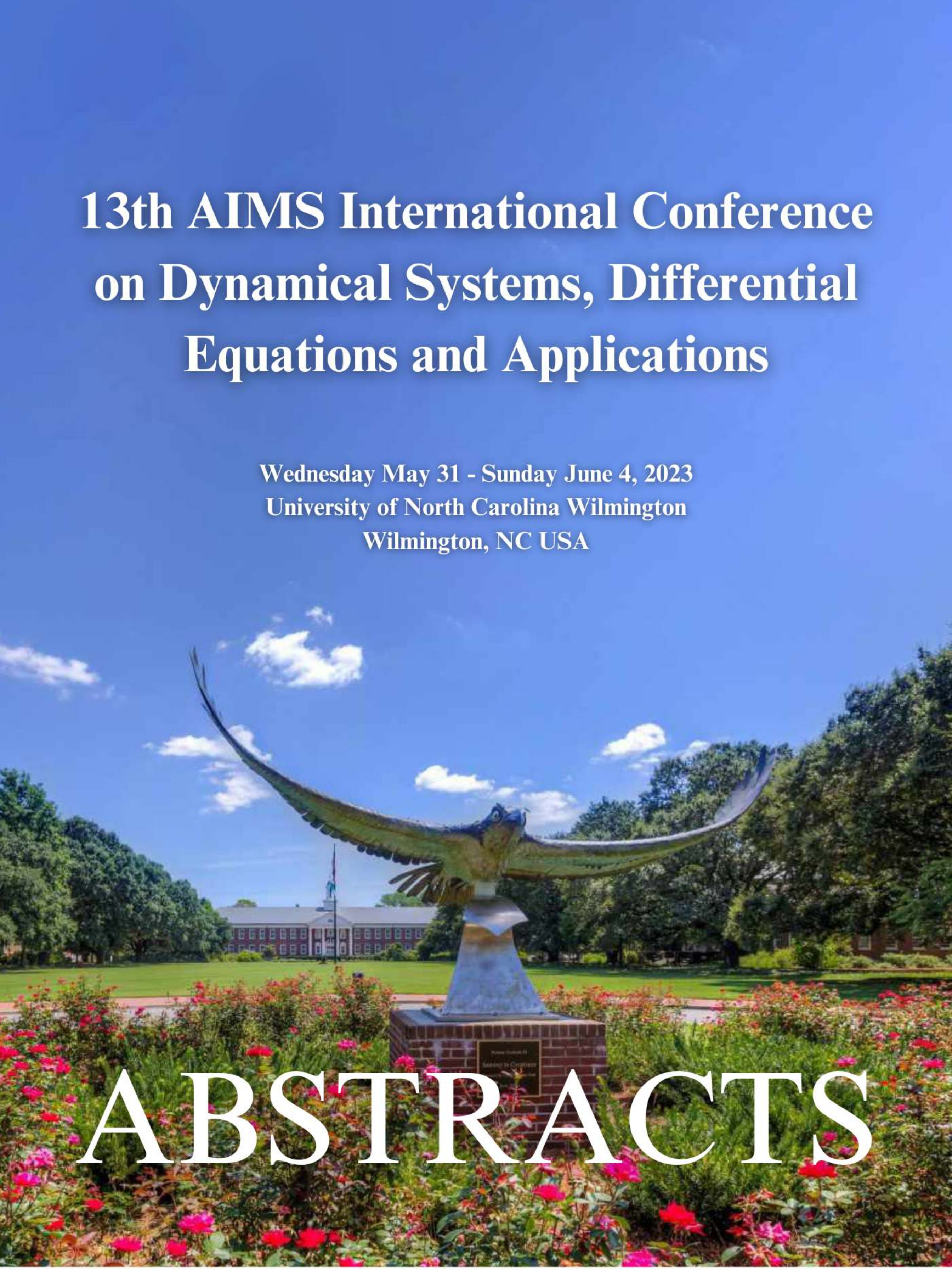
13th AIMS International Conference on Dynamical Systems, Differential Equations and Applications

Wednesday May 31 - Sunday June 4, 2023
University of North Carolina Wilmington
Wilmington, NC USA



ABSTRACTS

Department of Mathematics & Statistics
University of North Carolina Wilmington

American Institute of
Mathematical Sciences

The 13th AIMS Conference on Dynamical Systems, Differential Equations And Applications

May 31 – June 4, 2023
Wilmington, NC USA

ABSTRACTS

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The University of North Carolina Wilmington, USA

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Contributed Sessions

Invited Plenary Lectures

Tristan Buckmaster

University of Maryland, USA

Tristan Buckmaster is a Professor of Mathematics at the University of Maryland. He completed his Ph.D. at the University of Leipzig/Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany in 2014. He then spent three years as a Courant Instructor at New York University. He was appointed an Assistant Professor at Princeton University in 2017 and appointed a Professor at the University of Maryland in 2022. His main area of interest is partial differential equations with a particular focus on equations related to hydrodynamics. He was awarded the 2019 Clay Research Award, together with Vlad Vicol for their work regarding the non-uniqueness of weak solutions to the Navier-Stokes equations.

Singularities in Fluids

Abstract
In this presentation, I will delve into recent research pertaining to fluid singularities. Specifically, I will explore topics such as self-similar analysis, computer-assisted proofs, and physics-informed neural networks. The existence of singularities raises intriguing new questions that I will also address.

Maria Colombo

École Polytechnique Fédérale de Lausanna, Switzerland

Maria Colombo is an Italian mathematician who is active in the fields of calculus of variations, optimal transport and partial differential equations in fluid dynamics. She graduated in Scuola Normale Superiore in 2015 and, after a postdoc at ETH Zuerich, she is currently professor at the École polytechnique fédérale de Lausanne (EPFL). She received in 2022 the Peter Lax award and in 2023 the Collatz Prize of the International Council for Industrial and Applied Mathematics. Maria Colombo's contributions regard the regularity and the analysis of singularities in elliptic PDEs, geometric variational problems, transport equations, and incompressible fluid dynamics. One of her most recent contributons, in collaboration with Albritton and Brué, rigorously established non- uniqueness of Leray-Hopf solutions of the forced Navier-Stokes equations.

Instability and non-uniqueness for the Euler and Navier-Stokes equations

Abstract
The incompressible Navier-Stokes and Euler equations are fundamental PDEs in mathematical fluid dynamics and their well-posedness theory is nowadays largely open. The past decade has seen a surprising and remarkable progress, through various different attempts, in describing some non-unique solutions of these PDEs. The talk will survey some of the recent contributions in this direction, including works in collaboration with Albritton and Brué which show that Leray-Hopf solutions of the forced Navier-Stokes equations are not unique.

Ingrid Daubechies

Duke University, USA

Ingrid Daubechies has a Ph.D. in Theoretical Physics. She has spent most of her career on bringing mathematical techniques and analysis to bear on applications not only from physics, but also in signal processing, brain imaging, geophysics, biological morphology, and art conservation and analysis, as well as on research in mathematics. She is presently a professor at Duke University. During the isolation caused by the pandemic, she has spent part of her time on the realization, together with 23 other mathematicians and artists, of an art installation that seeks to communicate the wonder, the beauty and the whimsy of mathematics – see mathemalchemy.org

Computational Methods to Study Morphological Shapes

Abstract

The presentation will review work of a collaboration of applied mathematicians and biological morphologists, with some recent results.

Arnaud Debussche

École Normale Supérieure de Rennes, France

Arnaud Debussche is a professor of mathematics at the Ecole Normale Supérieure de Rennes. He received his Ph.D. from the University of Paris-Sud in 1989 and his “Habitation à diriger des recherches” in 1994. He was a researcher at CNRS from 1990 to 2000 and has been a professor since 2000. He was director of the mathematics department from 2005 to 2012, director of the Henri Lebesgue Center from its creation in 2012 to 2019, and vice-president in charge of research from 2015 to 2021. He is a specialist in stochastic partial differential equations and has obtained pioneering results in the study of singular SPDEs, dispersive SPDEs, numerical analysis and, more recently, the derivation of SPDEs by diffusion-approximation. He was a junior member of the Institut Universitaire de France from 2006 to 2011 and is currently a senior member. He held an Eisenbud Professorship at the Mathematical Sciences Research Institute in Berkeley in Fall 2015.

Multiscale Stochastic Fluid Models

Abstract

Recently, fluid models with transport noise have received a lot of attention. They are used in the modeling of turbulence for example. Transport noise is also known to have an important effect on the qualitative behavior of solutions of partial differential equations. In this talk, I will explain how fluid models with transport noise can be obtained rigorously from multiscale models. The main tool is the classical perturbed test function method, appropriately generalized to an infinite dimensional setting. We also recover other terms appearing in turbulence modeling, such as the Itô-Stokes drift.

Rafael de la Llave

Georgia Institute of Technology, USA

Rafael de la Llave received his undergraduate in Physics from Univ. Complutense in 1979. He obtained a Fulbright fellowship to obtain a Ph. D. in Mathematics from Princeton Univ. in 1983. He spent a semester in IMA (Minneapolis). After a postdoctoral position in IHES he returned to Princeton in 1984 as an assist prof. He moved to Univ of Texas at Austin in 1989 and to Georgia Inst. of Technology in 2010, where he is currently. He recieved a Centennial Fellowship of AMS and is a Fellow of the institute of Physics. He has supervised 26 Ph. D. thesis and has been a co-organizer of 6 special semester in several institutes.

How to Get Lots of Energy with Small Effort

Abstract

In systems with no friction (called Hamiltonian), the effects of external forces can sometimes accumulate and some times they average out. In systems with one particle in one dimension whether averaging takes place or not depends on whether the external frequency is an integer multiple or not of natural frequencies. In higher dimensional systems, whether there is accumulation or not depends on number theoretic properties of the combination of frequencies. Two powerful theorems from the 60's (KAM and Nekhoroshev) showed that, in many systems, averaging happened for many initial conditions and for a long time. However, an example of V. I. Arnold in 1964 showed that accumulation was still possible. This raised the mathematical question of how often it happened. Also the practical question of designing machines having accumulation or suppresing it according to whether it is desirable or not. Given the mathematical interest and the practical importance, the problem has been pursued by many people with many points of view and different tools. We will review some recent progress in a program to identify landmarks which organize the dynamics of many trajectories and lead to the accumulation of energy. At the end, one gets tools to analyze the presence of these behaviors in rather general systems or to construct systems with the desired properties as usual in engineering. Some recent developments by many people involve the theory of systems with some friction, the relations with control theory, the theory of generic properties (Baire sense) and the applications concrete models in celestial mechanics or in PDE's.

Manuel del Pino

University of Bath, UK

Manuel del Pino is a Chilean mathematician, specialist in asymptotic patterns in nonlinear elliptic and parabolic PDEs. After a degree in mathematical engineering at Universidad de Chile in 1988, he obtained his Ph.D. at the University of Minnesota in 1992, under the direction of Wei-Ming Ni. After postdoctoral positions at the Institute of Advanced Study and the University of Chicago, he became a faculty member at Universidad de Chile, a professor in 2002. In 2010 he was a speaker at the ICM Congress in Hyderabad and became a member of the Chilean Academy of Sciences. In 2018 he became a professor at the University of Bath and was awarded a University Research Professorship by The Royal Society. Among his main contributions are a counterexample to De Giorgi's conjecture in large dimensions and the construction of solutions with prescribed blow-up points in a planar domain for the harmonic map flow into the sphere. More recently, the construction of solutions with highly concentrated vorticity in incompressible Euler flows mathematically justifying the leapfrogging phenomenon for vortex rings observed by Helmholtz in 1858.

Solutions with Highly Concentrated Vorticity in Incompressible Euler Flows

Abstract

A classical problem that traces back to Helmholtz and Kirchhoff is the understanding of the dynamics of solutions to the Euler equations of an inviscid incompressible fluid when the vorticity of the solution is initially concentrated near isolated points in 2d or vortex lines in 3d. We discuss some recent results on these solutions' existence and asymptotic behaviour. We prove 1858's conjecture by Helmholtz on vortex ring leapfrogging dynamics. This is research in collaboration with J. Dávila, A. Fernández, M. Musso, S. Parmeshwar and J. Wei.

Thomas Yizhao Hou

California Institute of Technology, USA

Thomas Yizhao Hou is the Charles Lee Powell professor of applied and computational mathematics at Caltech. His research interests include singularity of 3D Euler and Navier-Stokes equations, multiscale problems and data analysis. He received his Ph.D. from UCLA in 1987, became a postdoc at Courant institute in 1987 and joined the Courant Institute as a junior faculty member in 1989. He moved to Caltech in 1993 and was named the Charles Lee Powell Professor in 2004. Dr. Hou has received a number of honors and awards, including a member of American Academy of Arts and Sciences in 2011, a member of the inaugural class of SIAM Fellows in 2009 and AMS Fellows in 2012, the Computational and Applied Sciences Award from USACM in 2005, the Morningside Gold Medal in Applied Mathematics in 2004, the SIAM Wilkinson Prize in Numerical Analysis and Scientific Computing in 2001, the Frenkiel Award from the Division of Fluid Mechanics of APS in 1998, the Feng Kang Prize in Scientific Computing in 1997, a Sloan fellow from 1990 to 1992. He was also the founding Editor-in-Chief of the SIAM Journal on Multiscale Modeling and Simulation from 2002 to 2007.

Stable and Nearly Self-Similar Blowup on the 3D Euler Incompressible Equations with Smooth Data

Abstract
Whether the 3D incompressible Euler equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in nonlinear PDEs. In this talk, I will present a recent result with Dr. Jiajie Chen in which we prove finite time blowup of the 2D Boussinesq and 3D Euler equations with smooth initial data. There are several essential difficulties in establishing such blowup result. We overcome these difficulties by decomposing the solution operator into a leading order operator that enjoys sharp stability estimates plus a finite rank perturbation operator that can be estimated by using energy estimates and space-time numerical solutions with rigorous error control. This enables us to establish nonlinear stability of the approximate self-similar profile and prove stable nearly self-similar blowup of the 2D Boussinesq and 3D Euler equations with smooth initial data and boundary. This provides the first rigorous justification of the Hou-Luo blowup scenario.

Benedetto Piccoli

Rutgers University - Camden, USA

Benedetto Piccoli is Distinguished Professor and the Joseph and Loretta Lopez Chair Professor of Mathematics at Rutgers University - Camden. He also serves as Vice Chancellor for Research. His research interests span various areas of applied mathematics, including control theory, traffic flow on networks, crowd dynamics, math finance and application to autonomous driving, population health and bio-medical systems. He is author of more than 300 research papers and 7 books and is the founding editor of Networks and Heterogeneous Media. Piccoli is the recipient of the 2009 Fubini Prize, Plenary speaker at ICIAM 2011, and 2012 inaugural Fellow of American Mathematical Society.

New Approaches for the Modeling and Control of Multi-Agent Systems

Abstract
We review some recent results on the modeling and control of large systems of agents, with particular focus on applications to social systems (as opinion dynamics), pedestrians' movements (also called crowd dynamics), and vehicular traffic. There is a natural parallel to gas and fluid dynamics, but also profound differences, for example agents inject energy into the system, thus preventing the typical conservation of momentum and energy. Similarly, the control problems posed by such systems are new and require innovative methods, such as sparse controls, bounded variation controls, mean-field limit, and defining new control problems for measures. After reviewing several approaches, we discuss some future research directions of potential interest. We conclude by illustrating a recent real-life experiment using autonomous vehicles on an open highway to smooth traffic waves. This opens the door to a new era of interventions to control in real-time multi-agent systems and to increase the societal impact of such interventions guided by control research.

Carola-Bibiane Schönlieb

University of Cambridge, UK

Carola-Bibiane Schönlieb is Professor of Applied Mathematics at the University of Cambridge. There, she is head of the Cambridge Image Analysis group and co-Director of the EPSRC Cambridge Mathematics of Information in Healthcare Hub. Since 2011 she is a fellow of Jesus College Cambridge and since 2016 a fellow of the Alan Turing Institute, London. She also holds the Chair of the Committee for Applications and Interdisciplinary Relations (CAIR) of the EMS. Her current research interests focus on variational methods, partial differential equations and machine learning for image analysis, image processing and inverse imaging problems. She has active interdisciplinary collaborations with clinicians, biologists and physicists on biomedical imaging topics, chemical engineers and plant scientists on image sensing, as well as collaborations with artists and art conservators on digital art restoration. Her research has been acknowledged by scientific prizes, among them the LMS Whitehead Prize 2016, the Philip Leverhulme Prize in 2017, the Calderon Prize 2019, a Royal Society Wolfson fellowship in 2020, a doctorate honoris causa from the University of Klagenfurt in 2022, and by invitations to give plenary lectures at several renowned applied mathematics conferences, among them the SIAM conference on Imaging Science in 2014, the SIAM conference on Partial Differential Equations in 2015, the SIAM annual meeting in 2017, the Applied Inverse Problems Conference in 2019, the FOCM 2020 and the GAMM 2021. Carola graduated from the Institute for Mathematics, University of Salzburg (Austria) in 2004. From 2004 to 2005 she held a teaching position in Salzburg. She received her Ph.D. degree from the University of Cambridge (UK) in 2009. After one year of postdoctoral activity at the University of Göttingen (Germany), she became a Lecturer at Cambridge in 2010, promoted to Reader in 2015 and promoted to Professor in 2018.

Learned Regularisation for Solving Inverse Problems

Gunther Uhlmann

University of Washington, USA

Gunther Uhlmann received the Licenciatura in Mathematics degree from the Faculty of Sciences of the University of Chile in 1973 and the Ph.D. in 1976 from MIT under the direction of Victor Guillemin. After postdocs at Harvard, MIT and the Courant Institute, he became Assistant Professor at MIT in 1980. In 1984 he moved to the University of Washington where he currently holds the Robert R. Phelps and Elaine F. Phelps Endowed Professorship. Uhlmann has received several honors including membership in the American Academy of Arts and Sciences. He is also a foreign member of the Finnish Academy of Science and a Corresponding member of the Chilean Academy of Sciences.. In 2011 he was awarded the Bocher Memorial Prize from the AMS and the Kleinman Prize from SIAM. In 2017 he received the Lefschetz medal from the Mathematical Council of the Americas. In 2021 he was awarded the Birkhoff Prize by the AMS and SIAM. He was named Doctor Honoris Causa of the University of Helsinki in 2022.

Inverse Problem: Seeing the Unseen

Abstract
In inverse problems one attempts to find the causes of an observed effect. They arise in most areas of science and technology. In many cases the physical situation is modelled by partial differential equations. We describe some recent results on inverse problems for nonlinear equations.

Enrico Valdinoci

University of Western Australia, Australia

Enrico Valdinoci is Professor and Australian Laureate Fellow at the University of Western Australia. His research interests include nonlocal and fractional differential equations, partial differential equations, and dynamical systems.

Long-Range Phase Coexistence Models and (Non)Local Minimal Surfaces

Abstract

The classical theory of phase transition describes the coexistence of two phases separated by a thin interface. Ferromagnetic materials aim at minimizing such an interface, thus producing a classical link with the theory of surfaces with minimal perimeter. Recently, phase coexistence models based on long-range particle interactions have been considered and their interface has been linked to either surfaces of minimal perimeter or of minimal nonlocal perimeter, depending on the type of interaction under consideration. These novel objects, namely the surfaces of minimal nonlocal perimeter, present new phenomena and many open problems of fundamental type. We will discuss some recent results on the theory of long-range phase transitions from the point of view of fractional elliptic equations and present some findings related to the nonlocal minimal surfaces which describe their interfaces at a large scale.

Special Session 1: Analysis of PDEs and Free Boundary Problems

Stefano Biagi, Polytechnic of Milan, Italy
Eugenio Vecchi, Università di Bologna, Italy
Serena Dipierro, University of Western Australia, Australia

The purpose of this special session is to present some recent trends and progress in the analysis of PDEs. Our aim is to cover as much "PDEs settings" as possible, and to create profitable exchanges of ideas among these settings. For example, the contributes that we have in mind focus on:

(1) Linear PDEs of integer and fractional orders in Euclidean settings (harmonic and poly-harmonic operators, fractional Laplacian...);

(2) Semi-linear and/or fully non-linear PDEs in Euclidean settings (p-Laplacian and fractional p -Laplacian);

(3) Linear PDEs on Riemannian and sub-Riemannian manifolds;

(4) Free boundary problems.

