Special Sessions

Special Session 1: Qualitative Studies of PDEs: Entire Solutions and Asymptotic Behavior

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The aim of this session is to discuss properties of entire solutions and asymptotic behavior of global solutions in various types of PDEs. Topics will include Liouville-type theorems, blow-up, traveling waves, convergence, concentration phenomena, etc.

Stability analysis of asymptotic profiles for fast diffusion equations

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Let Ω be a bounded domain of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. We are concerned with the Cauchy-Dirichlet problemfor fast diffusion equations of the form

$$\partial_t\left(|u|^{m-2}u\right) = \Delta u \qquad \text{in } \Omega\times(0,\infty), \tag{1}$$

$$u = 0 \qquad \text{on } \partial\Omega\times(0,\infty), \tag{2}$$

$$u(\cdot,0) = u_0 \qquad \text{in } \Omega, \tag{3}$$

where $\partial_t = \partial/\partial t$, $m \in (2, 2^*)$ with $2^* := 2N/(N-2)_+$ and u_0 might be sign-changing.Every solution $u = u(x,t)$ of (1)–(3) vanishes in finite time at a power rate. This talk is concerned with asymptotic profiles of vanishing solutions. We first introduce the notions of stability and instability of (possibly sign-changing) asymptotic profiles and present some stability criteria. Moreover, we also discuss annular domain cases, which do not fall within the criteria, by developing some perturbation method for radial symmetric functions.

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Finite Morse index solutions and asymptotics of weighted nonlinear elliptic equations

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We study the behavior of finite Morse index solutions of the equation

$$-\mathrm{div}(|x|^\theta \nabla v) = |x|^l |v|^{p-1}v \quad \text{in } \Omega \subset R^N \ (N \geq 2), \tag{1}$$

where $p > 1$, $\theta, l \in R^1$, and Ω is a bounded or unbounded domain. Through a suitable transformation of the form $v(x) = |x|^\sigma u(x)$, the following nonlinear Schrödinger equation with Hardy potential

$$-\Delta u = |x|^\alpha |u|^{p-1}u + \frac{\ell}{|x|^2}u \ \text{ in } \Omega \subset R^N \ \ (N \geq 3), \tag{2}$$

where $p > 1$, $\alpha \in (-\infty,\infty)$ and $\ell \in (-\infty, (N-2)^2/4)$, reduces to a special case of (1).

We demonstrate that the general form (1) is more natural to use even if one's main interest is in (2).

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Sharp decay estimates of L^q-norms for nonnegative Schrödinger heat semigroups

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Let $H = -\Delta + V$ be a nonnegative Schrödinger operator on $L^2(\mathbf{R}^N)$, where $N \geq 3$ and V is a radially symmetric nonpositive function in $\mathbf{R}^N$ decaying quadratically at the space infinity. For any $1 \leq p \leq q \leq \infty$, we denote by $\|e^{-tH}\|_{q,p}$ the operator norm of the Schrödinger heat semigroup e^{-tH} from $L^p(\mathbf{R}^N)$ to $L^q(\mathbf{R}^N)$. In this paper, under suitable conditions on V, we give the exact and optimal decay rates of $\|e^{-tH}\|_{q,p}$ as $t \to \infty$ for all $1 \leq p \leq q \leq \infty$. The decay rates of $\|e^{-tH}\|_{q,p}$ depend whether the operator H is subcritical or critical via the behavior of the positive harmonic function for the operator H.

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On the asymptotic behavior of solutions for semiliear parabolic equations involving critical Sobolev exponent

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In this talk, we are concerned with the asymptotic behavior of solutions for semilinear parabolic equations involving crirical Sobolev exponent. Particularly, we are interested in the behavior of threshold solutions and we will discuss the blow up rate, blow-up limit of such solutions.

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Existence and asymptotic stability of periodically growing solutions of nonlinear parabolic equations

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We study the behavior of unbounded global orbits in a class of strongly monotone semiflows and give a criterion for the existence of orbits with periodic growth. We also prove the uniqueness and asymptotic stability of such orbits. We apply our results to a certain class of nonlinear parabolic equations and show the convergence of the solutions to a periodically growing solution which grows up in infinite time with time-periodic profile.

