Special Session 13: Global Dynamics in Hamiltonian Systems

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This minisimposium will concentrate on global properties of Hamiltonian systems. Including the invariant objects and the global dynamics that they produce. We plan to bring together mathematical theory about these questions, numerical calculations and applications to realistic systems.

Zonal jets as transport barriers in the Earth's stratosphere

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Theoretical results relating to Kolmogorov–Arnold–Moser (KAM) theory have led to the expectation that associated with zonal (west–east) jet streams in the Earth's stratosphere should be barriers which inhibit meridional (south–north) transport. In this talk evidence will be provided for this expectation based on the analysis of stratospheric winds produced by a comprehensive general circulation model. This will follow a review of the relevant KAM theory results, namely, those dealing with the stability of a degenerate one-degree-of-freedom Hamiltonian system under a time-quasiperiodic perturbation.

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Poincare-Birkhoff theorem in quantum mechanics

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Quantum manifestations of the dynamics around resonant tori in perturbed Hamiltonian systems, dictated by the Poincare-Birkhoff theorem, are shown to exist. They are embedded in the interactions involving states which differ in a number of quanta equal to the order of the classical resonance. Moreover, the associated classical phase space structures are mimicked in the quasiprobability density functions and their zeros.

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"Theory and computation of quasi-periodic solutions of conformally symplectic systems

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Quasi-Periodic motions appear in many different physical systems (e.g. celestial mechanics, solid-state physics, non-equilibrium statistical mechanics, etc.) I will discuss an a-posteriori KAM theory for the existence an persistence of quasi-periodic solutions. The consequences of the a-posteriori format of KAM theory include local uniqueness, regularity results, and the rigorous justification of a criterion for the breakdown of analyticity. This format also suggests and validates very efficient numerical methods to approximate quasi-periodic solutions and their breakdown of analyticity and hyperbolicity.

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Some topological aspects of Aubry-Mather theory

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We re-construct the Aubry-Mather theory in terms of a foliation of the annulus, by expressing the twist condition and the tilt condition in terms of such a foliation. Then the existence of Aubry-Mather sets and of orbits that shadow prescribed families of Aubry-Mather sets can be proved using the diagonal method of G. R. Hall.

A motivation for this work is the study of Hamiltonian flows on strictly convex energy levels in $\mathbb{R}^4$. Such flows possess a system of open book decompositions whose pages are disk-like global surfaces of sections to the flow. Each disk-like global surface of section is foliated by the pages of the complementary open book decomposition. The first return map to the disk determines a symplectic homeomorphism of an open annulus. Thus we can apply Aubry-Mather theory via foliations to this setting, assuming that the annulus map satisfies a twist condition or a tilt condition. Even if such a condition is not satisfied, some topological aspects of Aubry-Mather theory remain valid.

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On stochastic sea of the standard map

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The standard map is one of the simplest and most famous conservative transformations that is still far from being completely understood. We prove that stochastic sea (the set of orbits with non-zero Lyapunov exponents) of the standard map has full Hausdorff dimension for sufficiently large topologically generic parameters. In the proof we consider a generic one-parameter unfolding of a homoclinic tangency of an area preserving surface diffeomorphism, and show that it give birth to a hyperbolic set of large (almost full) Hausdorff dimension. The last statement has also applications in celestial mechanics (this is a joint work with V.Kaloshin), namely can be used to show that in some versions of the three body problem for many parameters the set of oscillatory motions has full Hausdorff dimension.

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Generalized KAM cuves in time-aperiodic Hamiltonian systems

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I show how the recent geodesic theory of transport barriers can be used to compute invariant regions bounded by generalized KAM curves (elliptic transport barriers) in temporally aperiodic, planar Hamiltonian systems. When advected by the flow map over a finite time interval, these curves preserve both their arclength and the area they enclose, much the same way as KAM curves do under the iteration of the associated Poincare map in time-periodic Hamiltonians. These properties render elliptic transport barriers as ideal boundaries of coherent vortices in fluid mechanics. I will illustrate this by applications to geophysical flows and 2D forced turbulence.

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Singularity theory for non-twist KAM tori: a methodology

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We present a novel method to find KAM tori in degenerate (nontwist) cases. We also require that the tori thus constructed have a singular Birkhoff normal form. The method provides a natural classification of KAM tori which is based on Singularity Theory. The method also leads to effective algorithms of computation, and we present some preliminary numerical results.

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On the action-minimizing triple collision orbits in the planar three-body problem

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In this talk, we study the shape of the limiting configuration of the action-minimizing triple collision orbits in the planar Newtonian three-body problem with arbitrary masses. For a given non-collinear initial configuration, the minimizing triple collision orbit iscollision-free until a simultaneous collision, and its limiting configuration is the Lagrangian configuration with the same orientation as the initial configuration. For the collinear initial configuration, under a certain technical assumption, there exist two minimizing orbits. The limiting configurations of these orbits are the two opposite Lagrangian configurations.