As it regards the type of results that we expect, we are completely open-minded: existence and non-existence results; multiplicity of solutions and Gibbons-type results; maximum principles and applications to the symmetry of solutions; qualitative results such as Harnack's inequalities; regularity of the free boundary in various settings.

Hele-Shaw flow and parabolic integro-differential equations

Farhan Abedin
Lafayette College, USA
Russell Schwab

The Hele-Shaw flow is a model for the evolution of an ideal fluid occupying the narrow gap between two parallel plates and subject to an external pressure source. Assuming the fluid interface is given by the graph of a function, it can be shown that this function must solve a parabolic integro-differential equation of 1st order. I will discuss recent joint work with Russell Schwab (Michigan State University) on the regularization properties of such nonlocal parabolic equations. Our results allow us, in certain model scenarios and for short enough times, to conclude improvement-of-regularity for the fluid interface.

The nonlinear fractional relativistic Schrödinger equation

Vincenzo Ambrosio
Università Politecnica delle Marche, Italy

We consider the following class of fractional relativistic Schrödinger equations:

$$\begin{cases} (-\Delta+m^2)^s u + V(\varepsilon x)u = f(u) & \text{in } \mathbb{R}^N, \\ u \in H^s(\mathbb{R}^N), \quad u>0 & \text{in } \mathbb{R}^N, \end{cases}$$

where $\varepsilon > 0$ is a small parameter, $s \in (0,1)$, $m > 0$, $N > 2s$, $(-\Delta+m^2)^s$ is the fractional relativistic Schrödinger operator, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous potential satisfying a local condition, and $f : \mathbb{R} \to \mathbb{R}$ is a continuous subcritical nonlinearity. We first show that, for $\varepsilon > 0$ small enough, the above problem has a weak solution u_ε (with exponential decay at infinity) which concentrates around a local minimum point of V as $\varepsilon \to 0$. We also relate the number of positive solutions with the topology of the set where the potential V attains its minimum value. The main results will be established by using a penalization technique, the generalized Nehari manifold method and Ljusternik-Schnirelman theory.

Asymptotic mean-value formulas for nonlinear equations

Fernando Charro
Wayne State University, USA
Pablo Blanc, Juan J. Manfredi, Julio D. Rossi

In recent years there has been an increasing interest in whether a mean-value property, known to characterize harmonic functions, can be extended in some weak form to solutions of nonlinear equations. This question has been partially motivated by the surprising connection between Random Tug-of-War games and the normalized p-Laplacian discovered some years ago by Peres et al., where a nonlinear asymptotic mean-value property for solutions of a PDE is related to a dynamic programming principle for an appropriate game. In this talk we discuss asymptotic mean-value formulas for a class of nonlinear second-order equations that includes the classical Monge-Ampére and k-Hessian equations among other examples.

Maximum principles and related problems for a class of nonlocal extremal operators

Giulio Galise
Sapienza Università di Roma, Italy
Isabeau Birindelli, Delia Schiera

This talk is concerned with the validity of the comparison and maximum principles and their relation with principal eigenvalues, for a class of degenerate nonlinear integral operators that are extremal among operators with one-dimensional fractional diffusion.

Multiplicity and concentration results for some nonlinear Schrödinger equations with the fractional p-Laplacian

Teresa Isernia
Università Politecnica delle Marche, Italy

We consider a class of parametric Schrödinger equations driven by the fractional p-Laplacian operator and involving continuous positive potentials and nonlinearities with subcritical or critical growth. By using variational methods and Ljusternik-Schnirelmann theory, we study the existence, multiplicity and concentration of positive solutions for small values of the parameter.

Quasilinear critical equations with Hardy term on Carnot groups

Annunziata Loiudice
University of Bari, Italy

We present some recent results about quasilinear equations with critical exponent and Hardy perturbation for p-sub-Laplacians on stratified Lie groups. In particular, we provide existence and qualitative behavior, at the singularity and at infinity, for weak entire solutions and we study the associated Brezis-Nirenberg type problem on bounded domains.

On the critical p-Laplace equation

Dario Monticelli
Politecnico di Milano, Italy
Giovanni Catino, Alberto Roncoroni

In this talk we will consider the well-known generalized Lane-Emden equation $-\Delta_p u = |u|^{q-1}u$, involving the usual p-Laplace operator in the Euclidean space. I will discuss some non-existence and classification results for positive solutions of the subcritical and the critical p-Laplace equation. In particular in the critical case, using the moving planes method, it has been recently shown that positive solutions to the critical p-Laplace equation with finite energy can be completely classified. I will then present some recent results concerning the classification of positive solutions to the critical p-Laplace equation, possibly having infinite energy. If $n = 2$, or if $n = 3$ and p lies in $(3/2, 2)$, rigidity is obtained without any further assumptions. In the remaining cases the classification follows under energy growth conditions or suitable control of the solutions at infinity. These assumptions are much weaker than those already appearing in the literature. I will also discuss extensions of these results to the Riemannian setting. This is based on a recent joint work with G. Catino and A. Roncoroni (Politecnico di Milano).

The asymptotic p-Poisson equation as $p \to \infty$ in Carnot-Carathéodory spaces

Andrea Pinamonti
University of Trento, Italy
Luca Capogna, Gianmarco Giovannardi, Simone Vercellesi

In this paper we study the asymptotic behavior of solutions to the subelliptic p-Poisson equation as $p \to +\infty$ in Carnot Carathéodory spaces. In particular, introducing a suitable notion of differentiability, we prove that limits of such solutions solve in the sense of viscosity a hybrid first and second order PDE involving the $\infty-$Laplacian and the Eikonal equation.

Gauge balls and Heisenberg groups: Rigidity via horizontal curvature

Giulio Tralli
University of Padova, Italy

In this talk we consider the problem of characterizing gauge balls in the Heisenberg group by prescribing their (non-constant) horizontal mean curvature. We discuss two uniqueness results: in the lowest dimensional case under an assumption on the location of the singular set, and in higher dimensions in the proper class of horizontally umbilical hypersurfaces. This is a joint work with C. Guidi and V. Martino.

The Lévy flight foraging hypothesis

Enrico Valdinoci
University of Western Australia, Australia

Foraging theory tries to identify the most efficient strategy of a predator in different environments.
In the last years, more and more attention has been devoted to the analysis of Lévy-type patterns in animal searches.
We will present several efficiency functionals whose optimality is discussed in relation to the Lévy exponent of the corresponding evolution equation for the forager. We will also discuss whether or not the optimal Lévy exponent corresponds to a viable strategy and detect high-gain/high-risk foraging patterns.

Special Session 2: Hyperbolic Partial Differential Equations and Applications

Yachun Li, Shanghai Jiao Tong University, Peoples Republic of China
Yue-Jun Peng, Uni. of Clermont Auvergne, France; Shanghai Jiao Tong Uni., Peoples Republic of China
Ya-Guang Wang, Shanghai Jiao Tong University, Peoples Republic of China
Tong Yang, The Hong Kong Polytechnic University, Peoples Republic of China

The aim of this special session is to bring together experts in the area of partial differential equations to present their recent research results in theoretical analysis of hyperbolic PDEs and applications in some related fields like fluid dynamics. In this session, people are expected to exchange new ideas, to discuss challenging issues, to explore new directions and topics, and to foster new collaborations and connections.

Concentration phenomenon of weak solutions for compressible isentropic Navier-Stokes equations in dimensions three

Xianpeng Hu
City University of Hong Kong, Peoples Republic of China

We will discuss the concentration phenomenon of weak solutions for isentropic compressible Navier-Stokes equations. Except a closed set $\mathfrak{S}^c$ with zero parabolic Hausdorff measure, $\mathfrak{P}^{\gamma(3)}(\mathfrak{S}^c) = 0$, the weak limit $(\rho, \mathbf{u})$ of approximate solutions is a renormalized weak solution with finite energy of three dimensional compressible Navier-Stokes equations for $\gamma \in (6/5, 3/2]$ as constructed by Lions and Feireisl *et al* in the Leray sense. The key novelty is the improved integrability of pressure by localization, which is based on the faster decay of the gradient of velocity and the higher integrability of the Riesz potentials of both density and momentum.

A non-conservative and non-strictly hyperbolic diagonal system related to crystallography

Stephane Junca
LJAD, Inria & CNRS, Université Cote d'Azur, France
Maryam Al Zohbi

A $d \times d$ non-conservative and non strictly hyperbolic but hyperbolic diagonal system appearing in the theory of dislocations is studied in the framework of fractional BV spaces.
A definition of entropy solution is proposed for this non-conservative system. Existence of entropy solutions in BV^s for all 02. The infinite family of entropies for $d = 2$ is not enough to ensure uniqueness due to the loss of stric hyperbolicity. Entropy Riemann solvers that recover main properties of vanishing viscosity solutions exhibit more waves than usual, already for $d = 2$.

Non-uniqueness of weak solutions to the hypo-viscous compressible Navier-Stokes equations

Yachun Li
Shanghai Jiao Tong University, Peoples Republic of China

In this talk, I will discuss the Cauchy problem for the isentropic hypo-viscous compressible Navier-Stokes equations (CNS) under general pressure laws in all dimensions $d \geq 2$. For all hypo-viscosities $(-\Delta)^\alpha$ with $\alpha \in (0, 1)$, we prove that there exist infinitely many weak solutions with certain same initial data. This talk is the joint work with Prof. Peng Qu, Prof. Deng Zhang and Dr. Zirong Zeng.

Nonlinear instability in compressible fluids under gravity

Ronghua Pan
Georgia Institute of Technology, USA
Xulong Qin

It is known in physics that steady state of compressible fluids under the influence of uniform gravity is stable if and only if the convection is absent. In the context of incompressible fluids, such instability driven by convection is the well-known Rayleigh-Taylor instability, caused heavier fluids on top of lighter fluids. For compressible flow with heat transfer, the phenomenon is much closer to real world and more complicated. We will discuss our recent progress in these topics.

Boundary conditions for constrained hyperbolic systems of partial differential equations

Nicolae Tarfulea
Purdue University Northwest, USA

Many mathematical models in science and technology are based on first-order symmetric hyperbolic systems of differential equations whose solutions must satisfy certain constraints (e.g., in electromagnetism, magnetohydrodynamics, and general relativity). When the models are restricted to bounded domains, the problem of well-posed constraint-preserving boundary conditions arises naturally. However, for numerical solutions finding such

boundary conditions may represent just a step in the right direction. Including the constraints as dynamical variables of a larger, unconstrained system associated to the original one could provide better numerical results, as the constraints are kept under control during evolution. One of the main goals of this talk is to present this idea in the case of constrained first-order symmetric hyperbolic systems of differential equations. As an example of application, a vector-valued wave equation with the constraint that the solution be divergence free is considered. Interestingly enough, on a smooth bounded domain, the set of constraint-preserving boundary conditions for this model problem involves the geometry of the boundary.

On controllability of the incompressible MHD systems

Ya-Guang Wang
Shanghai Jiao Tong University, Peoples Republic of China
Manuel Rissel

In this talk, we shall introduce our recent study on the controllability of the initial boundary value problem for the incompressible magnetohydrodynamic systems. For the two-dimensional ideal incompressible MHD system, we obtained the global exact controllability by using the return method, and for the two and three dimensional viscous MHD systems with coupled Navier slip boundary condition, we deduced the global approximate controllability. This is a joint work with Manuel Rissel.

The global existence of solutions of the 3D Keller-Segel system with planar helical flow

Weike Wang
Shanghai Jiao Tong University, Peoples Republic of China

We introduce the planar helical flows on three dimensional torus and study the dissipation enhancement of such flows. Then use such flows as transport flows to solve the three dimensional Keller-Segel equations. The global classical solution of the three dimensional Keller-Segel is ensured for any size of the torus with arbitrarily large initial data.

Interface problem for the two-phase magnetohydrodynamic flows

Dehua Wang
University of Pittsburgh, USA
Tian Jing

The interface problem for the two immiscible incompressible viscous fluids in magnetohydrodynamics will be considered. Recent results on the solutions to this problem with surface tension in a three-dimensional bounded domain will be presented and discussed.

Global ill-posedness for a dense set of initial data to the isentropic system of gas dynamics

Cheng Yu
University of Florida, USA
Ming Chen, Alexis Vasseur

In dimension 2 or 3, we show that for any initial datum belonging to a dense subset of the energy space, there exist infinitely many global-in-time admissible weak solutions to the isentropic Euler system. This result can be regarded as a compressible counterpart of the one obtained by Szekelyhidi-Wiedemann (ARMA, 2012) for incompressible flows. Similarly to the incompressible result, the admissibility condition is defined in its integral form. Our result is based on a generalization of a key step of the convex integration procedure. This generalization allows, even in the compressible case, to convex integrate any smooth positive Reynolds stress. A large family of subsolutions can then be considered. These subsolutions can be generated, for instance, via regularization of any weak inviscid limit of an associated compressible Navier-Stokes system with degenerate viscosities.

Non-uniqueness of weak solutions to (stochastic) MHD equations

Deng Zhang
Shanghai Jiao Tong University, Peoples Republic of China

In this talk, we will show the non-uniqueness of weak solutions to 3D MHD equations, where the viscosity and resistivity can be larger than the Lions exponent. The non-uniqueness is sharp near one endpoint of the Ladyženskaja-Prodi-Serrin condition. Moreover, the constructed weak solutions admit the partial regularity outside a small fractal singular set in time with small Hausdorff dimension. At last, we will also present the non-uniqueness of probabilistic strong and analytic weak solutions in the stochastic setting.

Special Session 3: Dynamics of ODES and Nonlinear Parabolic Systems

Julian Lopez-Gomez, Complutense University of Madrid, Spain
Fabio Zanolin, University of Udine, Italy

Nonlinear parabolic equations model a huge variety of real-world models in all fields of science, technology and environmental and social sciences. In this session we plan to gather some of the world-leading experts in the dynamical behaviour of these prototypes.

Positive solutions of elliptic eigenvalue problems and heterogeneous logistic problems with glued Dirichlet-Neumann mixed boundary conditions

Santiago Cano-Casanova
Universidad Pontificia Comillas, Spain

In this talk we will deal with principal eigenvalues of elliptic boundary value problems with glued Dirichlet-Robin mixed boundary conditions, and we will see its application to the analysis of the existence of positive solutions of heterogeneous elliptic logistic problems with this kind of boundary conditions. Also, we will analyze the asymptotic behavior of the positive solutions of a very general class of heterogeneous logistic problems with nonlinear mixed boundary conditions, when the nonlinear term on the boundary condition blows up in the region where it does not vanish.

Resource matching in spatial ecology and evolutionary advantage

Robert Stephen Cantrell
University of Miami, USA

A convergence of concepts from game theory (evolutionary stable strategy), ecological theory (the ideal free distribution), and mathematics (line sum-symmetry and its functional analytic generalizations) combine to explain how resource matching in spatially heterogeneous but temporally constant habitats can convey evolutionary advantage robustly across a range of modeling formats. The ideal free distribution (IFD) is an ecological construct that posits that when organisms have full knowledge of the landscape they inhabit (ideal) and are able to locate themselves as they wish (free), they will locate themselves to maximize reproductive fitness. The IFD can readily be translated into mathematical terms for models wherein the environment is spatially varying but temporally constant. In this talk we will discuss how this is done across a range of modeling formats and how it consequently leads to predictions of evolutionary advantage in such modeling formats. We then report on ongoing efforts to define the ideal free distribution mathematically in cases when the environment varies in both space and time, focusing on the case wherein habitats vary periodically in time.

Nodal solutions of a paradigmatic class of semilinear 1-D BVP's

Pablo Cubillos
Universidad Complutense de Madrid, Spain
J. López-Gómez, A. Tellini

This talk deals with the analysis of 1-node solutions of the diffusive logistic equation. We first present the numerical simulations carried out in a recent paper concerning these solutions for a class of non-degenerate, or classical, diffusive logistic equation. Then, we present a substantial refinement of a multiplicity result of J. Lòpez-Gòmez and P. H. Rabinowitz. According to it, we can construct diffusive classical logistic equations with an arbitrarily large number of 1-node solutions for the appropriate ranges of values of the parameters. It is the first multiplicity result of this nature available in the literature. Finally, we describe numerically the global structure of this set of solutions for the degenerate case. Our main findings reveal that the number of such nodal solutions, as well as the number of components in the bifurcation diagrams, strongly depends on the number and position of the set components where the weight function in front of the nonlinearity vanishes. No technical device seems to be available to deal analytically with these problems.

Nodal solutions for a class of degenerate BVPs

J. López-Gómez
Complutense University of Madrid, Spain
P. H. Rabinowitz

In this talk I will characterize the existence of nodal solutions for a class of one-dimensional BVPs. The maximum principle and the topological degree will play a pivotal role in this characterization.

A spatially heterogeneous predator-prey model

Eduardo Munoz-Hernandez
Complutense University of Madrid, Spain
Julian López-Gómez

In this talk we will analyze the existence, uniqueness and multiplicity of coexistence states for the generalized spatially heterogeneous predator-prey model

$$(1)\quad \begin{cases} \mathfrak{L}_1 u = \lambda u - a(x)u^2 - b(x)\dfrac{uv}{1+\gamma m(x)u} & \text{in } \Omega, \\ \mathfrak{L}_2 v = \mu v + c(x)\dfrac{uv}{1+\gamma m(x)u} - d(x)v^2 & \text{in } \Omega, \\ \mathfrak{B}_1 u = \mathfrak{B}_2 v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\mathfrak{L}_1$ and $\mathfrak{L}_2$ are second order uniformly elliptic operators, and $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are general boundary operators of mixed type. In (1),

$$a > 0,\ d > 0,\ b \gneq 0,\ c \gneq 0,\ \gamma > 0 \text{ and } m \geq 0 \quad \text{in } \bar{\Omega},$$

while $\lambda, \mu \in \mathbb{R}$ are regarded as bifurcation parameters. The term $m(x)$ measures the level of saturation of the predator at any particular location $x \in \Omega$ where $m(x) > 0$, while saturation effects do not play any role if $m(x) = 0$. Thus, (1) is an homotopy in m which combines, within the same territory, the classical interactions of Lotka–Volterra type in the region $m^{-1}(0)$ with the Holling-Tanner functional responses in $\{x \in \Omega : m(x) > 0\}$.
During the talk, they will be ascertained the regions in the plane (λ, μ) in which coexistence states exist or could exist and the conditions for which model (1) in its one-dimensional counterpart has uniqueness of coexistence states. Then, after a comprehensive analysis of a shadow system appearing when $\gamma \uparrow +\infty$, it will be provided a generic multiplicity result ensuring the existence of, at least, two coexistence states of (1) for γ large enough.

Bifurcation theory for analytic operators

Juan Carlos Sampedro
Universidad Complutense de Madrid, Spain

In this talk we present some new results in bifurcation theory for analytic nonlinearities obtained by J. López-Gómez and the speaker. In essence, we use a number of tools from analytic geometry to determine the sharp local structure of the solution set of analytic nonlinear equations between Banach spaces. Moreover, in the context of global bifurcation theory, we generalize N. Dancer's bifurcation theorem to cover the general degenerate case, i.e., when the algebraic multiplicity of the eigenvalue is arbitrary and not necessarily equal to one. This is a breakthrough because in the case where the algebraic multiplicity is not one, the celebrated Crandall-Rabinowitz Theorem does not apply. We conclude the talk by applying these results to certain types of semi-linear elliptic equations, describing the exact topological behavior of the set of positive and negative solutions.

Wavefront analysis for reactive-convective Perona-Malik equations

Elisa Sovrano
University of Modena and Reggio Emilia, Italy
A. Corli, L. Malaguti

In this talk, we investigate a class of reaction-convection-diffusion equations, where the diffusion is driven by a nonlinear, bounded, and non-monotone function of the gradient that tends to zero at infinity. Our focus is on a Perona-Malik type operator, which is a paradigmatic example of this type of diffusion in image processing. We provide a comprehensive analysis of regular monotone wavefronts that connect two steady states when the reaction term is monostable, in terms of their wave speed. Our results reveal that the admissible speeds for these wavefronts form a closed half-line. Although the threshold speed cannot be computed explicitly, we provide an estimate for it. Furthermore, we show that these wavefronts are strictly monotone, and their slope is bounded by the critical values of diffusion. This research is based on joint work with A. Corli (University of Ferrara, Italy) and L. Malaguti (University of Modena and Reggio Emilia).

