

## Contributed Session 10: Bifurcation and Chaotic Dynamics

### Gursey instantons under the quantum fluctuation in phase space

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Gursey Model which has been assumed in 1956 as a possible basis for a unitary description of elementary particles is only possible four dimensional conformally invariant nonlinear pure spinor model. Recently, the behaviors of Gursey instantons in phase space were investigated depending on coupling constant. In this presentation, we shall investigate the behaviors of Gursey instantons in phase space under the quantum fluctuation to provide the better understanding quantum dynamics of spinor type instantons in vacuum.

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### Numerical normal forms for limit cycles

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**W. Govaerts, Yu. A. Kuznetsov, F. Della Rossa, H.G.E. Meijer**

In this talk we concentrate on the computation of the normal form coefficients for codim 2 bifurcations of limit cycles. Our formulation is independent of the dimension of the phase space and involves solutions of certain linear boundary value problems. In the Matlab numerical bifurcation software MatCont these BVPs are discretized by orthogonal collocation with piecewise polynomials. A great advantage of our method is that we are able to avoid the computation of Poincaré maps and their derivatives. The formulas allow us to distinguish between various bifurcation scenarios near codim 2 bifurcations of limit cycles. We discuss the theoretical derivation of the normal form coefficients, their implementation in MatCont and their practical use by means of examples.

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### The study of the standard families of unfoldings of the nilpotent saddle of codimension 2 and 3 whose x-axis is invariant.

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In this talk we study the standard families of unfoldings of the nilpotent saddle of codimension 2 and

3 whose x-axis is invariant. The goal of our study is to make a systematic analysis of the bifurcations of these models (which leads to the determination of their global bifurcation diagram): saddle-node bifurcations, Hopf bifurcation of codimension 1 or 2, heteroclinic bifurcation of codimension 1 or 2.

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### Bifurcations of piecewise smooth flows: perspectives, methodologies and open problems

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The theory of bifurcations in piecewise smooth flows is critically surveyed. The focus is on results that hold in arbitrarily (but finitely) many dimensions, highlighting significant areas where a detailed understanding is presently lacking. The clearest results to date concern equilibria undergoing bifurcations at switching boundaries and limit cycles undergoing grazing and sliding bifurcations. After discussing fundamental concepts such as topological equivalence of two piecewise smooth systems, discontinuity-induced bifurcations are defined for equilibria and limit cycles. Conditions for equilibria to exist in  $n$ -dimensions are given, followed by the conditions under which they generically undergo codimension-one bifurcations. The extent of knowledge of their unfoldings is also summarized. Codimension-one bifurcations of limit cycles and boundary-intersection crossing are described together with techniques for their classification. Codimension-two bifurcations are discussed with suggestions for further study.

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### Multi-state and multi-stage synchronization of Hindmarsh-Rose neurons with excitatory chemical and electrical synapses

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**Yu-Hao Liang**

The new phenomena of the multi-state synchronization of Hindmarsh-Rose(HR) neurons with both excitatory chemical and electrical synapses over the complex network are analytically studied. The regions for coupling strengths to achieve local synchronization are explicitly obtained. Such regions are characterized by the second largest eigenvalue  $\lambda_2$  of the electrical connection matrix and the number  $k$  of chemical signals each neuron receives. The dynamics

of the multi-state synchronization includes the coexistence of stable regular bursting and periodic/steady-state behaviors. These are in contrast with coupled oscillator systems or coupled map lattices where only single-state synchronization is found. It should also be noted that if the parameters of HR neurons are chosen resulting in an irregular (chaotic) bursting, then the coexistence state would contain chaotic attractor. Our method employed here is quite general. For instance, it can be immediately applied to other coupled nervous systems such as FitzHugh-Nagumo and Morris-Lecar nervous systems. The analytical tools and concepts needed include coordinate transformations, matrix measures, monotone dynamics and time averaging estimates.

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### Bifurcation and asymptotic analysis of a coupled gyroscope system

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A critical component of many navigation systems are the gyroscopes, a device used for detecting rotation rates and orientation. Gyroscopes are subject to material imperfections and manufacturing defects, potentially reducing their ability to detect signals and to minimize phase drift. This work explores the idea of performance enhancement by combining the input-output response of an ensemble of gyroscopes coupled in a directed ring fashion. Bifurcation and asymptotic analysis show the existence of a wide range of behaviors, including: chaos, quasi-periodicity, and of particular interest, a stable synchronization state in which all gyroscopes vibrate with the same phase and amplitude. In this state, the sum output of the sensing modes is significantly larger than any individual sensing mode, which leads to substantial reductions in phase drift caused by material imperfections, fabrication variations, and system noise.

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### A fold-Hopf-like bifurcation in piecewise linear continuous differential systems with symmetry

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**Enrique Ponce, Elisabet Vela**

Three-dimensional symmetric piecewise linear differential systems near the conditions corresponding to the fold-Hopf bifurcation are considered. By introducing one small parameter, we study the bifurcation of limit cycles in passing through its critical value, that is, when the three eigenvalues of the linear part

at the origin are at the imaginary axis of the complex plane. The simultaneous bifurcation of three limit cycles is proved. Conditions for stability of these limit cycles are provided, and analytical expressions for their period and amplitude are obtained. Finally, we apply the theoretical results of this work to a generalized version of Chua's circuit, showing that the fold Hopf bifurcation takes place for a wide range of the parameters.

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### Focus-center-limit cycle bifurcation in discontinuous planar piecewise linear systems without sliding

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A family of planar discontinuous piecewise linear systems with two linearity zones is considered. By using some changes of variables and parameters, a Liénard canonical form is obtained. This canonical form has seven parameters. In the particular case analyzed with focus-focus dynamic and without sliding, a reduced canonical form with only four parameters is obtained. Under certain hypotheses the existence of a limit cycle bifurcation is assured. Analytic expressions for the amplitude, period and characteristic multiplier of the bifurcating limit cycle are provided. To illustrate the appearance of this bifurcation in real world applications, a Wien bridge oscillator without symmetry is analyzed using the provided theoretical results.

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### Diffusions in chaotic billiards

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Lorentz gas is a simple model of diffusive billiards in studying the transport process of electrons in an ionized metal. Since the key aim in statistical mechanics is to characterize the diffusion matrix that appear in the CLT. We study 2-d periodic Lorentz gas in the presence of a twist force on the scatterers. In this system, particle orbits are still geodesics between collisions, but do not reflect elastically when reaching the boundary. When the horizon is finite, i.e. the free flights between collisions are bounded, the resulting current  $J$  is proportional to the strength of the twist force. We also prove the existence of a unique SRB measure, for which the Pesin entropy formula and Young's expression for the fractal dimension are valid. The classical CLT is verified and the diffusion matrix is calculated according to a formula involves the twist function.

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