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Extrapolation of frequencies of quasi-periodic orbits

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Frequency analysis was introduced by Laskar to study secular motions of the planets in the Solar system. A significant refinement of Laskar's method, based in the simultaneous improvement of the frequencies and the amplitudes of the signal, was given later by G. Gòmez, J.M. Mondelo and C. Simó. On another front, a methodology to compute rotation numbers of invariant curves (and more general objects) has been introduced recently in different works by A. Luque, T.M. Seara and J. Villanueva. The idea is to extrapolate the rotation number (and related quantities) from suitable averages of the iterates of an orbit. The goal of this talk is to present a methodology to compute the frequencies of a given quasi-periodic orbit with an arbitrary number of basic frequencies. The construction is a generalization of the mentioned averaging-extrapolation approach to study rotation numbers. We plan to describe informaly this construction, that allows us to compute with high precision the components of the frequency vector. The main advantage over other high precision methods is that we do not require to compute nor to refine the amplitudes of the signal. As an illustration, we will consider quasi-periodic motions close to the point L5 in a restricted three-body problem and also include quasi-periodic perturbations from another bodies.

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Hyperbolic cylinders and KAM tori in nearly integrable systems on A^3

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The aim of the talk is to present recent results on Arnold diffusion for three-degree-of-freedom systems. We will show how to construct a hyperbolic skeleton for diffusion, namely a generalized chain of hyperbolic cylinders connected by heteroclinic orbits, which is asymptotically dense on an energy level when the size of the perturbation tends to 0. All the conditions for constructing orbits shadowing this chain are fullfilled (torsion conditionon the cylinders and tame homoclinic connections), so the existence of diffusion orbits follows from classical shadowing lemmas.

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Generic super-exponential stability of invariant tori in Hamiltonian systems

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Morbidelli and Giorgilli have proved a result of stability over super-exponentially long times if one consider an analytic lagrangian torus, invariant for an analytic hamiltonian system, with diophantine translation vector which admit a sign definite torsion. More specifically, the solutions of the system moves very little over times which are super-exponentially long with respect to the inverse of the distance to the invariant torus. The proof is in two steps: first the construction of a Birkhoff normal form at a high order, then the application of Nekhoroshev theory. Bounemoura has shown that the second step of this construction remains valid if the Birkhoff normal linked to the invariant torus is in a generic set among the formal series. This is not sufficient to prove this kind of super-exponential stability results in a general setting. We should also establish that most strongly non resonant invariant torus in a Hamiltonian system admit a Birkhoff normal form in the set introduced by Bounemoura. We show here that this property is satisfied generically in the sense of the measure (prevalence) through methods similar to those developed by Kaloshin and Hunt in their work on the growth of the number of periodic points for prevalent diffeomorphisms.

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Geometric control and dynamical systems

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We show several applications of geometric control methods to Hamiltonian dynamics such as closing lemmas and Kupka-Smale like theorems.

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A saddle in a corner - a model of collinear triatomic chemical reactions

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A geometrical model which captures the main ingredients governing atom-diatom collinear chemical reactions is proposed. This model is neither near-integrable nor hyperbolic, yet it is amenable to analysis using a combination of the recently developed tools for studying systems with steep potentials and the study of the phase space structure near a center-saddle equilibrium. The nontrivial dependence of the reaction rates on parameters, initial conditions, and energy is thus qualitatively explained. Conditions under which the phase space transition state theory assumptions are satisfied and conditions under which they fail are derived.

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Weak integrability of Hamiltonians in the two torus and rigidity

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We show that a C^∞ k-basic Finsler metric in the two torus T^2 whose geodesic flow preserves a codimension one $C^{1,L}$ foliation is in fact flat. Although integrable high energy levels of Hamiltonians in the torus are not flat in general, the $C^{1,L}$ integrability of k-basic Finsler geodesic flows in T^2 implies flatness and in particular, C^∞ integrability. We also show that a codimension one C^1 foliation invariant by the geodesic flow of the k-basic Finsler metric is C^∞. A consequence of the above results is that the Hopf conjecture would be false for k-basic Finsler metrics in the two torus if and only if there exists a C^0 integrable k-basic Finsler geodesic flow that is not C^1 integrable.

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Dimension reduction for Hamiltonian dynamics

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We study the dimension reduction of Hamiltonian PDEs using Proper Orthogonal Decomposition (POD), which allows one to project the full evolution equations onto an optimal set of modes that determine the dynamics. The optimal set of modes is constructed from observations of the dynamics of the governing equations. We describe the properties of such a reduction in the Hamiltonian framework and show how the reduced model can be used for analytical and numerical bifurcation study. Examples of the application of the approach to solitondynamics described by the nonlinear Schrodinger equation and computation of periodic surface waves will be given.

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