Blowup rate near the boundary of boundary blowup solutions to k-Hessian equation and k-curvature equation

Kazuhiro Takimoto
Hiroshima University, Japan

We are concerned with the boundary blowup problem for the so-called k-Hessian equation of the form $F_k[u] = f(x)g(u)$ in a uniformly $(k-1)$-convex bounded domain $\Omega \subset \mathbb{R}^n$, where $f(x)$ behaves like $\operatorname{dist}(x, \partial\Omega)^\alpha$ as $\operatorname{dist}(x, \partial\Omega) \to 0$ and $g(u)$ behaves like u^p as $u \to \infty$. We establish the blowup rate of a solution near the boundary $\partial\Omega$. Also, we consider the boundary blowup problem for the so-called k-curvature equation of the form $H_k[u] = g(u)h(|Du|)$ in a uniformly k-convex (or uniformly convex) bounded domain $\Omega \subset \mathbb{R}^n$, where $g(u)$ behaves like u^p as $u \to \infty$ and $h(s)$ behaves like s^{-q} as $s \to \infty$, and establish the blowup rate of a solution near $\partial\Omega$.

On the multiplicity of positive even solutions to the one-dimensional Hénon type equation on some very narrow possible parameter set

Satoshi Tanaka
Tohoku University, Japan

The boundary value problem $u'' + (|x|^l + \lambda)u^p = 0$ for $x \in (-1, 1)$, $u(-1) = u(1) = 0$ is considered, where $l \geq 0$, $\lambda \geq 0$ and $p > 1$. This problem always has a positive even solution. At first, the uniqueness of positive even solutions is proved on the majority part of $(l, \lambda) \in [0, \infty) \times [0, \infty)$ for fixed $p > 1$. Then a very narrow set remains as the possible region for which the problem has at least two positive even solutions. Therefore it is natural to expect that the uniqueness of positive even solutions holds for each $(l, \lambda) \in [0, \infty) \times [0, \infty)$.
Contrary to this expectation, it is shown that there exists at least three positive even solutions with the aid of numerical verification method. This is a joint work with Naoki Shioji (Yokohama National University) and Kohtaro Watanabe (National Defense Academy).

On the number of steady-states for an indefinite problem arising in population dynamics

Andrea Tellini
Universidad Politécnica de Madrid, Spain
Guglielmo Feltrin, Elisa Sovrano

We consider a one-dimensional superlinear indefinite problem with a piecewise constant weight. Due to the shape of the nonlinearity, this kind of problems can be used to describe the spatial distribution of the steady-states in population genetics models.
We study the number of positive steady-states, which depends on the range of the parameters of the problem. We obtain results on the multiplicity of positive solutions that are sharper than those of previous works in the literature, e.g., P.H. Rabinowitz (Indiana Univ. Math. J., 1973/74), Y. Lou, W.-M. Ni and L. Su (DCDS, 2010), K. Nakashima, W.-M. Ni and L. Su (DCDS, 2010), and G. Feltrin and E. Sovrano (Nonlinearity, 2018).
This is a joint work with G. Feltrin (Univ. Udine, Italy) and E. Sovrano (U. Modena and Reggio Emilia, Italy).

An exact multiplicity result for some sublinear Robin problem with an indefinite weight

Kenichiro Umezu
Ibaraki University, Japan
Uriel Kaufmann, Humberto Ramos Quoirin

In this talk, we consider positive solutions of the sublinear elliptic equation $-\Delta u = a(x)u^q$ in a smooth bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 1$, under the Robin type boundary condition $\frac{\partial u}{\partial \nu} = \alpha u$ on $\partial\Omega$, where $q \in (0,1)$, $\alpha \geq 0$, and $a \in C^\theta(\overline{\Omega})$, $\theta \in (0,1)$, changes sign in $\overline{\Omega}$. We are interested in positive solutions belonging to $\mathcal{P}^\circ = \{u \in C(\overline{\Omega}) : u > 0 \text{ in } \overline{\Omega}\}$. First, we observe that if this problem, (P_α), has a positive solution, then $\int_\Omega a < 0$. Next, in some case of a and q, we show the existence of $\alpha_s > 0$ such that (P_α) has a unique positive solution in $\mathcal{P}^\circ$ for $\alpha = 0, \alpha_s$, exactly two (ordered) positive solutions in $\mathcal{P}^\circ$ for every $\alpha \in (0, \alpha_s)$, and no positive solutions in $\mathcal{P}^\circ$ for any $\alpha > \alpha_s$. We shall discuss it in the case when $q \simeq 1$ and $\int_\Omega a \simeq 0$.

Special Session 4: Qualitative and Quantitative Features of Delay Differential Equations and Their Applications

Fathalla Rihan, United Arab Emirates University, United Arab Emirates
Yang Kuang, Arizona State University, USA
Gennady Bocharov, Russian Academy of Sciences, Russia

In recent years, delay differential equations (DDEs) have received considerable attention for their ability to model many real-life problems, which are traditionally formulated using ordinary differential equations (ODEs), more accurately and naturally. In population dynamics, epidemiology, immunology, physiology, neural networks, and other biological and physical systems, such a class of DDEs is widely used. It is intrinsic to most biological and engineering systems that there are time delays or lags. As a consequence, differential equations with memory, represented by time delays (time lags), are more advantageous than models without memory. Adding memory to differential equations improves their stability and enriches their dynamics. It is the aim of this special session to explore the new trends and analytical insights of delay differential equations with biological systems, including the existence and uniqueness of solutions, their boundedness and persistence, their oscillatory behavior, their stability and bifurcation analyses, their parameter estimations and sensitivity analyses, and their numerical analysis.

Time-delayed two-Strain epidemic model with general incidence rates and therapy: Mathematical analysis

Karam Allali
Hassan II University of Casablanca, Morocco

In this work, we are interested in a time-delayed epidemic model with general incidence rates and therapy. The mathematical model contains four compartments that represent the susceptible, the first strain infected, the second strain infected and the recovered individuals. The time delays represent the needed time during the period of infection incubation for each strain. The wellposedness of the tackled model will be established in terms of proving the results of existence, positivity and boundedness. The global stability of the disease-free equilibrium, the first strain endemic equilibrium, the second strain equilibrium and the both strains endemic equilibrium is fulfilled. It was remarked that the global stability of the equilibria depends mainly on the each strain basic reproduction number. Numerical tests are performed to show the stability of the equilibria for two different incidence functions. In our numerical tests, we will restrict ourselves to only two cases of incidence functions, namely, two bilinear incidence functions and two non-monotonic incidence rates. It was concluded that therapy efficiency plays an essential role in reducing the infection severity. To control the spread of the infection in a two strain environment, it would be important to act on both strains treatment efficiencies.

Persistence and extinction for stochastic delay differential model of prey predator system with hunting cooperation in predators

Hebatallah Alsakaji
United Arab Emirates University, United Arab Emirates
F.A. Rihan

Stochastic differential models provide an additional degree of realism compared to their corresponding deterministic counterparts because of the randomness and stochasticity of real life. In this work, we study the dynamics of a stochastic delay differential model for prey-predator system with hunting cooperation in predators. Existence and uniqueness of global positive solution are investigated. Some sufficient conditions for persistence and extinction, using Lyapunov functional, are obtained. Illustrative examples and numerical simulations, using Milstein's scheme, are carried out to validate our analytical findings. It is observed that a small scale of white noise can promote the survival of both species; While large noises can lead to extinction to the predator population.

On stability and asymptotics of equations and systems with distributed unbounded delays

Elena Braverman
University of Calgary, Canada
Leonid Berezansky

Many differential equations of mathematical biology assume delayed production process and the instantaneous mortality. Introduction of delay can destabilize the unique positive equilibrium and even lead to chaos. However, for some types of equations and systems, lags in the reproduction term do not change stability properties. Consideration of variable, unbounded and distributed delays emphasizes robustness of this 'absolute stability' property. Influence of an infinite, not just unbounded, delay is also outlined.

Delay differential equations with dynamics and applications in biology

Fathalla Rihan
UAE University, United Arab Emirates
Bassel F. Rihan

The impact of infectious diseases on human and animal is enormous, both in terms of suffering and social and economic consequences. In this talk, we provide some mathematical models using delay differential equations in immunology, physiology, epidemiology, tumour-immune interactions in presence of treatments and optimal control. We investigate qualitative and quantitative features of such class of differential equations in dynamic diseases. We also study parameter estimations and sensitivity analysis of such models. Sensitivity analysis is an important tool for understanding a particular model, which is considered as an issue of stability with respect to structural perturbations in the model. Some numerical simulations are provided to show the consistency of delay differential equations with biological systems with memory.

R_0 and sensitivity analysis of a predator-prey model with seasonality and maturation delay

Xiunan Wang
University of Tennessee at Chattanooga, USA
Hao Wang, Michael Y. Li

Coexistence and seasonal fluctuations of predator and prey populations are common and well documented in ecology. In this talk, I will present a new predator-prey model that incorporates seasonality and predator maturation delay simultaneously. Both seasonality and time delay have been known as the main culprits for driving population cycles in addition to predator-prey interactions. We theoretically obtain the threshold result in terms of R_0 for the coexistence of predator and prey populations in a seasonally changing environment. We numerically explore the roles of seasonality, *Daphnia* maturation delay and *Daphnia*-algae interaction in determining seasonal *Daphnia*-algae cycles. The analytic method presented in this talk can be employed to prove the uniform persistence of many other periodic delay differential equations. The numerical results may help the study of algae blooms and the preservation of zooplankton in coastal areas. To our knowledge, this work is the first one that carries out sensitivity analysis of R_0 for a periodic system with time delay.

Dynamics on hepatitis B virus infection in vivo with delay interval

Kaifa Wang
Southwest University, Peoples Republic of China
Haonan Zhong

In view of the molecular biological mechanism of the cytotoxic T lymphocytes proliferation induced by Hepatitis B virus infection in vivo, a novel dynamical model with delay interval is proposed. The delay interval is determined by delay center and delay radius. We derive the basic reproduction number R_0 for the viral infection and obtain that the virus-free equilibrium (VFE) is globally asymptotically stable if $R_0 < 1$. When $R_0 > 1$, besides VFE, the unique virus-survived equilibrium (VSE) exists, and the conditions of its asymptotical stabilization are obtained. Moreover, we study the Hopf bifurcations induced by the two delay parameters. The results indicate that both these two delay parameters can lead to periodic fluctuations at VSE, but only the smaller delay radius will destabilize the system, which is different from the classical discrete delay or distributed delay. Numerical simulations indicate that the proposed model can capture the profiles of the clinical data of two untreated chronic hepatitis B patients. The ability of delay interval to destabilize the system is between discrete delay and distributed delay, and the delay center plays the primary role. Pharmaceutical treatment can affect the stability of VSE and induce the fast-slow periodic phenomenon.

Critical value in a SIR network model with heterogeneous infectiousness and susceptibility

Shuixian Yan
Gannan Normal University, Peoples Republic of China
Sanling Yuan

Using the technique of edge-based compartmental modelling (EBCM) for the spread of susceptible-infected-recovered (SIR) diseases in networks, in a recent paper (PloS One, 8(2013), e69162), Miller and Volz established an SIR disease network model with heterogeneous infectiousness and susceptibility. In this talk, Using the nature of irreducible cooperative system in the theory of monotonic dynamical system, we prove that the dynamics of the model are completely determined by a critical value p_0 : When $p_0 > 0$, the disease persists in a globally stable outbreak equilibrium; while when $p_0 < 0$, the disease dies out in the population and the disease free equilibrium is globally stable.

Special Session 5: Recent Results in Nonlinear PDEs

Vincenzo Ambrosio, Università Poliltecnica delle Marche, Ancona, Italy
Teresa Isernia, Università Poliltecnica delle Marche, Ancona, Italy

We aim to bring together senior and young researchers to interact and expose recent developments on topics in the thriving field of nonlinear PDEs related to variational problems, in particular that once arising in Mathematical Physics. We focus on existence, multiplicity and qualitative properties of solutions of nonlinear partial differential equations and systems.

Existence and regularity results for a class of parabolic problems with double phase flux of variable growth

Rakesh Arora
Indian Institute of Technology at Varanasi, India
Sergey Shmarev

This talk presents existence and regularity results for a class of double-phase parabolic problems with homogeneous Dirichlet boundary conditions. We find conditions on the source term and the initial data that guarantee the existence of a unique strong solution u. The solution possesses the property of global higher integrability of the gradient which is derived with the help of new interpolation inequalities in the variable Sobolev spaces. The second-order differentiability of the strong solution is also proven.

A Brezis-Oswald approach for mixed local and nonlocal operators

Stefano Biagi
Politecnico di Milano, Italy
Dimitri Mugnai, Eugenio Vecchi

In this seminar we present an existence and uniqueness result, in the spirit of the celebrated paper by Brezis and Oswald (Nonlinear Anal., 1986), for the following sublinear the Dirichlet problem

$$(\mathrm{P}) \qquad \begin{cases} \mathcal{L}_{p,s} u = f(x,u) & \text{in } \Omega, \\ u \gneqq 0 & \text{in } \Omega, \\ u \equiv 0 & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where $\mathcal{L}_{p,s}$ is the sum of a quasilinear local and a nonlocal operator, i.e.,

$$\mathcal{L}_{p,s} = -\Delta_p + (-\Delta)^s_p.$$

Under standard assumptions on the nonlinearity f, we show that if u solves (P), then $u > 0$ in Ω; moreover, we give precise conditions under which such a solution exists and is unique.

The Gelfand problem for the infinity Laplacian

Fernando Charro
Wayne State University, USA
Byungjae Song, Peiyong Wang

We study the asymptotic behavior as $p \to \infty$ of solutions to the p-Laplacian Gelfand-type problem

$$\begin{cases} -\Delta_p u = \lambda\, e^u & \text{in } \Omega \subset \mathbb{R}^n \\ \quad u = 0 & \text{on } \partial\Omega. \end{cases}$$

We identify a precise scaling between u and the bifurcation parameter λ that balances reaction and diffusion and produces a nontrivial limit problem. More precisely, under an appropriate rescaling on u and λ, we prove uniform convergence of solutions to the p-Laplacian Gelfand-type problem to solutions of

$$\begin{cases} \min\{|\nabla u| - \Lambda\, e^u, -\Delta_\infty u\} = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

We discuss existence, non-existence, and multiplicity of solutions of the limit problem in terms of the limit bifurcation parameter. Moreover, we prove a comparison principle for small solutions of the limit equation. This result is interesting because the limit equation is not proper (a basic requirement for comparison) and because one cannot expect comparison to hold in general, based on the multiplicity results in the literature for the p-Laplacian Gelfand-type problem. Remarkably, minimal solutions are small in the sense of the comparison principle and we can conclude they are the only ones under a certain threshold.

Nodal solutions for nonlinear elliptic equations

Michael Filippakis
University of Piraeus, Greece
Nikolaos S.Papageorgiou

We consider a nonlinear nonhomogeneous eliptic equation driven by the sum of a p-Laplacian and of q-Laplacian(a (p, q)-equation). The reaction term is a Caratheodory function which is resonant at $\pm\infty$ with respect to any nonprincipal variational eigenvalue of the Robin p-Laplacian. Morse theory (critical groups) with variational methods), help us to prove the existence of at least three nontrivial smooth solutions. [1]

[1]The publication of this paper has been partly supported by the University of Piraeus Research Center.

Gradient bounds/estimates for solutions to some nonlinear elliptic equations and parabolic equations

Zu Gao
Wuhan University of Technology, Peoples Republic of China
Cecilia Cavaterra, Serena Dipierro, Alberto Farina, Enrico Valdinoci

We give pointwise gradient bounds for solutions of (possibly non-uniformly) elliptic partial differential equations in the entire Euclidean space. The operator taken into account is very general and comprises also the singular and degenerate nonlinear case with non-standard growth conditions. The sourcing term is also allowed to have a very general form.
Besides, we also provide global gradient estimates for solutions to a general type of nonlinear parabolic equations, possibly in a Riemannian geometry setting. Our result is new in comparison with the existing ones in the literature, in light of the validity of the estimates in the global domain, and it detects several additional regularity effects due to special parabolic data.

Existence of normalized solutions for the planar Schrödinger-Poisson system with exponential critical nonlinearity

Olimpio Miyagaki
UFSCAR-Universidade Federal de Sao Carlos, Brazil
Claudianor O. Alves, Eduardo Boer

In the present work we are concerned with the existence of normalized solutions to the following Schrödinger-Poisson System, involving a nonlinearity with exponential critical growth. The problem contains a constant which stands as a Lagrange multiplier and it is part of the unknown. Our main results extend and/or complement some results found recently.

Variational convergences and applications to partial differential equations driven by vector fields

Andrea Pinamonti
University of Trento, Italy
Alberto Maione, Francesco Serra Cassano

Given a family of locally Lipschitz vector fields X on $\mathbb{R}^n$, we study functionals depending on X. We study the convergence of minimizers and we apply the results to prove a H-compactness theorem for linear differential operator of second order depending on X.

Normalized solutions to polyharmonic equations with Hardy-type potentials via a Nehari–Pohožaev approach

Jacopo Schino
North Carolina State University, USA
Bartosz Bieganowski, Jarosław Mederski

A novel approach is exploited to find solutions $(\lambda, u) \in \mathbb{R} \times H^m(\mathbb{R}^N)$ to

$$\begin{cases} (-\Delta)^m u + \frac{\mu}{|y|^{2m}} u + \lambda u = g(u), \\ \int_{\mathbb{R}^N} u^2 \, \mathrm{d}x = \rho^2, \end{cases}$$

where $\mathbb{R}^N \ni x = (y, z) \in \mathbb{R}^K \times \mathbb{R}^{N-K}$, $N \geq K \geq 2m$, $1 \leq m \in \mathbb{N}$, $\rho > 0$, and the growth of $g \colon \mathbb{R} \to \mathbb{R}$ is mass-supercritical and Sobolev-subcritical at infinity and at least mass-critical at the origin.
An important step in this approach is to show that all the solutions to the differential equation above that are radial in y satisfy the *Pohožaev identity*, which in the presence of a Hardy-type potential is only known in the case $m = 1$ and $N = K$.

Theory of stars in nonlinear PDEs

Jinmyoung Seok
Seoul National University, Korea
Younghun Hong, Juhi Jang, Sangdon Jin

In nonlinear PDE theories, there are several description of gaseous stars, depending on their scopes and equation of states. In microscopic scale, quantum mechanical descriptions of stars are given by Hartree or Hartree-Fock equations while the Euler-Poisson equations replace this in macroscopic scale. In this talk, we discuss about the existence, properties such as shape or stability of these stars and interconnection between them.

Regularity of solutions to boundary nonlocal equations

Pablo Raul Stinga
Iowa State University, USA
Luis A. Caffarelli, Mitchell Haeuser

We introduce the fractional normal derivative of a function on the boundary of a bounded domain as the fractional power of the Dirichlet-to-Neumann map for its corresponding harmonic extension to the interior. We prove Hölder regularity estimates of solutions to the boundary nonlocal problem depending on the regularity of the boundary manifold. This is joint work with Luis A. Caffarelli (UT Austin) and Mitchell Haeuser (Iowa State University).

Symmetry results for double phase operators

Eugenio Vecchi
University of Bologna, Italy

I will discuss qualitative properties of positive singular weak solutions of PDEs arising from double phase functionals. In particular, I will prove symmetry of solutions exploiting the celebrated moving plane method. The talk is based on joint works with S. Biagi (Politecnico di Milano) and F. Esposito (Università della Calabria).

Special Session 6: Fractal Geometry, Dynamical Systems, and Their Applications

Sangita Jha, National Institute of Technology Rourkela, India
Mrinal Roychowdhury, University of Texas Rio Grande Valley, USA
Saurabh Verma, Indian Institute of Information Technology Allahabad, India

The aim of this session is to bring together scientists including the young researchers to discuss and exchange ideas in the areas of fractal geometry, dynamical systems, and their recent advances and emerging applications.

The β-transformation with a hole at 0

Pieter Allaart
University of North Texas, USA
Derong Kong

The β-transformation $T_\beta(x) := \beta x \bmod 1$, where $\beta > 1$ is a real constant, has been studied by many authors due to its rich spectrum of dynamical properties. In 2018, Kalle, Kong, Langeveld and Li considered the β-transformation with $\beta \in (1, 2]$ and a hole $(0, t)$, and studied the survivor set $K_\beta(t)$ of points whose forward orbit never enters the hole. Extending a well-known result of Urbański (1987), they showed that for fixed β, the Hausdorff dimension of $K_\beta(t)$ is a decreasing devil's staircase as a function of t, but left open the question of where this function reaches the value 0. In recent work with D. Kong, we gave a complete answer to this question, which I will describe in this talk. I will also discuss a conjecture of Kalle *et al.* regarding the local dimension of the bifurcation set of the set-valued function $t \mapsto K_\beta(t)$ and, if time permits, explain a connection with topologically expansive Lorenz maps. (Joint work with D. Kong.)

