Special Session 12: Singular Perturbations and Boundary Layer Theory

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Roger Temam, Indiana University, USA

The main purpose of this workshop is to bring some new ideas and results of distinguished specialists in the field of singular perturbations and boundary layers for PDEs. The connection between applied mathematics with other fields constitutes a big challenge for the modern mathematics. In this direction, the studies focusing on the interface between fluid mechanics and PDEs are of great importance in our days. It is well-known that the convergence of the Navier-Stokes solutions to the ones of Euler on bounded domains is one of the challenging problems in fluid mechanics; this is an outstanding open problem because of the presence of boundary layers. The minisymposium is open both to young and established researchers and it constitutes a good platform to exchange the future of some relevant problems in fluid mechanics which are in connection with PDEs that develop boundary layers. It is also an opportunity to graduate students and post-docs to present and discuss their researches in this direction.

On the boundary value problem for the parabolic equation with a small parameter

Galina Bizhanova
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We consider the boundary value problem for the parabolic equation with a small parameter at the time derivative in a boundary condition. This problem is a linearized one of a Stefan type problem with a small parameter at the velocity of a free boundary. There are studied the properties of the solution of the perturbed problem, the convergence of it as a small parameter tends to zero.

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Boundary layer problem: Navier-Stokes equations and Euler equations

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This work is concerned with the boundary layer turbulence. We consider an incompressible viscous fluid in domains with permeable walls. The permeability is described by the Navier slip boundary conditions. The goal is to study the fluid behavior at vanishing viscosity. We show that the vanishing viscous limit is a solution of the Euler equations with the Navier slip boundary condition on the inflow region of the boundary.

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Boundary layers of the Navier-Stokes equations

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Makram Hamouda, Chang-Yeol Jung, James Kelliher, Roger Temam

We study boundary layers of the Navier-Stokes equations at small viscosity, in a curved domain, under various boundary conditions. In this talk, using the curvilinear system adapted to the boundary, we will focus on the construction of an incompressible corrector.

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Singular perturbations for the Primitive equations

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We present some convergence results related to the Linearized Primitive Equations for which the solution develops a boundary layer as the viscosity goes to zero. More precisely, as one of the difficulties is related with the ill-posedness of the limit solution with any choice of *local* boundary conditions, it is necessary to choose *nonlocal* ones. The boundary layer analysis allows us to confirm rigourously this choice.

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Boundary layer for a class of nonlinear pipe flow

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Anna Mazzucato, Dongjuan Niu, Xiaoming Wang

In this talk, we will consider a class of nonlinear pipe flow in which the Prandtl boundary layer theory can be verified in the Sobolev norm setting. In particular, we will derive the optimal $L^\infty(L^\infty)$ convergence and the $L^\infty(H^1)$ convergence under certain mild smoothness condition and compatibility condition on the data.

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Nonpolynomial spline finite difference scheme for nonlinear singular boundary value problems with singular perturbation and its mechanization

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Roger Temam

A general scheme for the numerical solution of nonlinear singular perturbation problems using nonpolynomial spline basis is proposed in the paper. The special nonequidistant formulation of mesh takes into account the boundary and interior layer structures. The proposed scheme is almost fourth order accurate and applicable to both singular and nonsingular cases. Convergence analysis of the scheme is briefly discussed. Maple program for the generation of difference scheme is presented. Computational illustrations characterized by boundary and interior layers show that-the practical order of accuracy is close to the theoretical order of the method.

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Singularly perturbed convection-diffusion equations on a circle domain

Chang-Yeol Jung
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Roger Temam

We discuss the boundary layers generated by a convection-diffusion equation in a circle. In the model problem that we consider two characteristic points appear. I will present how to treat boundary layers in a systematic way with certain simplifying compatibility conditions. We also discuss noncompatible data issues and how to handle them.

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Some recent results on an extended Navier-Stokes system

James Kelliher
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Mihaela Ignatova, Gautam Iyer, Arghir Zarnescu

We discuss recent results on the well-posedness and numerical stability of the extended Navier-Stokes equations. These results build on the earlier works of Liu, Liu, and Pego and Pego, Iyer, and Zarnescu.

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Vanishing viscosity limit for a certain class of channel flows

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Michael Taylor

We study the vanishing viscosity limit for certain Taylor-Couette flows in channels. We establish convergence of the Navier-Stokes solution to the corresponding Euler solution as viscosity vanishes in various norms without compatibility conditions on the data. In the process we obtain a detailed analysis of the small-diffusion limit for a heat equation with drift, using a parametric construction.

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Navier-Stokes equations in critical spaces: existence and stability of steady state solutions

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Nguyen Cong Phuc

In this talk, we discuss our recent results on the uniqueness existence of solutions to the stationary Navier-Stokes equations with small singular external forces belonging to a functional space introduced by Mazy'a and Verbitsky. The stability of the steady state solutions in such spaces is also obtained by a series of sharp estimates for resolvents of a singularly perturbed operator and the corresponding semigroup.

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From vortex layers to vortex sheets

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In this talk we shall consider the case of a plane flow with initial datum of a vortex layer type, i.e. with vorticity concentrated around a curve. We shall prove that, under analyticity hypotheses, the solution keeps the structure of vorticity layer, for a time that does not depend on the thickness. Moreover, when the thickness goes to zero, the layer moves according to the Birkhoff-Rott equation.

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Convection-diffusion equation with small viscosity in a circle

Roger Temam
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In this talk we will discuss singular perturbation problems for convection-diffusion equations in a circle when the viscosity is small. Highly singular behaviors can occur at the characteristic points which render the analysis difficult. A detailed analysis of these singularities has been conducted, and the corresponding boundary layers have been made explicit. This simplified model shows how singular and involved the behaviors can be in incompressible fluid mechanics when the viscosity is small.

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