of delayed Hopf bifurcation when a focus-node transition is present. With this result, we can have a new point of view on some mixed mode oscillations.

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Blow-up of vector fields and Painleve equations

Hayato Chiba
Kyushu university, Japan
chiba@imi.kyushu-u.ac.jp

Painleve property is one of the most fundamental concept in the study of ODEs in the complex plane. In this talk, it is shown that Painleve equations are obtained from blow-up of singularities in slow-fast systems. Many properties of the Painleve transcendents are given by means of the dynamical systems theory.

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Slow-fast cycles with singular contact points

Peter De Maesschalck
Hasselt University, Belgium
peter.demaesschalck@uhasselt.be

We consider slow-fast cycles on an orientable two-dimensional manifold, consisting of an arbitrary (finite) number of slow and fast branches, but where the contact points of the slow branches are either of jump type or of singular type. Depending on the order of the singularity of the contact points, and depending on a so-called slow divergence integral computed along the slow-fast cycle, we deal with the unicity of nearby periodic orbits. This talk is a continuation of the talk of Freddy Dumortier, and is based on recent results obtained in collaboration with Freddy Dumortier and Robert Roussarie.

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Numerical continuation techniques for planar slow-fast systems

Mathieu Desroches
INRIA Paris-Rocquencourt, France
mathieu.desroches@inria.fr
Peter De Maesschalck

Continuation techniques have been known to successfully describe bifurcation diagrams appearing in slow-fast systems with more than one slow variable. In this talk we will investigate the usefulness of numerical continuation techniques dealing with both open and solved problems in the study of planar singular perturbations. More precisely, we first verify known theoretical results (thereby showing the reliability of the numerical tools) on the appearance of multiple limit cycles of relaxation-oscillation type and on the existence of multiple critical periods in well-chosen annuli of slow-fast periodic orbits in the plane. We then apply the technique to study a notion of maximal canard, in the sense of maximal period.

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Relaxation oscillations near common slow-fast cycles

Freddy Dumortier
Hasselt University, Belgium
freddy.dumortier@uhasselt.be

The talk deals with slow-fast cycles on orientable two-dimensional surfaces. The slow-fast cycles are common, in the sense that the slow curves are all attracting or all repelling. The contact points have a finite order, which can be arbitrary. Results are presented about the existence and unicity of relaxation oscillations near such slow-fast cycles. Attention is given to the proofs of the results. The talk is based on a recent paper by P.De Maesschalck, F.Dumortier and R. Roussarie.

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Gasless combustion fronts with heat loss

Anna Ghazaryan
Miami University, USA
ghazarar@muohio.edu
Stephen Schecter, Peter Simon

We consider a model of gasless combustion with heat loss, with the heat loss from the system to the environment modeled according to Newton's law of cooling. For the regime when the system contains two small parameters, a diffusion coefficient for the fuel and a heat loss parameter, we use geometric singular perturbation theory to show existence of traveling combustion fronts. We also study their spectral and nonlinear stability.

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Using geometric singular perturbation techniques to analyse models of intracellular calcium dynamics

Emily Harvey
Montana State University, USA
emily.harvey@coe.montana.edu
Vivien Kirk, James Sneyd, Martin Wechselberger

Oscillations in free intracellular calcium concentration are known to act as signals in almost all cell types, controlling a huge range of cellular processes including muscle contraction, cellular secretion and neuronal firing. Due to the almost universal nature of calcium oscillations, understanding the mechanisms underlying them is of great physiological importance. A key feature of intracellular calcium dynamics is that some physiological processes occur much faster than others. This leads to models with variables evolving on very different time scales. This separation in time scales suggests that geometric singular perturbation techniques (GSPT) may be useful in explaining the observed dynamics, including mixed-mode oscillations. In this talk the results from analysing a range of representative models of intracellular calcium dynamics using GSPT will be presented. We will describe the important steps and parameters in the analysis and demonstrate the usefulness of these techniques and their limitations in this context.