Compound poisson statistics for dynamical systems via spectral perturbation

Jason Atnip
University of Queensland, Australia
Gary Froyland, Cecilia Gonzalez-Tokman, Sandro Vaienti

In this talk we discuss recent results concerning the return time statistics for deterministic and random dynamical systems. Taking the perturbative approach á la Keller-Liverani, we consider a decreasing sequence of holes in phase space which shrink to a point. For systems satisfying a spectral gap, we show that limiting distribution of return times to these shrinking holes is a compound Poisson distribution. We provide a specific example of a class of β-transformations for which the limiting distribution is the Pólya-Aeppli distribution.

Geodesic taxicab paths in generalizations of the Sierpinski carpet

Ethan Berkove
Lafayette College, USA
Elene Karangozishvili, Derek Smith

The Sierpinski carpet and Menger sponge are well-studied generalizations of versions of the Cantor set. They are also members of a two-parameter family of connected higher-dimension fractals that can be constructed iteratively from the n-cube. In this talk we focus on determining taxicab paths–piece-wise linear paths that always travel parallel to an axis, possibly with non-trivia limiting behavior at the endpoints–between any two points x and y in members of this fractal family. In particular, given points x and y in a fractal, we explicitly construct taxicab geodesics between them. We then use these data to compare the taxicab metric to the standard Euclidean metric in the fractals we consider. This is joint work with Elene Karangozishvili and Derek Smith.

Analysis on fractal geometry

Raffaela Capitanelli
Sapienza University of Roma, Italy

Aim of this talk is to present some recent results on fractal and fractional problems, that is problems having some non-integer order features. Fractal refers to geometrical structures with non-integer dimension, while fractional is the non-integer order of differential operators. More precisely, the present talk is concerned with the study of fractional problems arising as models of fractional processes on irregular structures: domains with non-smooth boundaries, fractal boundaries and fractal layers. Several physical and natural phenomena are characterized on one hand by the presence of different temporal and spatial scales, on the other by the presence of contacts among different components through rough (prefractal and fractal) interfaces. Our aim is to propose mathematical models to investigate these phenomena from an analytical point of view as well as their numerical approximation.

A universal Fatou component

Mark Comerford
University of Rhode Island, USA

The possibilities for limit functions on a Fatou component for the iteration of a single polynomial or rational function are well understood and quite restricted. In non-autonomous iteration, where one considers compositions of arbitrary polynomials with suitably bounded degrees and coefficients, with so much freedom in choosing the particular polynomial sequence, one expects to observe a far greater range of behaviour. We show this is indeed the case and that it is possible to obtain the *whole* of the classical Schlicht family of normalized univalent functions on the unit disc as limit functions on a *single* Fatou component for a *single* bounded sequence of quadratic polynomials.
The main ideas behind this are quasiconformal surgery and the feature of dynamics on Siegel discs where suitable high iterates of a single polynomial with a Siegel disc U approximate the identity closely on compact subsets of U. This allows us both to approximate many functions from the Schlicht family on a Fatou component and to correct the small but inevitable errors arising from these approximations. Do almost nothing and you can do almost anything!

The partial derivative of Okamoto's functions with respect to the parameter

Nathan Dalaklis
University of North Texas, USA
Kiko Kawamura, Tobey Mathis, Michalis Paizanis

The differentiability of the one parameter family of Okomoto's functions as functions of x has been analyzed extensively since their introduction in 2005. In this talk, we motivate why one might choose to study the partial derivative with respect to the parameter before then considering this partial derivative for Okomoto's functions. We place a significant focus on $a = 1/3$ as an analogue to our motivation and describe the properties of a nowhere differentiable function derived from this setting, $K(x)$, for which the set of points of infinite derivative produces an example of a measure zero set with Hausdorff dimension 1. Time allowing, we will look to future questions stemming from this work.

Attracting basins and capture components in polynomial limits

Joanna Furno
University of South Alabama, USA
Devin Becker, Lorelei Koss

In joint work with Devin Becker and Lorelei Koss, we explore the convergence of polynomial families to families that are a product of a power map and the exponential. These families have several critical points but only one free critical point that can change behavior with the parameter. We leverage this fact to describe the structure of the attracting basins in dynamical space and the capture components in parameter space, where the orbit of the free critical point converges to the fixed critical point at 0.

Fractal geometry of sea ice structures

Kenneth Golden
University of Utah, USA

Polar sea ice is a critical component of Earth's climate system. As a material it exhibits composite structure on length scales ranging over 10 orders of magnitude. Tiny brine inclusions inside sea ice, large melt ponds on its surface, and even the ice pack itself are all fractals. Here we discuss how the fractal dimension of these structures depends on the parameters that characterize their evolution. For example, we find that box-counting computations of the fractal dimension of liquid brine in sea ice as a function of its porosity agree almost perfectly with the corresponding exact formula for the famous Sierpinski triangle. We also examine the fractal dimension of melt pond boundaries, which transitions from 1 to about 2 as the ponds grow and coalesce, and explore the role of saddle points of the sea ice surface in driving the fractal transition. Such questions lead us into Morse theory, and then into topological data analysis, where we will discuss persistent homology and the Euler characteristic curve for sea ice topography as a function of the water level.

Exact Hausdorff and packing measures of some sets of digital and Luroth expansions

Daniel Ingebretson
Ben-Gurion University of the Negev, Israel

We calculate the exact values of the Hausdorff and packing measures of some fractal sets arising in numeration systems. Specifically, sets of digital expansions and Luroth expansions that are missing digits beyond a given threshold.

Understanding the limit sets generated by general iterated function systems on unbounded space

Kanji Inui
Keio University, Japan

The limit sets generated by iterated function systems (for short, IFSs) with finitely many contractive mappings have been studied and the ones generated by general IFSs (in some sense) also have been studied gradually.

Note that most of the papers consider IFSs defined on some bounded set in some sense, which deduces that the limit sets are always uniformly bounded with respect to the base points.
In this talk, we consider general IFSs defined on (possibly unbounded) complete metric spaces.
Then, under the natural condition, we show the existence and uniqueness (in some sense) of the limit sets which are not uniformly bounded with respect to the base points in general.
In addition, we discuss some basic properties of the limit sets.

Global graphs of the metric entropy of SRB measures

Yunping Jiang
The City University of New York, USA

The metric entropy of the SRB measure measures the level of complexity and is a smooth invariant. It defines a function defined in the space of smooth conjugacy classes. In this talk, I will mention a joint work with Huyi Hu and Miaohua Jiang about constructing two smooth paths in Axiom A (or Anosov) dynamical systems and area-preserving Anosov dynamical systems. The metric entropy takes any value from 0 to the topological entropy on these two paths. Our results lead to the recent Katok flexibility and rigidity program. I will discuss constructing a smooth path in expanding Blaschke products, with an almost expanding Blaschke product as a limit. It turns out that this almost expanding Blaschke product is smoothly conjugate to the famous Boole map on the real line. Thus, this gives a new explanation of Boole's formula, discovered more than one hundred years ago. By modifying this path, we construct another smooth path of area-preserving expanding Blaschke products with a totally degenerate map as a limit. These two paths represent the same smooth path in the space of all smooth conjugacy classes. We then obtain a global graph of the metric entropy, which looks like a bell. Finally, I will apply this global graph to the main cardioid of the Mandelbrot set.

Compositions of scaling involutions and measure-preserving transformations, and some infinite IET's

Chris Johnson
Western Carolina University, USA

Understanding the dynamics of flows on infinite-area translation surfaces is complicated by the fact that the flow need not be conservative and thus may not return to a transverse geodesic segment, and so we may not be able to apply standard tools from the theory of interval exchanges to study the dynamics of these surfaces. Motivated by observations made in a project with Rob Niemeyer, though, we can describe an ergodic-theoretic construction that generalizes a special case of such infinite area translation surfaces where self-similarity can be exploited to study dynamics using infinite interval exchanges. In this talk I will describe this construction and mention some preliminary dynamical results.

Rational-linear anticompetitive systems of difference equations

Chris Lynd
Bloomsburg University, USA

There are 112 systems of two rational-linear difference equations where, for all positive values of the parameters, the corresponding map is anticompetitive and the square of the map is strongly competitive. We present a theorem that describes the global behavior of the solutions for all 112 systems.

Inverse limits of unimodal maps on dendrites

Jonathan Meddaugh
Baylor University, USA
Cordell Hammon, Brian Raines

In this talk, we will discuss the topological structure of inverse limits of unimodal maps on self-similar dendrites.
In particular, for a fixed dendrite D and unimodal map $f : D \to D$ under which D is self-similar, we demonstrate that there is a countable collection $\{g_i : D \to D\}$ of unimodal maps under which D is self-similar, which share the same critical point and Hubbard tree (the convex hull of the critical orbit in D) but have mutually non-homeomorphic inverse limits.

Hausdorff dimension of sets with restricted, slowly growing partial quotients in the semi-regular continued fraction

Yuto Nakajima
Keio University, Japan
Hiroki Takahasi

We consider sets of irrational numbers whose partial quotients $a_{\sigma,n}$ in the semi-regular continued fraction expansion obey certain restrictions and growth conditions. Our main result asserts that, for any sequence $\sigma \in \{-1,1\}^{\mathbb{N}}$ in the expansion, any infinite subset B of $\mathbb{N}$ and for any function f on $\mathbb{N}$ with values in $[\min B, \infty)$ and tending to infinity, the set of irrationals in $(0,1)$ such that

$$a_{\sigma,n} \in B,\ a_{\sigma,n} \leq f(n) \text{ for all } n \in \mathbb{N}$$

and $a_{\sigma,n} \to \infty$ as $n \to \infty$ is of Hausdorff dimension $\tau(B)/2$, where $\tau(B)$ is the exponent of convergence of B. We also prove that for any $\sigma \in \{-1,1\}^{\mathbb{N}}$ and any $B \subset \mathbb{N}$, the set of irrationals in $(0,1)$ such that

$$a_{\sigma,n} \in B \text{ for all } n \in \mathbb{N} \text{ and } a_{\sigma,n} \to \infty \text{ as } n \to \infty$$

is also of Hausdorff dimension $\tau(B)/2$. To prove these results, we construct non-autonomous iterated function systems well-adapted to the given restrictions and growth conditions, and then apply the dimension theory developed by Rempe-Gillen and Urbański.

Quantization for probability distributions

Mrinal Roychowdhury
University of Texas Rio Grande Valley, USA

Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability supported by a set with no more than n points. It has broad application in signal processing and data compression. Quantization dimension gives the speed how fast the specified measure of the error goes to zero as n approaches infinity. Quantization dimension is also connected with other dimensions of dynamical systems such as Hausdorff, packing and box counting dimensions. I will talk about it.

Recent progress in periodic tiling conjecture

Shrey Sanadhya
Ben Gurion University of the Negev, Israel
Tom Meyerovitch, Yaar Solomon

Given a finite set (a tile) F in $\mathbb{Z}^d$, we say that a set A in $\mathbb{Z}^d$ is a co-tile of F if the collection of sets $F+a$, for a in A, form a tiling of $\mathbb{Z}^d$. Namely, these sets are disjoint and their union in $\mathbb{Z}^d$. Given a k-tuple of tiles $F_1, ..., F_k$, we say that A is a joint co-tile of them if A is a co-tile of each F_i. In this talk I will discuss the structure of joint co-tiles in $\mathbb{Z}^d$, particularly, their periodicity. As a result, we extend an old theorem of Newman, stating that every tiling of $\mathbb{Z}$ by a single tile is period. If time allows, we will discuss the connections of our new notions to the periodic tiling conjecture, whose $\mathbb{Z}^2$ case was resolved by Bhattacharya, and a counterexample in high dimension was recently given by Greenfeld and Tao. The talk is based on a joint work with Tom Meyerovitch and Yaar Solomon (arXiv:2301.11255).

Average geodesic distances in generalized Sierpinski carpets

Derek Smith
Lafayette College, USA
Ethan Berkove

The problem of determining the average geodesic distance between two points in a fractal has been addressed by many authors for many different fractals. In this talk, we will survey some of the methods and results in this area, present new results for 2-dimensional generalized Sierpinski carpets, and discuss some of the impediments to extending current methods to higher-dimensional generalizations. This work is joint with Ethan Berkove.

On the stochastic bifurcations regarding random iterations of polynomials of the form $z^2 + c_n$

Takayuki Watanabe
Chubu University, Japan

We consider random iterations of polynomial maps $z^2 + c_n$ where c_n are complex-valued independent random variables following the uniform distribution on the closed disk with center c and radius r. The aim of this talk is twofold. First, we study the (dis)connectedness of random Julia sets. Here, we reveal the relationships between the bifurcation radius and connectedness of random Julia sets. Second, we investigate the bifurcation of our proposed random iterations and give quantitative estimates of bifurcation parameters. In particular, we prove that for the central parameter $c = -1$, almost every random Julia set is totally disconnected with much smaller radial parameters r than expected.

Unmating of expanding Thurston maps with Julia set $\mathbb{S}^2$

Mary Wilkerson
Coastal Carolina University, USA

Every expanding Thurston map f without periodic critical points is known to have an iterate which is the topological mating of two polynomials. This has been examined by Kameyama and Meyer; the latter who has offered an explicit construction for finding two polynomials that can be mated to yield the iterate. The initialization of this algorithm depends on an invariant Jordan curve through the postcritical set of f–but we propose adjustments to this unmating algorithm for the case where there exists a curve which is fully f-invariant up to homotopy and not necessarily simple. We demonstrate that when f is a critically pre-periodic expanding Thurston map, extending the algorithm to accommodate non-Jordan curves allows us to unmate without iterates.

Ergodic theory of coded shift spaces: Measures of maximal entropy and equilibrium states

Christian Wolf
The City College of New York, USA
Tamara Kucherenko, Martin Schmool, Y. Yang

In this talk we discuss recent results concerning the ergodic properties of coded symbolic systems. These systems include S-gap shifts, generalized gap shifts, and Beta-shifts. Specifically we present generalizations of results by V. Climenhaga and R. Pavlov about the uniqueness of measures of maximal entropy and equilibrium states for codes systems. We also re-visit the classical example of the Dyck shift to illustrate our results. The topics discussed in this talk are joint work with T. Kucherenko, M. Schmoll and Y. Yang.

Dynamical systems represented as graphs

James Yorke
University of Maryland, USA
Roberto De Leo

Dynamical systems represented as graphs with special emphasis on the logistic map and on partial differential equations. We have recently described the graph of the logistic map. For some parameter values, the graph has infinitely many nodes that are chaotic saddles. Now we approach graphs more generally. Our approach is based on feedback controls for ordinary and partial differential equations. We develop the general theory. In particular, under mild conditions, we show the graphs are connected.

Special Session 7: Emergence and Dynamics of Patterns in Nonlinear Partial Differential Equations and Related Fields

Danielle Hilhorst, CNRS and University Paris-Sud Paris-Saclay, France
Yoshihisa Morita, Ryukoku University, Japan
Junping Shi, College of William and Mary, USA

The solution structures of many nonlinear partial differential equations reveal the emergence and the evolution of very exciting patterns. Such nonlinear models come from various fields of mathematical science, including material science as well as life sciences. In this session, we will bring together recent studies on solutions of nonlinear partial differential equations related to pattern formation, dynamics, and bifurcations, presenting new aspects of solutions capturing nonlinear phenomena together with underlying solution structures.

Predator-prey systems with prey-dependent diffusion in spatially heterogeneous habitat

Inkyung Ahn
Korea University, Korea
Wonhyung Choi, Kwangjoong Kim

In this talk, we consider a predator-prey system with a certain type of prey-dependent diffusion for predators where the source of prey population depends on location in a habitat with spatial heterogeneity distributed within a bounded domain. It is assumed that the spread rate of predators can change depending on the satisfaction of predators according to the amount of available prey in the vicinity of predators in the habitat. We demonstrate that predators can invade a habitat region through prey-dependent diffusion by analyzing the stability of the system's semitrivial solution when the predator is absent. We also explore the existence and uniqueness of a positive steady state using fixed point index theory in a positive cone in a Banach space. We conclude that the coexistence state is unique if the prey's diffusion rate is above a specific threshold and the average of the prey's resource function is within a particular range represented by the prey's equilibrium value.

Reaction-diffusion models for dispersal in a stage structured population

Chris Cosner
University of Miami, USA
Robert Stephen Cantrell, Salome Martinez, Rachidi Salako

This talk will describe some results for reaction-diffusion models of populations with stage structure. Reaction diffusion models for populations in heterogeneous environments where all individuals are considered to be identical and subject to logistic type growth predict that there is selection for slower diffusion. For stage structured populations that is not always true. We consider a model introduced by Brown and Zhang (J. Math. Anal. Appl. 2003). We show that if the regions favorable for reproduction by adults and for survival by juveniles are sufficiently similar, then slow diffusion is still favored. However, if the regions are different, sometimes fast diffusion is necessary for survival of the population. When the model includes competition between juveniles and adults it is cooperative at low densities but competitive at high densities. Also, in many populations, either juveniles or adults do not disperse, so there is no diffusion in one of the equations. Both of those situations require some novel mathematical approaches. In the case of strong competition between adults and juveniles we do not know if it is possible for the system to have multiple equilibria, which might correspond to multiple spatial patterns if they exist.

Spreading profile of two invading competitors

Yihong Du
University of New England, Australia
Chang-Hong Wu

In this talk I will report some recent results on the invading profiles of a two species competition model with free boundaries. We are interested in a complete classification of the invading profiles of the system in the weak-strong competition case, where a strong competitor and a weak competitor invade the spatial environment simultaneously. We show that there are exactly five different types of long-time invading profiles for this system. This talk is based on theoretical work with Chang-Hong Wu, and numerical work with K. Khan, Shuang Liu and T. Schaerf.

The Rayleigh–Bénard problem for compressible fluid flows

Eduard Feireisl
Mathematical Institute, Czech Academy of Sciences, Czech Republic
Agnieszka Swierczewska-Gwiazda

We consider the physically relevant fully compressible setting of the Rayleigh–Bénard problem of a fluid confined between two parallel plates, heated from the bottom, and subjected to the gravitational force. We show that this open system is dissipative in the sense of Levinson, meaning there exists a bounded absorbing set for any global–in–time weak solution. In addition, global–in–time trajectories are asymptotically compact in suitable topologies and the system possesses a global compact trajectory attractor. The standard technique of Krylov and Bogolyubov then yields the existence of an invariant measure – a stationary statistical solution sitting on the attractor. In addition, the Birkhoff–Khinchin ergodic theorem provides convergence of ergodic averages of solutions belonging to the attractor a.s. with respect to the invariant measure.

Stability of KPP-type fronts in Rosenzweig-MacArthur model

Anna Ghazaryan
Miami University, USA
Vahagn Manukian, Stephane Lafortune, Yuri Latushkin

We consider a diffusive Rosenzweig-MacArthur predator-prey model in the situation when the prey diffuses at the rate much smaller than that of the predator. Earlier, the existence of fronts in the system was proved using the Geometric Singular Perturbation Theory. The underlying dynamical system in a singular limit is reduced to a scalar Fisher-KPP equation and the fronts supported by the full system are small perturbations of the Fisher-KPP fronts. The current project is to investigate whether the stability of the fronts is also governed by the scalar Fisher-KPP equation. The techniques of the analysis include a construction of unstable augmented bundles and their treatment as multi-scale topological structures.

Spiky patterns for the Schnakenberg model on a star shaped graph

Yuta Ishii
National Institute of Technology, Ibaraki College, Japan

A star shaped graph is a domain composed by a single junction and a finite number of edges. In this talk, spiky stationary solutions to the Schnakenberg model on a star shaped graph is considered. The location, amplitude, and stability of spikes are decided by the effect of the geometry of the graph. Moreover, the lengths of edges are classified when spikes with same amplitude are arranged on edges one by one. In particular, the lengths of edges are not classified into three or more in this case.