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Convergence for asymptotically autonomous parabolic equations on R^N

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We consider parabolic equation on R^N whose nonlinearities are asymptotically autonomous, in both space and time, as time approaches infinity. We will present a convergence result for positive solutions of such equations and discuss the key tools of its proof.

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Ground state solutions of the nonlinear Schroedinger equation with interface

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Tomas Dohnal, Kaori Nagatou, Michael Plum, Wolfgang Reichel

We consider the nonlinear Schroedinger equation $-\Delta u + V(x)u = \Gamma(x)|u|^{p-1}u$ in R^n with $V(x) = V_1(x)$, $\Gamma(x) = \Gamma_1(x)$ where $x_1 \geq 0$ and $V(x) = V_2(x)$, $\Gamma(x) = \Gamma_2(x)$ where $x_1 \leq 0$ with functions $V_1, V_2, \Gamma_1, \Gamma_2$ which are periodic in each coordinate direction. This problem describes the interface of two periodic media, e.g. photonic crystals. We study the existence of ground state solutions and provide an existence criterion. Examples of interfaces satisfying these criteria are provided. In 1D it is shown that the criteria can be reduced to conditions on the linear Bloch waves of the operators $-\frac{d^2}{dx^2} + V_1(x)$ and $-\frac{d^2}{dx^2} + V_2(x)$.

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On the number of maximum points of least energy solutions to a two-dimensional Hénon equation with large exponent

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In this talk, we prove that least energy solutions of the two-dimensional Hénon equation

$$-\Delta u = |x|^{2\alpha} u^p \ (x \in \Omega), \qquad u > 0 \ (x \in \Omega),$$
$$u = 0 \ (x \in \partial\Omega),$$

where Ω is a smooth bounded domain in $\mathbb{R}^2$ with $0 \in \Omega$, $\alpha \geq 0$ is a constant and $p > 1$, have only one global maximum point when $\alpha > e - 1$ and the nonlinear exponent p is sufficiently large. This answers positively to a recent conjecture by C. Zhao (preprint, 2011).

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On Toda system: classification and applications

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We give a complete classification of the solutions to the following $SU(N+1)$ Toda system with a single source:

$$\Delta u_i + \sum_{j=1}^{N} a_{ij} e^{u_j} = 4\pi\gamma_i\delta_0$$

in R^2. here the matrix (a_{ij}) is the $SU(N+1)$ cartan matrix. Then we apply these classification and nondegeneracy result to give a rigorous proof of the existence of non-topological solutions for $SU(3)$ Chern-Simons system.

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Slowly traveling waves, blow-up at spatial infinity and homoclinic orbits in nonlinear parabolic equations of fast diffusion type

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We consider the spatially one-dimensional nonlinear diffusion equation $u_t = u^p u_{xx}$ for $p > 0$. We construct positive classical solutions, defined for all negative times, which are of the form $u(x,t) = (-t)^{-\frac{1}{p}} F\Big(x + \frac{1}{p\alpha}\ln(-t)\Big)$, with arbitrary $\alpha > 0$, by solving an associated ODE for F. These 'ancient slowly traveling wave solutions' have the following properties: 1.) If $p \leq 2$ then u blows up at time $t = 0$ with empty blow-up set. 2.) If $p > 2$ then u can be extended so as to become an entire positive classical solution $\bar{u}$, defined on $\mathbb{R} \times \mathbb{R}$, such that $\bar{u}_x > 0$ on $\mathbb{R}$, but such that $\bar{u}$ connects the trivial equilibrium to itself in the sense that $\bar{u}(x, t) \to 0$ as $t \to \pm\infty$, locally uniformly with respect to $x \in \mathbb{R}$.

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Asymptotic behavior of singular solutions for a semilinear parabolic equation

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We consider singular solutions of a semilinear parabolic equation with a powernonlinearity. It is known that in some range of parameters, this equation has a family of singular steady states with a separation property. In this case,we show the existence of time-dependent singular solutions and study their asymptotic behavior. In particular, we prove the convergence of solutions to the singular steady states and give exact convergence rates.

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