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Limit cycles in slow-fast codimension 3 saddle and elliptic bifurcations

Renato Huzak
Hasselt University, Belgium
renato.huzak@uhasselt.be
Peter De Maesschalck, Freddy Dumortier

In this talk we present singular perturbation problems occuring in planar slow-fast systems

$$\begin{cases} \dot{x} = y \\ \dot{y} = -xy + \epsilon\Big(b_0 + b_1 x \\ \qquad + b_2 x^2 \pm x^3 + x^4 + x^5 H(x,\lambda)\Big) + y^2 G(x,y,\lambda), \end{cases}$$

where G and H are smooth, $\epsilon > 0$ is the singular parameter that is kept small, (b_0, b_1, b_2) are regular perturbation parameters close to 0 and $\lambda \in \Lambda$, with Λ a compact subset of some euclidian space.

We investigate the number of limit cycles that can appear near the origin $(x, y) = (0, 0)$. When the sign in front of x^3 is positive, we deal with slow-fast saddle bifurcation. If the sign in front of x^3 is negative, the slow-fast systems under consideration are referred to as slow-fast elliptic bifurcations. In the saddle case we encounter canard-type relaxation oscillations of small amplitude. Hence the limit cycles in the saddle case are confined to small neighbourhood of the origin in the phase space and their size tends to 0 for $(b_0, b_1, b_2) \to (0, 0, 0)$. The slow-fast elliptic bifurcations allow detectable limit cycles to be present.

Using geometric singular perturbation theory, including blow-up, both saddle end elliptic case can be restricted to the well known jump case, slow-fast Hopf bifurcations and slow-fast Bogdanov-Takens bifurcations. The most difficult problem to deal with concerns an upper bound on the number of limit cycles that appears in slow-fast Hopf case.

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Reinjected horseshoes

Vincent Naudot
Florida Atlantic University, USA
vnaudot@fau.edu
William D.Kalies, Markus Fontaine

Horseshoes play a central role in dynamical system and are observed in many chaotic systems. These are invariant sets for both the forward and the backward iterations of the map that defines the dynamics. However most all the points outside this set escape from a neighborhood of the Horseshoe after finite iterations. In this work we construct a system that possesses a Horseshoe together with a nearby nontrivial attractor, i.e., this latter is transitive, stable and possesses a positive Lyapunov exponent. This system is obtained after reinjecting the Horseshoe thanks to the unfolding of a degenerate double homoclinic orbit.

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Interval mappings for slow-fast models of neurons

Andrey Shilnikov
GSU, USA
ashilnikov@gsu.edu
Jeremy Wojcik, Alex Neiman

We present a thorough bifurcation analysis of transformations of bursting activity in slow-fast models of neurons through the computer-assisted reduction to one and two-dimensional Poincare mappings of an voltage interval. We were able to examine in detail typical bifurcations that underlie the complex activity transitions between: tonic spiking and bursting, bursting and mixed-mode oscillations, including torus breakdown in generic slow-fast models.

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Global attractors for damped semilinear wave equations with a Robin–acoustic boundary perturbation

Joseph Shomberg
Providence College, USA
jshomber@providence.edu
Sergio Frigeri

Under consideration is the damped semilinear wave equation

$$u_{tt} + u_t - \Delta u + u + f(u) = 0$$

on a bounded domain Ω in $\mathbb{R}^3$ with a perturbation parameter $\varepsilon > 0$ occurring in an acoustic boundary condition, limiting ($\varepsilon = 0$) to a Robin boundary condition. With minimal assumptions on the nonlinear term f, the existence and uniqueness of global weak solutions is shown. Also, the existence of a family of global attractors is shown to exist. After proving a general result concerning the robustness of a one-parameter family of sets, the result is applied to the family of global attractors. Because of the complicated boundary conditions for the perturbed problem, fractional powers of the Laplacian are not well-defined; moreover, because of the restrictive growth assumptions on f, the family of global attractors is obtained from the asymptotic compactness method developed by J. Ball for generalized semiflows.

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Mixed-mode oscillations in a multiple time scale phantom bursting system

Alexandre Vidal
University of Evry, France
alexandre.vidal@univ-evry.fr
Martin Krupa, Mathieu Desroches, Frédérique Clément

During the past 20 years, studies have focused on the lowest dimension dynamics (two slow and one fast variables) that may display Mixed-Mode Oscillations (MMOs). However, the complex transition that adds a small oscillations to a periodic MMO orbit as a parameter varies has never been studied in detail since, in this context, one expects chaos to be the main underlying mechanism. I will present a new kind of MMOs in the case of a four dimensional system with three different time scales. I will show, using geometric singular perturbation theory, that the system admits a limit cycle generating these MMOs even during the canard-induced transition that adds a small oscillation.

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