Instability of planar waves in a combustion problem

Hirofumi Izuhara
University of Miyazaki, Japan

It is reported that combustion in a narrow channel shows a variety of char patterns depending on the airflow rate. In this talk, we consider a mathematical model which describes the combustion experiment, and numerically study the instability of planar combustion wave which is the onset of the variety of char patterns.

Front propagation in the shadow wave-pinning model

King-Yeung Lam
The Ohio State University, USA
Daniel Gomez, Yoichiro Mori

In this paper we consider a nonlocal bistable reaction-diffusion equation arising from cell polarization. A typical solution of this model exhibits an interface with velocity regulated by the total mass. The feedback between mass-conservation and bistablity causes the interface to approach a fixed limit. In the limit of a small diffusivity $\epsilon^2 \ll 1$, we prove that the interface can be estimated within $O(\epsilon^\gamma)$ of the location as predicted using formal asymptotics, for any γ less than 1/2. This is joint work with D. Gomez and Y. Mori at University of Pennsylvania.

Monotone properties of the eigenfunction of Neumann problems

Yi Li
John Jay College of Criminal Justice, CUNY, USA

In this paper, we prove the hot spots conjecture for long rotationally symmetric domains in $\mathbb{R}^n$ by the continuity method. More precisely, we show that the odd Neumann eigenfunction in x_n associated with lowest nonzero eigenvalue is a Morse function on the boundary, which has exactly two critical points and is monotone in the direction from its minimum point to its maximum point. As a consequence, we prove that the Jerison and Nadirashvili's conjecture 8.3 holds true for rotationally symmetric domains and are also able to obtain a sharp lower bound for the Neumann eigenvalue.

Nonlinear Stefan problem with a certain class of multi-stable nonlinearity

Hiroshi Matsuzawa
Kanagawa University, Japan
Yuki Kaneko, Yoshio Yamada

In this talk, I will discuss the long-time dynamical behavior of solutions to a nonlinear Stefan problem for a reaction diffusion equation.
I will show that when the nonlinearity is a positive bistable type nonlinearity which is a certain kind of multi-stable type nonlinearity, the asymptotic behavior of the solutions is classified into four cases: vanishing, small spreading, big spreading, and transition. In particular, I will show that when the transition occurs, the solution converges to an equilibrium solution which is radially symmetric and radially decreasing and centered at some point as time goes to infinity locally uniformly.

Stability of sharp traveling waves for degenerate Burgers-Fisher-KPP equations

Ming Mei
McGill University and Champlain College, Canada
Shanming Ji, Tianyuan Xu, Jingxue Yin

This talk is concerned with the stability of sharp traveling wave for Burgers-Fisher-KPP equations with degenerate diffusion. We show the evolution of free boundary of the solution to Cauchy problem and the convergence to sharp traveling wave with almost exponential decay rates. Here, a key difficulty lies in the intrinsic presence of nonlinear advection effect. After providing an delicate analysis for the relations between the nonlinear advection and the critical wave speed, we develop an equivalent transformation of the evolution estimation of the semi-supported free boundary by analysis the compact supported boundary behavior of the solutions. The new method overcomes the diffculties of the non-integrability of the generalized derivatives of sharp traveling waves near the compacted supported boundary.

The Cahn-Hilliard equation with a source term

Alain Miranville
University of Poitiers, France

Our aim in this talk is to discuss the Cahn-Hilliard equation with a (nonlinear) source term and a logarithmic potential. In particular, we discuss the existence of weak solutions and additional regularity. Such an equation has applications in image processing, biology, tumor growth, ...

Stability and complete bifurcation diagram for a shadow Gierer-Meinhardt system in one space dimension

Yasuhito Miyamoto
University of Tokyo, Japan
Yuki Kaneko, Tohru Wakasa

We are concerned with a shadow system of the Gierer-Meinhardt model in a finite interval. A stationary problem is studied and we consider the diffusion coefficient as a bifurcation parameter. A complete bifurcation diagram of the stationary solutions is obtained, and a stability of every stationary solution is determined. In particular, we show that all 1-mode solutions are stable for a small time constant and that other nontrivial solutions are unstable. The system is known for having stationary spiky patterns with large amplitude. Then, asymptotic expansions of maximum and minimum values of a stationary solution are also obtained.

Discretization of a model governing the motion of two cell populations

Hideki Murakawa
Ryukoku University, Japan

We consider a model governing the motion of two cell populations. In this talk, we approximate the model by a discrete-time or a fully discrete scheme and analyze the convergense of the scheme. This talk is based on a joint work with Julia Hauser and Markus Schmidtchen.

Phase-field approximation model for a self-propelled motion

Masaharu Nagayama
Hokkaido University, Japan
Masshara Nagayama, Koya Sakakibara, Harunori Monobe, Ken-Ichi Nakamura, Yasuaki Kobayashi, Hiroyuki Kitahata

Many experimental systems of self-propelled motion have been reported, and many mathematical models have been proposed for the motion of objects with fixed shapes. On the other hand, there are not many mathematical models of self-propelled motion with deformations. This study suggests a volume-conserving phase-field model that approximates the deformation of self-propelled motion. A physical model describing the self-propelled motion can be derived by taking the singular limit for the mathematical model. Finally, using the computer-assisted analysis, we show that the phase-field model can approximate the self-propelled motion model.

The sign of the speed of bistable traveling waves for the Lotka-Volterra competition-diffusion system

Ken-Ichi Nakamura
Kanazawa University, Japan
Toshiko Ogiwara

In this talk, I will report some results on the speed of bistable traveling waves for the 2-component Lotka-Volterra competition-diffusion system. Under strong competition conditions, the propagating speed of a bistable traveling wave is uniquely determined, but there are few results on its sign. Recently, some new results on the sign have been obtained by constructing suitable comparison functions. I will introduce a new comparison function and show that if the interspecific competition coefficients are sufficiently different, the sign of the speed of a bistable traveling wave may not change no matter how the diffusion coefficients and growth rates are changed.

Pattern dynamics appearing on compact metric graph

Toshiyuki Ogawa
Meiji University, Japan
Shunsuke Kobayashi

The study of reaction-diffusion equations on metric graph has been drawing attention recently. Here, we focus on pattern dynamics on the compact metric graph. There are eight different types of compact metric graphs which are constructed from two or three finite intervals. And we consider systems of reaction-diffusion equations on these compact metric graphs with natural boundary conditions. Suppose additionally the system has Turing or Wave instability. Then, by choosing the length of the original intervals appropriately we have a degenerate situation, where we can use Fourier expansion. This enables us the normal form analysis to determine the local bifurcation structure around the bifurcation point.

Convergence to equilibrium for time and space discretizations of the Cahn-Hilliard equation

Morgan Pierre
Université de Poitiers, France
Matthieu Brachet, Philippe Parnaudeau

We review space and time discretizations of the Cahn-Hilliard equation which are energy stable. In many cases, we prove that a solution converges to a steady state as time goes to infinity. The proof is based on Lyapunov theory and on a Lojasiewicz type inequality. In a few cases, the convergence result is only partial and this raises some interesting questions.

Modelling and analysis of grounded shallow ice sheets

Paolo Piersanti
Indiana University Bloomington, USA
Roger Temam

In this talk, which is the result of a joint work of the speaker with Roger Temam (IU), we formulate a model describing the evolution of thickness of a grounded shallow ice sheet. The thickness of the ice sheet is constrained to be nonnegative, rendering the problem under consideration an obstacle problem. A rigorous analysis shows that the model is thus governed by a set of variational inequalities that involve nonlinearities in the time derivative and in the elliptic term, and that it admits solutions, whose existence is established by means of a semi-discrete scheme and the penalty method.

Nonlocal operators in cancer models

Mabel Lizzy Rajendran
Queen's University Belfast, Northern Ireland

The aim of the talk is to present the motivation for considering nonlocal operators in cancer models; address the challenges these operators pose in terms of analysis and numerical simulation; and showcase the observations and results of the numerical experiments.
The tumour microenvironment has a strong influence on tumour cell proliferation and migration. For instant, the cell-cell adhesion induces subdiffusivity; hapotaxis and chemotaxis induces superdiffusivity; and the viscoelasticity of the extracellular matrix substantially affect the spreading, growth, proliferation, migration of the cancer cells.
Fractional operators have undoubtedly exercised their efficiency in modelling complex and intermediate processes like anomalous diffusion (sub and super diffusion) and viscoelasticity. At the same time, the very nature of nonlocality of the fractional operators introduces computational complexities and challenges in obtaining existence results. This calls for the analysis of these equations and efficient memory management algorithms.
Fractional operators have shown to outperform the integer order differential equations in capturing effects which are crucial in cancer modelling. Adopting these operators in the modelling could help in better understanding of the development of tumour cells and their dynamics. These in turn will help in devising appropriate treatment methods for cancer patients.

Mathematical analysis of the dynamics of some reaction-diffusion models for infectious diseases

Rachidi Salako
University of Nevada Las Vegas, USA
Yuan Lou, Rachidi B Salako

The possibility of coexistence of several strains for an infectious disease can be a major concern for policy makers in taking informed decisions to alleviate its effect on the population. For instance, the emergence of different strains of the Coronavirus disease (COVID-19) has generated significant concerns due to its attendant waves after waves of infected populations across the world. So a compelling question is whether some of these mutated strains could become locally adapted to coexist. In this talk, we study some reaction-diffusion systems for infectious diseases and investigate how the dynamics is impacted by the movement of populations and spatial heterogeneity of the environment. General conditions for the existence, uniqueness and stability of the coexistence steady states are established. Our analysis revealed two mechanisms for the coexistence of strains: (i) when the ratio of the transmission probability of two strains falls into some intermediate ranges; (ii)

when the diffusion rate of two strains falls into proper ranges. Interestingly, when there is no coexistence of strains, it is possible for the "weak" strain to be dominant for some intermediate range of diffusion rates, in strong contrast to small and large diffusion cases where the "weak" strain always go extinct.

Spiral solutions of the Koppel-Howard lambda-omega reaction-diffusion equations

William Troy
University of Pittsburgh, USA

Our investigation of logarithmic spirals is motivated by disparate experimental results:
(i) the discovery of logarithmic spiral shaped precipitate formation in chemical garden experiments. Understanding precipitate formation in chemical gardens is important since analogous precipitates form in deep ocean hydrothermal vents, where conditions may be compatible with the emergence of life.
(ii) the discovery that logarithmic spiral shaped waves of spreading depression can spontaneously form and cause macular degeneration in hypoglycemic chick retina. The role of reaction-diffusion mechanisms in spiral formation in these diverse experimental settings is poorly understood. To gain insight we use topological shooting to prove existence of 0-bump stationary logarithmic spiral solutions, and rotating logarithmic spiral wave solutions, of the Kopell-Howard lambda-omega reaction-diffusion model.

Global dynamics of a reaction-diffusion epidemic model with nonlinear incidence mechanism

Yixiang Wu
Middle Tennessee State University, USA
Rui Peng

I will talk about an epidemic reaction-diffusion system with nonlinear incidence mechanism of the form $S^qI^p\,(p,\,q>0)$. The coefficients of the system are spatially heterogeneous and time dependent (particularly time periodic). We first establish the L^∞-bounds of the solutions of a class of systems. Based on such estimates, we then study the long-time behavior of the solutions of the system. Our results reveal the delicate effect of the infection mechanism, transmission rate, recovery rate and disease-induced mortality rate on the infection dynamics. Our analysis can be adapted to models with some other types of infection incidence mechanisms.

Special Session 8: Propagation Phenomena in Reaction-Diffusion Systems

Hirokazu Ninomiya, Meiji University, Japan
Masaharu Taniguchi, Okayama University, Japan

This special session is concerned with mathematical analysis on propagation phenomena or pattern formation appearing in reaction-diffusion systems. Related topics are traveling waves, entire solutions and asymptotic behaviors of solutions in reaction-diffusion systems.

On the interaction of a pair of coaxial circular vortex filaments

Masashi Aiki
Tokyo University of Science, Japan

We consider the motion of a pair of coaxial vortex rings immersed in an incompressible and inviscid fluid. The motion is formulated as the motion of a pair of coaxial circular vortex filaments. The motion of such vortex rings have been studied for a very long time, dating back to the work by Helmholtz in 1858, and many motion patterns are known to exist for interacting coaxial vortex rings. One such pattern that attracted the attention of many is leapfrogging. Leapfrogging was suggested to occur by Helmholtz, but it took until 1971 for leapfrogging motion to be experimentally confirmed in the work by Yamada and Matsui. Since then, more laboratory experiments and numerical experiments of leapfrogging motion has been done, but the mathematical analysis of the leapfrogging motion is scarce. To this end, we propose a system of partial differential equations describing such motion and prove that solutions corresponding to leapfrogging exist. We further give necessary and sufficient conditions for leapfrogging to occur.

Propagation properties for SIR reaction-diffusion system

Romain Ducasse
Université Paris Cité, France

We present some results concerning the propagation phenomenon that appears in SIR models. These models are reaction-diffusion systems that describes the spread of a disease in a population. We shall consider in particular a multi-species model, describing the interaction between the propagation of different strains of a disease. Our model is a natural generalisation of the classical Kermack-McKendrick model. We shall study the long-time behavior of solutions, and prove that the model exhibits a "selection via propagation" phenomenon.

Center manifold theory for the motions of camphor boats in L^2-framework

Kota Ikeda
Meiji University, Japan

The collective motion of camphor boats in the water channel exhibits both a homogeneous and an inhomogeneous state, depending on the number of boats. In order to analyze those phenomena, we have developed the center manifold theorem proposed by S.-I. Ei et al. (2002) and derived a reduced system from the original model in our previous works (K. Ikeda et al., 2019, K. Ikeda and S.-I. Ei, 2020). As shown in those works, the existence of delta functions in our original system raises two difficulties. One is that our theory is established in $(H^1)^*$-framework, where $(H^1)^*$ is the dual space of H^1. Recall that S.-I. Ei et al. has developed their theory in L^2-framework in their previous work. The other is that the delta functions cause the lack of regularity of functions and then make it impossible to derive higher order terms. To avoid the difficulties mentioned above, we introduce a function which can exclude the step function in the original system. In this talk, we develop a center manifold theory in a new system without delta functions in L^2-framework.

Traveling waves to a nonlocal scalar equation with sign-changing kernel

Hiroshi Ishii
Kyoto University Institute for the Advanced Study of Human Biology, Kyoto University, Japan
Shin-Ichiro Ei, Jong-Shenq Guo, Chin-Chin Wu

In recent years, mathematical models with nonlocal effects described by convolution with suitable integral kernels have been proposed in various fields: material science, neural fields, and so on. Recent studies have shown that those nonlocal scalar equations can be derived as certain reduced systems of reaction-diffusion systems. One of the studies has pointed out that the properties of reaction-diffusion networks can be described by the shape of the kernel, and especially that structures such as Turing instability can be obtained using a kernel with local activation and long-range inhibition. Such sign-changing kernels have been pointed out for their importance from the viewpoint of pattern formation problems, but there have been few studies on traveling waves. This talk will consider the existence of traveling waves connecting the unstable state and the stable state of a nonlocal scalar equation with a sign-changing kernel. We first describe the problem setup and then present numerical results on traveling waves and other spatio-temporal patterns. Next, we introduce new upper-lower solutions to prove the existence of traveling waves. Finally, we construct them and analyze asymptotic profiles of traveling waves.

On the stability and behavior of solutions of the Allen-Cahn-Nagumo equation

Toru Kan
Osaka Metropolitan University, Japan

We consider the Allen-Cahn-Nagumo equation in the whole plane. It is known that V-shaped travelling wave solutions exist and are stable under perturbations decaying at infinity. In this talk, we first consider conditions for a solution to converge to a V-shaped travelling wave solution, and then discuss the asymptotic behavior of solutions whose initial values are close to indicator functions with noncompact support.

Coexistence-segregation dichotomy in the full cross-diffusion limit to the stationary SKT model

Kousuke Kuto
Waseda University, Japan
Jumpei Inoue, Homare Sato

This talk is concerned with the Dirichlet problem of a Lotka-Volterra competition system with cross-diffusion terms. The purpose is to derive the limiting behavior of positive steady-state solutions as equal two cross-diffusion coefficients tend to infinity (so-called the full cross-diffusion limit). The main result shows that almost all limits of positive steady-states can be classified into two types: small coexistence or complete segregation, and the limit of small coexistence can be characterized by positive solutions to a single elliptic equation, while the limit of complete segregation can be characterized by nodal solutions to another single elliptic equation. By the perturbation of solutions of these two limiting equations, we construct the set of positive solutions when equal two cross-diffusion coefficients are sufficiently large. From the viewpoint of the bifurcation diagram, the branch of the small coexistence bifurcates from the trivial solution, and many branches of the segregation bifurcate from solutions on the branch of the small coexistence.

Logarithmic shiftings of a radial propagating terrace in a free boundary problem for a multi-stable reaction diffusion equation in high space dimensions

Hiroshi Matsuzawa
Kanagawa University, Japan
Yuki Kaneko, Yoshio Yamada

In this talk, I will present a recent result on a free boundary problem for a multi-stable reaction-diffusion equation of the form $u_t = \Delta u + f(u)$ with a radially symmetric setting in high space dimensions. In particular, I will focus on positive bistable nonlinearity f introduced by [Kawai-Yamada, 2016]. Recently [Kaneko-Matsuzawa-Yamada, 2020] revealed that for the one-dimensional problem, under certain conditions, a solution approaches a so-called propagating terrace that consists of a semi-wave and a traveling wave.
In this talk, I will present that for higher dimensional case, under certain conditions, the solution generates a propagating terrace with logarithmic shiftings. In particular, I will show that the solution has two kinds of logarithmic shifting which come from the free boundary problem and Cauchy problem, respectively.

Sharp traveling waves: Burgers equations vs Fisher-KPP equations

Ming Mei
McGill University and Champlain College, Canada

In this talk, we present the existence, regularities and oscillations of sharp traveling waves for Burgers equations with degenerate diffusion and time-delay. These sharp waves usually lose their regularities, and possess stronge singularitites (the derivatives can be infinity at the sharp points) once the degenrate diffusion is strong, and occur oscillations once the time-delay is big. We then make a comparation to the related studies for the degenerate Fisher-KPP equations, and show the essential differences between these two types of sharp traveling waves.

Dynamics of area-preserving curvature flow of a plane curve in an inhomogeneous medium

Hirokazu Ninomiya
Meiji University, Japan

Gage studied the area-preserving curvature flow of a plane curve in 1986 and showed that an initially convex interface remains convex and converges to a stationary circle. However, in applications, the medium is often not homogeneous and the interface moves towards a more favorable environment. In this talk, I will extend the area-preserving curvature flow to an inhomogeneous medium and will explain the dynamics of an interface of the area-preserving curvature flow in an inhomogeneous medium.

Axially asymmetric traveling fronts in balanced bistable reaction-diffusion equations

Masaharu Taniguchi
Okayama University, Japan

For a balanced bistable reaction-diffusion equation, an axisymmetric traveling front has been well studied. We prove that an axially asymmetric traveling front with any positive speed does exist in a balanced bistable reaction-diffusion equation. Our method is as follows. We use a pyramidal traveling front for an imbalanced reaction-diffusion equation whose cross section has a major axis and a minor axis. Preserving the major axis and the minor axis to be given constants and taking the balanced limit, we obtain an axially asymmetric traveling front in a balanced bistable reaction-diffusion equation. This traveling front is monotone decreasing with respect to the traveling axis, and its cross section is a compact set with a major axis and a minor axis.

Propagation for heterogeneous reaction-diffusion systems in cylinders with general sections

Andrea Tellini
Universidad Politécnica de Madrid, Spain
Beniamin Bogosel, Thomas Giletti

We consider a cylinder whose section is an arbitrary (regular) domain of $\mathbb{R}^N$. We study a reaction-diffusion system consisting of two coupled equations, one posed in the interior of the cylinder and one on the boundary. Reaction and diffusion heterogeneities are present. We show how to study the asymptotic speed of propagation of the solutions by means of an eigenvalue problem. Then, we analyze the dependence of the speed of propagation on the parameters of the problem and on the shape of the domain. This is a joint work with Beniamin Bogosel (CMAP, Ecole Polytechnique, France) and Thomas Giletti (University of Lorraine, France).

Special Session 9: Stochastic Analysis and Large Scale Interacting Systems

Danielle Hilhorst, CNRS and University Paris-Sud Paris-Saclay, France
Sunder Sethuraman, University of Arizona, USA
Bin Xie, Shinshu University, Japan

The interaction between the theory of stochastic analysis and large scale interacting systems is an important and fruitful research field, not only because of its intrinsic mathematical challenges, but also the wide range of its applications in physics, biology, mathematical finance, engineering and so on. In this session, we aim to bring together experts in the research area of stochastic analysis on interacting particle systems arising in physics and biology, as well as in phase transitions, and related research topics including stochastic partial differential equations, rough paths, random surfaces, quantum field theory, and stochastic homogenization. It will involve methods for their stochastic analysis and will also deal with numerical algorithms for explicit computations.

Extinction and persistence thresholds of stochastic e-rumor models

Severine Bernard
Université des Antilles, Guadeloupe
Alain Pietrus, Kendy Valmont

In recent years, social networks became new spaces of discussions which can be dangerous for our societies because it can destabilize human behaviors and collective feelings. Namely e-rumor, the propagation of informations on social networks is a multi-dimensional diffusion process mainly driven by socio-psychological and geographical elements.
The first e-rumor models are based on the epidemic ones and some control studies of fakes news have been made for deterministic ones. Then a stability analysis has been done on a new model better taking into account the dynamic within the social network. But this phenomenom is a random one and the use of stochastic models proved useful.
From a deterministic e-rumor model, we first propose a stochastic one with only white noises. But e-rumor systems can suffer social perturbations such as election period, social movement, etc. Such events can be described by Lévy jump process. Consequently, we propose then a stochastic e-rumor model with jumps, for which we give the extinction and persistence thresholds of an information on a social network.

Moment growth and intermittency for SPDEs in the sublinear-growth regime

Le Chen
Auburn University, USA
Panqiu Xia

In this paper, we investigate stochastic heat equation with sublinear diffusion coefficients. By assuming certain concavity of the diffusion coefficient, we establish non-trivial moment upper bounds for the solution. These moment bounds shed light on the *smoothing intermittency effect* under *weak diffusion* (i.e., sublinear growth) previously observed by Zeldovich *et al.* The method we employ is highly robust and can be extended to a wide range of stochastic partial differential equations, including the one-dimensional stochastic wave equation. This is a joint-work with Panqiu Xia (Auburn University).

Large deviations of slow-fast systems driven by fractional Brownian motion

Ioannis Gasteratos
Imperial College London, England
Siragan Gailus

We consider a multiscale system of stochastic differential equations in which the slow component is perturbed by a small fractional Brownian motion with Hurst index H¿1/2 and the fast component is driven by an independent Brownian motion. Working in the framework of Young integration, we use tools from fractional calculus and weak convergence arguments to establish a Large Deviation Principle (LDP) in the homogenized limit, as the noise intensity and time-scale separation parameters vanish at an appropriate rate. Our approach is based in the study of the limiting behavior of an associated controlled system. We show that, in certain cases, the nonlocal rate function admits an explicit non-variational form. The latter allows us to draw comparisons to the case H=1/2 which corresponds to the classical Freidlin-Wentzell theory. Moreover, we study the asymptotics of the rate function as H-¿1/2+ and show that it is discontinuous at H=1/2.

Fluctuations in conservative systems and SPDEs

Benjamin Gess
MPI MIS Leipzig, Germany

Fluctuations are ubiquitous in non-equilibrium conservative systems. The analysis of their large deviations lead to macroscopic fluctuation theory (MFT), a general framework for non-equilibrium statistical mechanics. MFT is based on a constitutive formula for large fluctuations around thermodynamic variables, and can be justified from fluctuating hydrodynamics. The latter postulates conservative, singular SPDEs to describe fluctuations in systems out of equilibrium. Both theories are informally linked via zero noise large deviations principles for SPDEs. In this talk, we will introduce several examples demonstrating these relations, and leading to a class of conservative SPDEs. The analysis of large deviations of interacting particle systems and conservative SPDEs will then lead to intricate and open problems for PDEs with irregular coefficients. In the last part of the talk, we present one positive result in this direction, solving a long-standing open problem in the proof of a full large deviation result for the zero-range process.

Stability of waves for SPDEs

Manuel Gnann
TU Delft, Netherlands
Katharina Eichinger, Christian Kuehn, Rik Westdorp, Joris van Winden

In this talk, I will discuss recent results on the stability of waves for two examples of stochastic partial differential equations. The first example is the FitzHugh-Nagumo equation driven by additive noise, for which stable traveling pulse solutions in the deterministic case exist. The second example is the parametrically forced nonlinear Schrödinger equation (PFNLS) driven by multiplicative noise, allowing for stable bright solitary waves if the noise is zero. I will discuss how the deterministic stability results can be lifted to the stochastic setting, leading to a phase correction due to noise meeting a stochastic ordinary differential equation.

Schemes for SDE/SPDE with singular drifts

Ludovic Goudenege
CNRS, France
E. M. Haress, A. Richard

The numerical schemes for the approximation of stochastic processes rely on classical temporal schemes for stochastic differential equations. In the case of stochastic partial differential equations, time and space discretizations must be considered to obtain a fully discretized scheme.
In this talk, I will present implementations of numerical schemes for stochastic (partial) differential equations, in the case of singular drift and with fractional Brownian motion as additive noise. I will prove that we can define solutions using a regularization effect from the fractional noise. Next, I will demonstrate how this approach can be used to define solution for stochastic partial differential equations with additive space-time white noise. I will also present some numerical simulations of the stochastic heat equation with Dirac drift or penalization drift to obtain reflected nonnegative solution.

Industrial mathematics, hypocoercivity and SPDEs

Martin Grothaus
University of Kaiserslautern-Landau, Germany

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

Directed polymers, a nonlocal reaction-diffusion equation, and Fisher-KPP

Christopher Henderson
University of Arizona, USA
Yu Gu

Beginning from a directed polymer in a Gaussian random environment, we derive a hierarchical PDE system from the annealed endpoint density. A natural approximation of this in the annealed setting yields a simple nonlocal reaction diffusion equation. An open conjecture in probability that has been checked in certain exactly solvable cases is that, in one dimension, this directed polymer is in the KPZ universality class, which exhibits scaling in space with a 2/3 power law in time. We show that our approximate model exhibits non-trivial behavior on the same spatial scales (that is, a 2/3 power law in time) and identify the limiting distribution by connecting to the Fisher-KPP equation via a careful rescaling and change-of-coordinates.

A regularity structure for the quasilinear generalized KPZ equation

Masato Hoshino
Osaka University, Japan
Ismael Bailleul, Seiichiro Kusuoka

We prove the local well-posedness of a regularity structure formulation of the quasilinear generalized KPZ equation and give an explicit form of the renormalized equation in the full subcritical regime. Our formulation is different from the ones by Gerencsér and Hairer and by Otto, Sauer, Smith, and Weber.

Support theorem for pinned diffusion processes

Yuzuru Inahama
Kyushu University, Japan

In this paper we prove a support theorem of Stroock-Varadhan type for pinned diffusion processes. To this end we use two powerful results from stochastic analysis. One is quasi-sure analysis for Brownian rough path. The other is Aida-Kusuoka-Stroock's positivity theorem for the densities of weighted laws of non-degenerate Wiener functionals.

Long-time behavior of stochastic heat equations

Kunwoo Kim
POSTECH, Korea

The long-time behavior of stochastic heat equations perturbed by space-time white noise depends on the spatial domain and the initial data. In this talk, we consider stochastic heat equations on a one-dimensional torus and the real line and investigate how the long-time behavior depends on the spatial domain and the initial data. This is based on joint work with Davar Khoshnevisan and Carl Mueller.

Optimal parameterizing manifolds and reduced systems for stochastic transitions

Honghu Liu
Virginia Tech, USA
Mickael D. Chekroun, James C. McWilliams

A general, data-informed, and theory-guided variational approach based on analytic parameterizations of unresolved variables will be presented to address the closure problem of stochastic systems. It relies on the Optimal Parameterizing Manifold (OPM) framework which allows, for deterministic chaotic systems away from the instability onset, to derive useful analytic formulas for such parameterizations. These are obtained as homotopic deformations of parameterizations near criticality such as e.g. arising in center manifold reduction, and whose homotopy parameters are optimized away from criticality using data from the full model. Unlike other nonlinear-parameterization approaches such as those based on invariant/inertial or slow manifolds, the superiority of the OPM approach lies in its ability to alleviate the constraining spectral gap or timescale separation conditions.
In this talk, we present an extension of this program to stochastic partial differential equations (SPDEs) driven by additive noise, either white or of jump type. Analytic formulas of stochastic OPMs are derived. These parameterizations are optimized using a single solution path and are shown to represent efficiently the interactions between the noise and nonlinear terms in a given reduced state space, which are valid beyond the training solution path. Path-dependent non-Markovian coefficients depending on the noise history are shown to play a key role in these parameterizations especially when the noise is acting along the “orthogonal direction” of the reduced state space. Applications to stochastic transitions in SPDEs will be presented.

Least squares estimators for stochastic logistic model driven by small Lévy noises

Hongwei Long
Florida Atlantic University, USA
Bikram Bhusal

We study parameter estimation for a stochastic logistic model driven by small Lévy noises under discrete observations. Note that the stochastic logistic equation has a unique positive global solution under certain conditions on the underlying Lévy measure. We use the least squares method to obtain consistent estimators of the drift parameters. Then we establish the rate of convergence and asymptotic distributions of the least squares estimators when a small dispersion coefficient tends to zero and the sample size tends to infinity simultaneously. We present some simulation study to examine the efficacy of the proposed estimators.

Scaling limit for moderately interacting particle systems with environmental noise

Dejun Luo
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Republic of China
Shuchen Guo

We consider moderately interacting particle systems with singular interaction kernel and environmental noise. The particles evolve in a common random field while interacting with each other through some regularized kernels. It is shown that the mollified empirical measures converge in strong norms to the unique (local) solutions of nonlinear Fokker-Planck equations. The approach works for the Biot-Savart and repulsive Poisson kernels.

Smoothness of the diffusion coefficients for particle systems in continuous space

Maximilian Nitzschner
New York University, USA
Arianna Giunti, Chenlin Gu, Jean-Christophe Mourrat

We consider a class of particle systems with local interactions in continuous space, which are reversible with respect to the Poisson measures with constant density. A natural quantity of interest capturing the large-scale behavior of particles in this set-up is the bulk diffusion matrix. Recent work by Giunti, Gu, and Mourrat has established that finite-volume approximations of this diffusion matrix converge at an algebraic rate. We show that the bulk diffusion matrix is an infinitely differentiable function of the density of particles, and obtain relatively explicit expressions for the derivatives in terms of the corrector, an object which already appeared in the description of the bulk diffusion matrix itself.
Based on joint work with Giunti, Gu, and Mourrat.

Strong Feller semigroups and Markov processes: A counterexample

Michael Röckner
University of Bielefeld, Germany
Lucian Beznea, Iulian Cimpean

In this talk we present a general method how to construct counterexamples that prove that the strong Feller property and the joint (space-time) continuity for a semigroup of Markov kernels on a Polish space are not enough to ensure the existence of an associated càdlàg Markov process on the same space. The analogous result with càdlàg Markov process replaced by right Markov process also holds, even if one allows to change the Polish topology to another Polish topology with the same Borel σ-algebra.

Convergence of a finite-volume scheme for a heat equation with multiplicative Lipschitz noise

Kerstin Schmitz
University of Duisburg-Essen, Germany
Caroline Bauzet, Flore Nabet, Aleksandra Zimmermann

We study an approximation by a finite-volume scheme in space and a semi-implicit discretization in time for a stochastic heat equation with convection and a multiplicative Lipschitz noise. In the passage to the limit the main difficulty is to identify the limit of the non-linear terms. In the special case of a heat equation without convection we use a stochastic compactness method based on Skorokhod's representation theorem to get first a martingale solution and then, by a pathwise uniqueness argument, the unique variational solution. In the general case with convection we are able to prove convergence of the scheme adapting well-known monotonicity methods to the stochastic framework.

Hydrodynamic limit in a 'Sinai'-type random environment

Sunder Sethuraman
University of Arizona, USA
C. Landim, C.G. Pacheco, J. Xue

We consider a zero-range system of interacting random walks moving in a one dimensional 'Sinai'-type random environment. We show a hydrodynamic limit for the space time evolution of the mass in the system. Interestingly, the limit equation includes a random transport term arising from the random environment.

Scaling limits for Glauber–Kawasaki processes

Kenkichi Tsunoda
Faculty of Mathematics, Kyushu University, Japan

We discuss scaling limits for Glauber–Kawasaki dynamics. The Glauber–Kawasaki dynamics has been introduced by De Masi et al. to study a reaction-diffusion equation from a microscopic particle system. In fact, they derived a reaction-diffusion equation as a limiting equation of the density of particles. This limit is usually called hydrodynamic limit. In this talk, we focus on several scaling limits related to this hydrodynamic limit.

A martingale formulation for stochastic compartmental susceptible-infected-recovered (SIR) models to analyze finite size effects in COVID-19 case studies

Chuntian Wang
University of Alabama, USA
Xia Li, Hao Li, Andrea L. Bertozzi

Deterministic compartmental models for infectious diseases give the mean behaviour of stochastic agent-based models. These models work well for counterfactual studies in which a fully mixed large-scale population is relevant. However, with finite size populations, chance variations may lead to significant departures from the mean. In real-life applications, finite size effects arise from the variance of individual realizations of an epidemic course about its fluid limit. In this article, we consider the classical stochastic Susceptible-Infected-Recovered (SIR) model, and derive a martingale formulation consisting of a deterministic and a stochastic component. The deterministic part coincides with the classical deterministic SIR model and we provide an upper bound for the stochastic part. Through analysis of the stochastic component depending on varying population size, we provide a theoretical explanation of finite size effects. Our theory is supported by quantitative and direct numerical simulations of theoretical infinitesimal variance. Case studies of coronavirus disease 2019 (COVID-19) transmission in smaller populations illustrate that the theory provides an envelope of possible outcomes that includes the field data.

Modulated free energy and mean field limit

Zhenfu Wang
Peking University, Peoples Republic of China
Didier Bresch, Pierre-Emmanuel Jabin

We proves the mean field limit and quantitative estimates for many-particle systems with singular attractive interactions between particles. As an important example, a full rigorous derivation (with quantitative estimates) of the Patlak-Keller-Segel model in optimal subcritical regimes is obtained for the first time. To give an answer to this longstanding problem, we take advantage of a new modulated free energy and we prove some precise large deviation estimates encoding the competition between diffusion and attraction. This modulated free energy approach can also treat the systems with a wide range of repulsive kernels, including the vanishing viscosity case.

Reflected stochastic partial differential equations driven by space-time white noise

Bin Xie
Shinshu University, Japan

In this talk, I will discuss recent results on the reflected stochastic partial differential equation driven by space-time white noise based on the approach of the dimension-free Harnack inequalities. The reflected SPDE is one kind of stochastic parabolic obstacle problems and is also regarded as the infinite-dimensional Skorokhod problem, which is closely connected with $\nabla\phi$-interface model. I first state the results on the Harnack inequalities for the reflected SPDEs and then apply them to the study of various properties of Markov semigroup associated with the reflected SPDEs.

Well-posedness and Lewy-Stampaccia inequalities for nonlinear stochastic evolution equations

Aleksandra Zimmermann
University of Duisburg-Essen and TU Clausthal, Germany
Guy Vallet, Niklas Sapountzoglou

We consider nonlinear stochastic evolution equations such as the stochastic p-Laplace equation with a nonlinear first-order perturbation. More precisely, the leading operator in our equation is a nonlinear, second order pseudomonotone operator of Leray-Lions type. In our setting, we may also add Lipschitz continuous perturbations of zero order. The multiplicative noise term is given by a stochastic integral with respect to a Q-Wiener process. We show well-posedness of the associated initial value problem for random initial data on a bounded domain with a homogeneous Dirichlet boundary condition. Therefore we consider a singular perturbation of our problem by a highter order operator. Through the a-priori etimates for the approximate solutions of the singular perturbation, only weak convergence is obtained. This convergence is not compatible with the nonlinearities in the equation. Therefore we use the theorems of Prokhorov and Skorokhod to establish existence of martingale solutions. Then, pathwise uniqueness follows from a L1-contraction principle and we may apply the method of Gyøngy-Krylov to obtain stochastically strong solutions. These well-posedness results serve as a basis for the study of variational inequalities and Lewy-Stampaccia inequalities for our problem.

Special Session 10: Sharp Inequalities and Nonlinear Differential Equations

Bernhard Ruf, University of Milano, Italy
Futoshi Takahashi, Osaka City University, Japan
Federica Sani, University of Modena and Reggio Emilia, Italy

The role played by sharp functional inequalities in the analysis of nonlinear partial differential equations (PDE) is fundamental in the calculus of variations. On the one hand, this special session will be focused on recent developments in limiting and critical inequalities describing subtle relations between function spaces. On the other hand, the purpose will be to emphasize the effect of sharp inequalities in the understanding of characteristic properties of solutions to nonlinear elliptic and evolution equations (mainly of parabolic type) such as existence/nonexistence, uniqueness/multiplicity, blowup, singularity formation and bubbling phenomena. We hope to encourage vital discussions between groups of researchers from real analysis and PDE, new collaborations and connections.

On the solutions of Poisson equation in $\mathbb{R}^n$

Luigi Fontana
Università di Milano-Bicocca, Italy

The Poisson equation $-\Delta u = f$ in R^n has a (obviously non unique) distributional solution u for any given distribution f. Not much can be said about u in such a general setting. However, when f is a L^p function, u can be selected in the corresponding homogeneous Sobolev space. We focus on the critical case when p is $n/2$.

Asymptotic properties of critical points for Trudinger-Moser functional involving scale parameter

Masato Hashizume
Hiroshima University, Japan

In this talk, we consider asymptotic behavior of critical points for the Trudinger-Moser functional. We derive several properties of the critical points both for large scale parameter and for small scale parameter. In particular, we prove that asymptotic behavior of maximizers depends on the exponent in the Trudinger-Moser functional. If the exponent is close to the critical exponent, then sequence of maximizers concentrates at one point, while if the exponent is small, then the limit of sequence of maximizers vanishes.

Non-uniqueness for a critical heat equation in two dimensions with singular data

Norisuke Ioku
Tohoku University, Japan
Yohei Fujishima, Bernhard Ruf, Elide Terraneo

We consider non-uniqueness for nonlinear heat equations in two dimensions. For higher dimensions, uniqueness and non-uniqueness is well understood by Brezis-Cazenave and Ni-Sacks. In this talk we focus on exponential nonlinearities as a critical growth in two dimensions and reveal that non-uniqueness result arises from the singular stationary solution.

Pseudo-traveling wave decomposition of time-global solutions for semilinear parabolic equations and its applications

Michinori Ishiwata
Osaka University, Japan

This talk is concerned with the large-time asymptotics for general time-global solutions of semilinear parabolic problems defined in $\mathbb{R}^N$. The orbit of a solution could be noncompact in the natural energy space $H^1(\mathbb{R})$ due to the unboundedness of $\mathbb{R}$. It is proved in this talk that every time-global solution decomposes into a superposition of "pseudo-traveling waves" whose profiles are stationary solutions.
The analysis is done within the energy space together with the profile decomposition argument. Some applications to the analysis of the asymptotic behavior of time-global solutions which are treated e.g. in Chill-Jendoubi (2003), Cortazar-del Pino-Elgueta (1999) and Feireisl-Petzeltova (1997) are also given.

On the existence of solutions of the Backus problem

Toru Kan
Osaka Metropolitan University, Japan

The Backus problem is the problem of finding a harmonic function on a given domain such that the modulus of its gradient coincides with prescribed values on the boundary of the domain. This problem appears when we try to determine the magnetic field of the Earth from the measurements of its intensity on the Earth's surface. In this talk, we discuss the existence of solutions of the Backus problem in a neighborhood of a dipole, and observe that a solution is constructed if the given boundary values have symmetry. Deriving appropriate a priori estimates for the linearized problem around the dipole is the key to constructing the solution.

The Choquard logarithmic equation involving a nonlinearity with exponential growth

Olimpio Miyagaki
Universidade Federal de Sao Carlos, Brazil
Eduardo Boer

In the present work we are concerned with the Choquard Logarithmic equation involving a nonlinearity with exponential critical growth. We prove the existence of a nontrivial solution at the mountain pass level and a nontrivial ground state solution. Also, we provide these results under a symmetric setting, taking into account subgroups of O(2). We handle the lack of compactness of the associated energy functional due to the unboundedness of the domain and the presence of a limiting case embedding.

Adams inequalities with exact growth conditions on metric measure spaces, and applications

Carlo Morpurgo
University of Missouri, USA
Liuyu Qin

I will present a recent general result on Adams inequalities for Riesz-Like potentials on metric measure spaces. I will then discuss applications of these results on various spaces, such as $\mathbb{R}^n$, the Heisenberg group and Riemannian manifolds with controlled curvature.

Concentration phenomena on radial solutions to semilinear elliptic equations with the Trudinger-Moser growth

Daisuke Naimen
Muroran Institute of Technology, Japan

We study the asymptotic behaviour of radial solutions to semilinear elliptic equations with Trudinger-Moser critical nonlinearities. Via the blowup analysis, we classify the concentration compactness behaviour of them. Three different behaviour; concentration with zero weak limit, concentration with nontrivial weak limit, and strong convergence, are observed. We also deduce the limit equation and energy of them.

On the Moser-Trudinger inequality in fractional Sobolev-Slobodeckij spaces

Enea Parini
Aix Marseille Université, France

In this talk we consider the problem of finding the optimal exponent in the Moser-Trudinger inequality

$$\sup\left\{\int_\Omega \exp\left(\alpha\,|u|^{\frac{N}{N-s}}\right)\,\middle|\, u\in \widetilde{W}_0^{s,p}(\Omega),\ [u]_{W^{s,p}(\mathbb{R}^N)}\le 1\right\}<+\infty.$$

Here Ω is a bounded domain of $\mathbb{R}^N$ $(N \geq 2)$, $s\in(0,1)$, $sp=N$, $\widetilde{W}_0^{s,p}(\Omega)$ is a Sobolev-Slobodeckij space, and $[\cdot]_{W^{s,p}(\mathbb{R}^N)}$ is the associated Gagliardo seminorm. We exhibit an explicit exponent $\alpha^*_{s,N}>0$, which does not depend on Ω, such that the Moser-Trudinger inequality does not hold true for $\alpha\in(\alpha^*_{s,N},+\infty)$. The results have been obtained in collaboration with Bernhard Ruf (Milan).

Adams inequalities with exact growth condition for Riesz-like potentials on $\mathbb{R}^n$

Liuyu Qin
Hunan University of Finance and Economics, Peoples Republic of China

In this talk, I will present some of my recent results on Adams inequalities with exact growth condition for general Riesz-like potentials on $\mathbb{R}^n$. I will then present some consequences of such results, such as sharp Moser-Trudinger inequalities with exact growth condition for the fractional Laplacian and for homogeneous elliptic differential operators.

Extremal functions for Adams inequalities with Navier boundary conditions

Federica Sani
University of Modena and Reggio Emilia, Italy

We consider the problem of existence of extremal functions for second order Adams' inequalities with Navier boundary conditions on balls in $\mathbb{R}^n$ in any dimension $n\geq 4$. We also discuss some sharp weighted versions of Adams' inequality on the same spaces. The weights that we consider determine a supercritical exponential growth, except in the origin, and the corresponding inequalities hold for spherically symmetric functions only.

Improvements and generalizations of two Hardy type inequalities and their applications to the Rellich type inequalities

Megumi Sano
Hiroshima University, Japan

I will give improvements and generalizations of both the classical Hardy inequality and the geometric Hardy inequality including the best constants. Next, I will mention about a relation between virtual minimizers of these Hardy type inequalities. Finally, we will generalize them to Rellich type inequalities and will mention about open problems in term of best constants of Rellich type inequalities.

Deficit estimates for the logarithmic Sobolev inequality for the Tsallis entropy and its application

Takeshi Suguro
Osaka Metropolitan University, Japan

The logarithmic Sobolev inequality is a functional inequality that means control of the Boltzmann–Shannon entropy by the Fisher information of probability density functions. It is well-known that the optimizer for this inequality is the Gauss function. In this talk, we consider the logarithmic Sobolev inequality for the Tsallis entropy, a one-parameter extension of the Boltzmann–Shannon entropy. We obtain deficit estimates for the logarithmic Sobolev inequality corresponding to the Tsallis entropy and consider its application to the generalized Heisenberg uncertainty relation.

Sharp Hardy-Leray and related inequalities for vector fields with differential constraints

Futoshi Takahashi
Osaka Metropolitan University, Japan

In this talk, we show several weighted Hardy type inequalities for vector fields with curl-free conditions. We prove the sharpness and non-attainability of constants involved in the inequalities. This talk is based on the joint works with Naoki Hamamoto (Osaka Metropolitan University).

Geometric properties of sliced Wasserstein metric

Asuka Takatsu
Tokyo Metropolitan University, Japan
Jun Kitagawa

The Wasserstein metric is a metric on the space of probability measures on a complete separable metric space. Even on Euclidean space, the Wasserstein metric is not easy to compute except for the one-dimensional case. To reduce the computational complexity, the sliced Wasserstein metric is introduced. In this talk, I introduce a two-parameter family of metrics on Euclidean space, including the sliced Wasserstein metric. I discuss its geometric properties emphasis on the difference from the Wasserstein metric. This is joint work with Jun Kitagawa (Michigan State University).

Some remarks on mass-weighted Trudinger-Moser type inequalities

Cristina Tarsi
Università degli Studi di Milano, Italy

We discuss some recent results on Trudinger-Moser type inequalities on the whole space $\mathbb{R}^N$ with an *increasing radial weight*, in the framework of Sobolev mass-weighted spaces. The presence of an increasing weight prevents us us to use rearrangement arguments: we perform instead a suitable change of variable, which allow us to prove non sharp Trudinger-type inequalities and to determine the sharp Moser exponents. We also address the problem of the corresponding sharp Moser type inequalities and related concentration/vanishing phenomena.

Concentrating bound states for singularly perturbed nonlinear Dirichlet problems involving critical growth

Jianjun Zhang
Chongqing Jiaotong University, Peoples Republic of China

We consider the following singularly perturbed elliptic problem

$$-\varepsilon^2\Delta u + u = f(u) \text{ in } \Omega,\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega,$$

where Ω is a domain in $\mathbb{R}^N (N \geq 3)$, not necessarily bounded, with boundary $\partial\Omega \in C^2$ and the nonlinearity f is of critical growth. In this paper, we construct a family of bound state solutions to the equation given above which concentrates around the local maxima of the distance function from the boundary $\partial\Omega$.

Special Session 13: Nonlinear Differential and Difference Equations with Applications to Population Dynamics

Kunquan Lan, Toronto Metropolitan University, Canada
Elena Braverman, University of Calgary, Canada
Gunog Seo, Colgate University, USA

This session will bring together researchers working on nonlinear differential and difference equations, as well as their applications to population dynamics modeling. It will not only provide an opportunity to present and discuss recent developments and future directions in this area, but also enable participants to identify primary ideas and open questions and encourage future interdisciplinary collaborations.

Analysis of a competitive reaction-diffusion population dynamics model with density dependent dispersal on the boundary

Ananta Acharya
University of North Carolina Greensboro, USA
S. Bandyopadhyay, J. Goddard II, A. Muthunayake, R. Shivaji

We study a competitive reaction-diffusion population dynamics model with density dependent dispersal on the boundary. To analyze the model using computational methods in one dimension we take positive and negative density dependent dispersals separately. Namely, for positive density dependent dispersal case, we take $h(u) = 1 + \varepsilon u$ and for negative case we take $h(u) = \frac{1}{1+\varepsilon u}$. We obtain bifurcation curves using the combination of quadrature method and shooting method for various values of ε in each case.

Maximum, anti-maximum principles and monotone methods for boundary value problems for Riemann-Liouville fractional differential equations in neighborhoods of simple eigenvalues

Paul Eloe
Univeristy of Dayton, USA
Jeffrey T. Neugebauer

It has been shown that, under suitable hypotheses, boundary value problems of the form, $Ly + \lambda y = f$, $BCy = 0$ where L is a linear ordinary or partial differentiable operator and BC denotes a linear boundary operator, then there exists $\Lambda > 0$ such that $f \geq 0$ implies $\lambda y \geq 0$ for $\lambda \in [-\Lambda, \Lambda] \setminus \{0\}$, where y is the unique solution of $Ly + \lambda y = f$, $BCy = 0$. So, the boundary value problem satisfies a maximum principle for $\lambda \in [-\Lambda, 0)$ and the boundary value problem satisfies an anti-maximum principle if $\lambda \in (0, \Lambda]$. Let $\alpha \in (1, 2]$. In an abstract result, we shall provide suitable hypotheses such that boundary value problems of the form, $D_0^\alpha y + \beta D_0^{\alpha-1} y = f$, $BCy = 0$ where D_0^α is a Riemann Liouville fractional differentiable operator of order α and BC denotes a linear boundary operator, then there exists $\mathcal{B} > 0$ such that $f \geq 0$ implies $\beta D_0^{\alpha-1} y \geq 0$ for $\beta \in [-\mathcal{B}, \mathcal{B}] \setminus \{0\}$, where y is the unique solution of $D_0^\alpha y + \beta D_0^{\alpha-1} y = f$, $BCy = 0$. Two examples are provided in which the hypotheses of the abstract theorem are satisfied to obtain the sign property of $\beta D_0^{\alpha-1} y$. The boundary conditions are chosen so that with further analysis a sign property of βy is also obtained. One application of monotone methods is developed to illustrate the utility of the abstract result.

Seasonal influence on age-structured invasive species with yearly generation

Jian Fang
Harbin Institute of Technology, Peoples Republic of China

How do seasonal successions influence the propagation dynamics of an age-structured invasive species? We investigate this problem by considering the scenario that the offsprings are reproduced in spring and then reach maturation in fall within the same year. For this purpose, a reaction-diffusion population model is proposed, with yearly periodic time delay and spatially nonlocal response caused by the periodic developmental process. Further, we find some complex spatiotemporal dynamics for this model.

Solutions for a class of nonlinear fractional difference equations at resonance

Wenying Feng
Trent University, Canada
H. Chen, Y. Cui, S. Kang, Y. Lu

We study a class of nonlinear fractional difference equations with nonlocal boundary conditions at resonance. The system is inspired by the three-point boundary value problem for differential equations that have been extensively studied. It is also an extension to a fractional difference equation arising from real-world applications. Applying the coincidence degree theory for semi-linear operators, we are able to obtain sufficient conditions for the existence of solutions. In addition, we prove a new property of the Gamma function and construct a family of examples to illustrate the applications of the results.

Elimination, permanence, and exclusion in a competition model under Allee effects

Wei Feng
University of North Carolina Wilmington, USA
Yaw Chang, Michael Freeze, Xin Lu

We study a system of nonlinear partial differential equations that models the population dynamics of two competitive species both under Allee effects. The consideration of the model includes Logistic growth with Allee effects, Lotka-Volterra competition, diffusion, initial density and boundary conditions on the habitat. In the reaction-diffusion system, we employ the method of upper and lower solutions to address questions on self-elimination or persistence, as well as permanence or competitive exclusion. Specific conditions on biological parameters are explicitly given for extinction, coexistence and competitive exclusion of the species under various boundary conditions. Numerical simulations for the model are demonstrated to illustrate our results from mathematical analysis.

Monotonicity and convexity for discrete fractional operators with applications to fractional difference equations

Christopher S. Goodrich
University of New South Wales, Australia

I will discuss the connections between the sign of a discrete fractional operator evaluated at a point n, such as

$$\left(\Delta^{\beta} \circ \Delta^{\alpha} u\right)(n),$$

and the monotonicity and convexity of the function u. Some representative results in the area will be discussed. Applications to the analysis of fractional difference equations will be mentioned.

On the effects of density-dependent dispersal on ecological models with logistic and weak Allee type growth terms

Alketa Henderson
University of North Carolina Greensboro, USA
A. Acharya, N. Fonseka, J. Goddard II, R. Shivaji

We analyse positive solutions to the steady states reaction diffusion equation:

$$\begin{cases} -\Delta u = \lambda \frac{1}{a} u(1-u)(a+u); \ \Omega \\ \frac{\partial u}{\partial \eta} + \gamma\sqrt{\lambda} g(u) u = 0; \ \partial\Omega \end{cases}$$

where $a > 0, \gamma > 0$, and $\lambda > 0$ are parameters, Ω is a bounded domain in $\mathbb{R}^N$; $N > 1$ with smooth boundary $\partial\Omega$ or $\Omega = (0,1)$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of u. In this paper, we study three emigration forms. Namely, we consider density independent emigration ($g = 1$), a negative density dependent emigration of the form $g(s) = \frac{1}{1+\beta s}$, and a positive density dependent emigration of the form $g(s) = 1 + \beta s$, where $\beta > 0$ is a parameter. We establish existence, nonexistence, and multiplicity results for ranges of λ depending on the choice of the function g.

The role of demographic and environmental stochasticity in a single population model with component Allee effects

Yun Kang
Arizona State University, USA
Feng Tao, Hongjuan Zhou

Different types of stochasticity play essential roles in shaping complex population dynamics. This paper presents a novel approach to model demographic and environmental stochasticity in a single-species model with cooperative components that are measured by component Allee effects. Our work provides rigorous mathematical proof on stochastic persistence and extinction, ergodicity (i.e., the existence of a unique stationary distribution) and the existence of a nontrivial periodic solution to study the impacts of demographic and environmental stochasticity on population dynamics. The theoretical and numerical results suggest that stochasticity may affect the population system in a variety of ways, specifically: (i) In the weak Allee effects case (e.g., strong cooperative efforts), the demographic stochasticity from the attack rate contributes to the expansion of the population size, while the demographic stochasticity from the handling rate and the environmental stochasticity have the opposite role, and may even lead to population extinction; (ii) In the strong Allee effects case (cooperative efforts not strong enough), both demographic and environmental stochasticity play a similar role in the survival of population, and are related to the initial population level: if the initial population level is large enough, demographic stochasticity and environmental stochasticity may be detrimental to the survival of population, otherwise if the initial population level is small enough, demographic stochasticity and environmental stochasticity may bring survival opportunities for the population that deterministically would extinct indefinitely; (iii) In the extinction case, demographic and environmental stochasticity cannot change the trend of population extinction, but they can delay or promote population extinction.

Inertial effects in variational inequality problems and fixed point problems

Somayya Komal
University of Mianwali, Mianwali, Punjab, Pakistan
Poom Kumam, Auwal Bala Abubakar, Wachirapong Jirakitpuwapat

In this article, we introduce methods for finding the common element of the set of fixed points of a quasi nonexpansive mapping and the set of solutions of the variational inequality problem with inertial effects. Also, we establish a weak convergence theorem and strong convergence theorem for proposed algorithms. Finally, we describe the performance of our proposed algorithm with the help of numerical experiment and we show the efficiency and advantage of the inertial proposed methods.

User traffic dynamics in online social networks

Lingju Kong
University of Tennessee at Chattanooga, USA
Min Wang

We propose a susceptible-infected-recovered-susceptible (SIRS) model to study the user adoption and abandonment dynamics of online social networks (OSNs). Two types of abandonment dynamics are considered in our model: (1) infectious abandonment due to internal influences by individuals who already quit the network and (2) noninfectious abandonment due to mass media, commercials, or emergence of new products. The well-poseness of the model is first verified and the existence and stability of its equilibria are then discussed. Extensive numerical simulations are performed to validate the theoretic results. The effectiveness of the model is demonstrated by fitting it to the historical Facebook daily active user data, and finally, the fitted model parameter values are used to predict the numbers of future Facebook active users.

Rich and realistic dynamics of resource quality based population models

Yang Kuang
Arizona State University, USA

All organisms are composed of multiple chemical elements such as nitrogen (N), phosphorus (P), and carbon (C). P is essential to build nucleic acids (DNA and RNA) and N is needed for protein production. To keep track of the mismatch between P requirement in the consumer and the P content in the producer, stoichiometric models have been constructed to explicitly incorporate food quality and quantity. Most stoichiometric models have suggested that the consumer dynamics heavily depend on P content in the producer when the producer has low nutrient content (low P:C ratio). Motivated by recent lab experiments, researchers explored the effect of excess producer nutrient content (extremely high P:C ratio) on the grazer dynamics. This phenomenon is called the stoichiometric knife edge. However, the global analysis of these resource quality based models is challenging because the phase plane/space is separated into many regions in which the governing nonlinear equations are different. The aim of this talk is to present an overview of the rich and novel dynamics embodied in these stoichiometric population models and its many biological implications and present an alternative framework to build mathematically more tractable and biologically more plausible models.

Pause of larval development and their consequences for stage-structured populations

Rongsong Liu
University of Wyoming, USA
Hermann Brunner, Stephen A. Gourley, Yanyu Xiao

We consider some novel predictions of a mathematical model for a stage-structured insect species that undergoes diapause if faced with strong intraspecific competition among larvae. The model consists of a system of two delay differential equations with a state-dependent time delay of threshold type. When the model has an Allee effect, we show that diapause may cause extinction in some parameter regimes even where the initial population is high. We also demonstrate that the model can have diapause-induced periodic solutions that can arise even if the birth function is strictly increasing, a situation in which solutions for the constant delay case always converge to an equilibrium.

Modeling the effects of trait-mediated dispersal on coexistence of competitors and coexistence of predator-prey species

Dustin Nichols
University of North Carolina Greensboro, USA
Ananta Acharya, Emily Cosgrove, James T. Cronin, Jerome Goddard II, Eddie Lindsey, Amila Muthunayake, Ratnasingham Shivaji

We analyze positive solutions (u, v) to the reaction-diffusion system:

$$\begin{cases} -\Delta u = \lambda u(1-u);\ \Omega \\ -\Delta v = \lambda r v(1-v);\ \Omega \\ \frac{\partial u}{\partial \eta} + \sqrt{\lambda} g(v) u = 0;\ \partial\Omega \\ \frac{\partial v}{\partial \eta} + \sqrt{\lambda} h(u) v = 0;\ \partial\Omega \end{cases}$$

where Ω is a bounded domain (patch) in $\mathbb{R}^N$ of unit length, area, or volume; $N \in \{1, 2, 3\}$ with smooth boundary $\partial\Omega$, $\lambda > 0$ is a parameter proportional to the square of patch size and $\frac{\partial z}{\partial \eta}$ is the outward normal derivative of z. Here u and v represent the normalized densities of two species which inhabit the patch, surrounded by a hostile matrix, where the level of hostility is determined by the functions $g, h \in C^1([0,1],(0,\infty))$. Finally, $r > 0$ compares the two species by the ratio of patch intrinsic growth to patch diffusion rate. We explore two cases: (1) u and v are competing species (g and h are increasing) and (2) v competes with u while u cooperates with v (g increasing, h decreasing). We establish coexistence and nonexistence results analytically for certain ranges of λ depending on the characteristics of g and h. We prove our coexistence results via the method of subsolutions and supersolutions. We discuss some interesting ecological phenomena observed in our numerical simulations.

A non-spatial host-parasitoid model for pest control: Insights from bifurcation theory

Gunog Seo
Colgate University, USA
Gail S. K. Wolkowicz

The horse-chestnut leaf miner is a pest that has spread throughout Europe, and controlling its population is a challenge. In a previous study, a non-spatial host-parasitoid model was proposed for controlling the leaf miner, where a generalist parasitoid preys on the leaf miners with a Holling type II functional response. The model identified up to six equilibrium points and discussed their local stability. Here, we revisit the non-spatial model and identify cases that were not explored in the previous investigation. Using a bifurcation theoretical approach, we consider both local stability and global properties of the model. We provide analytical expressions for fold and Hopf bifurcations and the criticality of Hopf bifurcations. Our numerical results reveal interesting dynamics resulting from various bifurcations such as Hopf, fold, transcritical, cyclic-fold, and homoclinic bifurcations of codimension one and Bautin and Bogdanov-Takens bifurcations of codimension two, as well as a Bogdanov-Takens bifurcation of codimension three. Our findings have significant implications for potential pest control strategies.

Population dynamics in heterogeneous networks

Zhisheng Shuai
University of Central Florida, USA

Population dynamics are heavily influenced by spatial heterogeneity and movement. When dispersal occurs in a discrete environment, a connectivity matrix and corresponding network can be used to represent the movement. This results in a mathematical model that is a coupled dynamical system on a network. Our research focuses on the impact of the coupling strength and topological structure of the dispersal network on population dynamics. In collaboration with Shanshan Chen (Harbin Institute of Technology), Junping Shi (College of William & Mary), and Yixiang Wu (Middle Tennessee State University), our recent findings highlight the dual nature of population persistence or extinction (infectious disease invasion or eradication) in relation to the strength of dispersal.

Impact of threshold harvesting on the dynamics of age-structured populations

Elisa Sovrano
University of Modena and Reggio Emilia, Italy
E. Liz

In this talk, we investigate the impact of threshold harvesting rules on discrete-time population models with age structure. Specifically, we analyze how different thresholds for each age class influence the dynamics of the population. We discuss the existence of equilibria and their stability. We establish precise conditions for hydra effects and demonstrate that harvesting does not destabilize globally stable equilibria. In addition, our research yields insights into the emergence of unusual bifurcations, such as border-collision bifurcations. This is joint work with E. Liz (University of Vigo).

Two examples of using phase plane analysis in nonlinear RDA models

Olga Vasilyeva
Memorial University of Newfoundland, Canada

Steady states of nonlinear reaction-diffusion-advection (RDA) models can be viewed as solutions of a system of two first order ODEs (subject to appropriate boundary conditions). Geometrically, they are represented by orbits in the phase plane, generated by the corresponding flow operator. In this talk, I will discuss applications of the phase plane technique in two extensions of a logistic reaction-diffusion-advection model. In one setting, we increase the complexity of the habitat by considering a binary river network. Here a steady state is represented by a configuration of orbits in the corresponding phase plane satisfying geometric constraints induced by junction and boundary conditions. Proving a certain concavity preservation property of the flow of the corresponding system of ODEs allows us to establish uniqueness of the positive steady state solution, as well as explicit existence conditions. In the second setting (joint work with Abby Anderson), we increase the complexity of the reaction term. Namely, we study an RDA version of the classical Ludwig-Aronson-Weinberger spatial spruce budworm model (where reaction term combines logistic growth and predation by generalist), with advection term describing biased movement of larvae due to prevailing winds. We use phase plane analysis to determine conditions for the existence of the outbreak solutions. In particular, we observe that increasing advection can prevent outbreaks while allowing persistence in the form of an endemic state. We obtain upper and lower bounds for the critical advection for outbreak steady state solutions.

Online social network models with varying population size

Min Wang
Kennesaw State University, USA
Lingju Kong

In this talk, ordinary differential equation (ODE) and stochastic differential equation (SDE) models are developed to describe the dynamics of online social network user adoption and abandonment. A series of criteria on the uniqueness and positivity of solutions as well as the stability of the user-free equilibrium for both ODE and SDE models are obtained. Numerical simulations are then carried out to verify the qualitative analysis results.

Different derivation techniques for population growth difference equation models and their dynamics

Gail Wolkowicz
McMaster University, Canada
Sabrina H. Streipert

Depending upon the underlying assumptions and reason for including delay in a model of population growth, we propose different strategies for deriving discrete models predicting growth of a single population. We then analyze the resulting models. The dynamics of the models that we introduce differ from existing logistic delay difference equations, such as the delayed logistic difference equation that was formulated as a discretization of the Hutchinson model. In all cases, we identify an important critical delay threshold that depends on the length of the delay and other parameters in the model. If the length of the time delay exceeds this threshold, the models predict that the population will go extinct for any non-negative initial conditions. Below this threshold, there is at least one equilibrium at which the population survives. We discuss the number of survival equilibria, their stability, and how their magnitude depend on the length of the delay.

Population die-outs in ecosystems

James Yorke
University of Maryland, USA
Naghmeh Akhavan, Sana Jahedi, Tim Sauer

The understanding of the structure of ecological networks is inadequate in many areas. We need tools to explain why some species persist. Networks are often presented as graphs where each species is a node, and edges represent species interactions. Which links are crucial for the robustness of the network, links without which the system is fragile and species are lost? How many and which species must be removed from a fragile network for it to become robust? We develop a theory of die-outs where many Lyapunov die-out functions are needed.

Special Session 14: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

Shaohua Chen, Cape Breton University, Canada
Ming Mei, McGill University, Canada

This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include fluid dynamics, delay, localized, nonlocal, degenerate evolution equations, traveling waves, steady states and their properties. It will be a great opportunity to bring the experts in this field to exchange their recent results and to enhance collaborations.

Global solutions for the 1-D compressible Euler equations with time-dependent damping

Shaohua Chen
Cape Breton University, Canada

In this talk we investigate the Cauchy problem for the 1-D compressible Euler equations with time- dependent damping. We prove the existence of global solutions under the assumptions that the derivatives of initial data are suitable small and the initial volume is large without the condition of small perturbations to the constant initial data. Our approach is based on estimates of the derivatives of Riemann invariants along two characteristic curves.

Blowup in damped abstract nonlinear equations

Jorge Esquivel-Avila
Universidad Autónoma Metropolitana, México

We present our analysis in an abstract framework. A typical example is an evolution equation in viscoelasticity, linearly damped and with a nonlinear source term. We give sufficient conditions on the initial data to conclude nonexistence of global solutions for any positive value of the initial energy, in particular for high energies. We compare our results with those in the literature and we give more examples to illustrate the applicability of the abstract formulation. Our talk is based in the following papers.

- Esquivel-Avila, Jorge A., Blowup in damped abstract nonlinear equations, *Electronic Research Archive*, Vol 28, no2, 2020, 549-567.
- Esquivel-Avila, Jorge A., Nonexistence of global solutions for a class of viscoelastic wave equations, *Discrete and Continuous Dynamical Systems Series S*, Vol 14, 2021, 4213-4230.

Impact of nonlocal dispersal and time-periodicity on the global exponential stability of bistable traveling waves

Chunhua Ou
Memorial University, Canada
Manjun Ma, Wentao Meng

Global exponential stability of bistable traveling wave solutions to a periodic Lotka-Volterra system with nonlocal dispersal is investigated. We first establish a refined squeezing upper and lower solutions based on the property of the bistable wavefront. This important squeezing technique has been successfully applied to study the exponential convergence of solutions.

Gradient integrability for parabolic p-Laplace type equations with measure data

Jung-Tae Park
Korea University of Technology and Education, Korea

In this talk, we consider a parabolic p-Laplace type equation when the right-hand side is a signed Radon measure with finite total mass, whose model is

$$u_t - \operatorname{div}\left(|Du|^{p-2}Du\right) = \mu \quad \text{in } \Omega \times (0,T) \subset \mathbb{R}^n \times \mathbb{R}.$$

In the singular range $\frac{2n}{n+1} < p \leq 2 - \frac{1}{n+1}$, we discuss integrability results for the spatial gradient of a solution in the Marcinkiewicz space, under a suitable density condition of the right-hand side measure μ.

Regularity and long term dynamics of some nonlinear evolution equations

Maria Michaela Porzio
Sapienza University of Rome, Italy

We will present some recent advances for a class of nonlinear evolution equations.
Particular attention will be given to the qualitative properties, regularity and behavior in time of the solutions to some nonlinear parabolic equations appearing in many physical applications and including the p-Laplacian equation and nonlinear versions of the heat equation.

We will show cases when the influence of the solutions to suitable elliptic problems appears. Finally, we will discuss what happens in presence of non-zero forcing terms.

Existence of weak solutions to a nonlocal reaction-diffusion-advection system

Yurij Salmaniw
University of Alberta, Canada
D. Liu, J. Potts, J. Shi, H. Wang

Reaction-Diffusion equations have been applied rather successfully to model various biological/ecological phenomena. More recently, authors have begun to consider the influence of cognition (e.g., perception, memory, learning) in animal movement models. In many cases, this leads to nonstandard reaction-diffusion-advection equations and systems that are both nonlocal (in space) and nonlinear at higher order. In this talk, I will explore some recent advances made in a single species animal movement model paired with a so-called cognitive map. For this session, I will focus on the existence of global weak solutions for two biologically reasonable scenarios featuring slightly different hypotheses. This is joint work with colleagues Di Liu, Jonathan Potts, Junping Shi and Hao Wang.

On the stability with asymptotic phase of semi-wavefronts

Abraham Solar
Catholic University of the Most Holy Conception, Chile
Sergei Trofimchuk

In this talk we preset some stability results of semi-wavefronts to the equation $u_t(t,x) = u_{xx}(t,x) + f(u(t,x), u(t-h,x)), \quad t>0, x\in\mathbb{R}$ where $h>0$ is a delay. Unlike non delayed case, we show that the generated solution by an initial datum which is similar to a semi-wavefront $\phi_c(\cdot)$ at time $t=0$ will converge to $\phi_c(x\cdot+ct+a)$, as $t\to+\infty$, where the phase $a\in\mathbb{R}$ is not equal to zero when $f(u,v) = -u+g(v)$.

Finite-energy self-similar solutions describing singularity formation in the nonlinear Schrödinger equation in dimension $N = 3$

William Troy
University of Pittsburgh, USA

In dimension $N = 3$ the cubic nonlinear Schrödinger (NLS) equation has solutions which become singular, i.e. at a spatial point they blow up to infinity in finite time. In 1972 Zakharov famously investigated finite time singularity formation in the cubic nonlinear Schrödinger equation as a model for spatial collapse of Langmuir waves in plasma, the most abundant form of observed matter in the universe. Zakharov assumed that (NLS) blow up of solutions is self-similar and radially symmetric, and that singularity formation can be modeled by a solution of an associated self-similar, complex ordinary differential equation (ODE). A parameter $a > 0$ appears in the ODE, and the dependent variable, Q, satisfies $(Q(0), Q'(0)) = (Q_0, 0)$, where $Q_0 > 0$. A fundamentally important step towards putting the Zakharov model on a firm mathematical footing is to prove, when $N = 3$, whether values $a > 0$ and $Q_0 > 0$ exist such that Q also satisfies the physically important 'zero-energy' integral constraint.
Here, we discuss this issue and present several open problems.

Special Session 15: Recent Advances on Population Models in Ecology and Epidemiology

Junping Shi, William and Mary, USA
Zhisheng Shuai, University of Central Florida, USA
Yixiang Wu, Middle Tennessee State University, USA

Mathematical tools such as ordinary and partial differential equation theory, dynamical system theory, matrix theory, network theory, have been used extensively to study population models arising from ecology and epidemiology. Such studies have promoted our understanding of many natural phenomena such as species competition and cooperation and the transmission infectious diseases. In this session, we will bring several studies related to these topics and discuss recent advances on theoretical investigations of population dynamics.

The diffusive Lotka-Volterra competition model in fragmented patches I: Coexistence

Ananta Acharya
University of North Carolina Greensboro, USA
S. Bandyopadhyay, J. Goddard II, A. Muthunayake & R. Shivaji

We study the positive solutions to the reaction diffusion model

$$(*)\begin{cases} -\Delta u = \lambda u(1-u-b_1 v);\ \Omega \\ -\Delta v = \lambda r v(1-v-b_2 u);\ \Omega \\ \frac{\partial u}{\partial \eta} + \gamma_1\sqrt{\lambda}u = 0;\ \partial\Omega \\ \frac{\partial v}{\partial \eta} + \gamma_2\sqrt{\lambda}v = 0;\ \partial\Omega \end{cases}$$

which describes the steady states of two species u and v competing in a habitat Ω. Here b_1, b_2 represent the strengths of competition, λ represents a patch size measure, and γ_1, γ_2 are related to the hostility of the exterior domain. We analyze the positive solutions of (*) as the parameters b_1, b_2 and γ_1, γ_2 vary.

The effect of global warming on plant population dynamics on barrier islands

David Chan
VCU, USA
Julie Zinnert, Reed Ogrosky, Kiran Shrestha, Beth Thomas, Kezie Osei

We present a cellular model of barrier island evolution, and will focus on how plants evolve under the stress of global warming. The effects of global warming include sea level rise and more frequently and stronger storms. These storms often include overwash events that can effect the vegetation located on the island. This vegetation in turn also impacts the resistance of these islands to evolve over time.

A mathematical model to assess the efficacy of Wolbachia transinfection in mosquitoes in controlling dengue fever outbreaks

Vinodh Kumar Chellamuthu
Utah Tech University, USA
Jisun Otterson

Dengue is a viral infection transmitted by mosquitoes that primarily occurs in tropical and subtropical regions worldwide. The use of Wolbachia bacterium to infect mosquitoes has been shown to disrupt the cycle of dengue transmission, reducing the level of the virus in mosquitoes and shortening their lifespan. Studies have demonstrated that infecting and releasing Wolbachia-infected mosquitoes through Transinfection can spread the bacterium to local mosquito populations and mitigate the disease's impact. To investigate the potential for Wolbachia to reduce Dengue virus transmission in humans, a mathematical model was developed that considers local temperature data, which affects mosquito reproduction and growth. The model simulation suggests that the outbreak of Dengue fever can be mitigated by releasing a small number of Wolbachia-infected mosquitoes at the appropriate time.

The impact of travel restriction on patterns of disease dynamics for multi-patch models

Seoyun Choe
University of Central Florida, USA
Zhisheng Shuai

After the beginning of the COVID-19 pandemic around the world, travel restriction policies internationally and domestically have been significant issues. In mathematical epidemiology, there are several mathematical modelings about the impact of varying residence times or travel restrictions, such as lockdowns on the infectious disease dynamics in a heterogeneous environment. We set two kinds of multi-patch models: Lagrangian and Euler models. For the Euler model, we explored how the travel frequency (restriction) affects the pattern of disease dynamics for a multi-patch model. For the Lagrangian model, we proved that the basic reproduction number is monotonically decreasing with respect to the travel restriction factor. Also, we derived the final size relation by using the weighted geometric mean. Numerical simulations illustrate that the final size of the outbreak depends on the travel restriction measure as well as the transmissibility. Moreover, we investigated patch-specific optimal treatment strategies.

Analysis of two-group Malaria model incorporating vaccination and optimal control

Chidozie Chukwu
Wake Forest University, USA
S. Y. Tchoumi, M. L. Diagne, H. Rwezaura, M. L. Juga, J. M. Tchuenche

Malaria is a vector borne disease that poses significant health challenges globally with the highest burden in children under 5 years old. Prevention and treatment have been the main intervention measures until the recent groundbreaking highly recommended malaria vaccine by WHO for children below five. A two group malaria model structured by age with vaccination of individuals aged below 5 years old is formulated and theoretically analyzed. The disease free equilibrium is globally asymptotically stable when the disease-induced death rate in both human groups is zero. Descarte rule of signs discusses the possible existence of multiple endemic equilibria. By construction, mathematical models inherit the loss of information that could make the prediction of model outcomes imprecise. Thus, a global sensitivity analysis of the basic reproduction number and the vaccination class as response functions using Latin Hypercube Sampling in combination with partial rank correlation coefficient are graphically depicted. The most sensitive parameters relate to children under 5 years old. Applying optimal control theory, the best combination of intervention measures to mitigate the spread of malaria is investigated. Simulation results show that concurrently applying the three intervention measures, namely: personal protection, treatment and vaccination of children under five is the best strategy for fighting against the malaria epidemic in any community relative to using either single or any dual combination of interventions at a time.

Impact of population dispersal on disease prevalence

Daozhou Gao
Cleveland State University, USA
Yuan Lou, Ailing Wang, Xin Li

Human movement not only facilitates disease spread but also poses a serious challenge to disease control and eradication. In reality, disease eradication is rather difficult or even impossible for any infectious diseases. Thus, reducing disease prevalence (proportion of people being infected) to a low level is more feasible and cost-effective goal. The basic reproduction number can serve as a threshold for disease persistence and extinction, but it usually cannot measure the endemic level. In this talk, based on an SIS patch model, I will explore the impact of the movement of susceptible and infected populations on the local and global disease prevalence. In particular, for the two-patch case, two complete classifications of the model parameter space are given: one addressing when dispersal leads to larger or smaller global disease prevalence than no dispersal, and the other concerning how the global disease prevalence varies with dispersal rate. Some results on an SIAR patch model will also be given and compared. This is a joint work with Yuan Lou, Ailing Wang and Xin Li.

Ecological release and patch geometry can cause nonlinear density-area relationships

Jerome Goddard II
Auburn University Montgomery, USA
J. T. Cronin, R. Shivaji

A primary driver of species extinctions and declining biodiversity is loss and fragmentation of habitats owing to human activities. Many studies spanning a wide diversity of taxa have described the relationship between population density and habitat patch area, i.e., the density-area relationship (DAR), as positive, neutral, negative or some combination of the three. However, the mechanisms responsible for these relationships remain elusive. In this talk, we will discuss implementation of a reaction-diffusion framework with absorbing boundary conditions to model a habitat specialist dwelling in islands of habitat surrounded by a hostile matrix. We consider patches with both a convex and non-convex geometry. Our results show overall DAR structure can be either 1) positive, 2) positive for small areas and neutral for large, or 3) hump-shaped, i.e., positive for area below a threshold and negative for area above. We will also discuss comparison of our theoretical results with two empirical studies. Close qualitative agreement between theoretical and observed DAR indicates that our model gives a reasonable explanation of the mechanisms underpinning DAR found in those studies.

Traveling pulses and their bifurcations in a diffusive Rosenzweig MacArthur model

Xiaojie Hou
University of North Carolina Wilmington, USA
Yi Li

The existence of three different types of traveling pulses with monotone tails in a diffusive Rosenzweig MacArthur model with a small parameter is studied, and we further show that the pulses are connected by a bifurcation surface.

Dispersal-induced growth: A mathematical analysis

Guy Katriel
Braude College, Israel

The effect of dispersal of organisms among different habitats on survival and growth of populations is an important issue in ecological research. Dispersal-induced growth (DIG), also known as the inflationary effect, is a surprising phenomenon which has been identified by theoretical ecologists using mathematical models, and has also been experimentally confirmed. DIG occurs when populations of a species inhabiting different sites with growth rates which fluctuate in time, and with dispersal among them, are able to persist and grow, despite the fact that each site is of such a low quality that it would not be able to sustain a population in the absence of dispersal. The work to be presented offers a mathematical analysis of the DIG phenomenon, with the aim of identifying the conditions under which this phenomenon occurs, in the case of periodic (seasonal) variation of growth rates. The analysis relies on the study of periodic linear dynamics systems (Floquet theory), taking advantage of recent results of Liu, Lou, and Song on the principal eigenvalue associated with time-periodic patch models.

Optimal impulse control of a late-season model of a West Nile virus epidemic

Rachel Leander
Middle Tennessee State University, USA
Folashade Agusto, Daniel Bond, Adira Cohen, Wandi Ding, Allis Royer

This presentation concerns a late-season model of a West Nile virus epidemic that includes transmission between bird hosts and mosquito vectors, mosquito life stages, and pesticide dynamics. Our model describes three mosquito life stages; eggs, larvae, and adults, and three pesticides; two types of larvicide and one adulticide. The basic reproduction number for the model epidemic is analyzed in the absence of control, and impulsive optimal control problems are constructed. Objective functions are designed to balance the cost of the insecticide application schedule with the benefit of (1) vector control: reducing the number of mosquitoes or (2) disease control: reducing the disease burden. The resulting impulsive optimal control problems are then reformulated as nonlinear optimization problems in order to derive necessary conditions for the characterization of optimal controls. Numerical simulations are used to address three questions: How does the control and its impact on the system vary with the objective type? Is it beneficial to optimize the treatment timing? How does the control and its impact on the population vary with the type of pesticide used?

Modeling the impacts of temperature during nesting seasons on Loggerhead sea turtles populations in South Florida

Suzanne Lenhart
University of Tennessee at Knoxville, USA

Loggerhead sea turtles (Caretta caretta) are a threatened sea turtle species that nests on beaches along the northwestern Atlantic Ocean. Our goal is to mathematically explore the relationship between air temperature and emergence success of hatchings across multiple nesting seasons to better anticipate the potential impact of climate change on subsequent loggerhead populations. Using data from southeast Florida, we build a statistical model representing the relationship of air temperature and hatchling emergence success. The results of our statistical model of emergence success feed into the dynamics during the nesting seasons of the eggs and hatchlings in a submodel on daily time scale (in summer). The submodel is connected to an age-structure model with two juvenile and one adult classes on a yearly time scale. We illustrate the effect of temperature changes across these life stages. This application is a discrete time model with hybrid features; the nesting submodel with a daily time scale is embedded in the population model on a yearly time scale.

An approach to model the bird migration and the transmisssion dynamics of bird flu among migration birds

Rongsong Liu
University of Wyoming, USA
Rachel Jennings, Stephen A. Gourley

An approach to modelling bird migration is proposed, in which there is a region where birds do not move but spend time breeding. Birds leave this breeding region and enter a migration flyway which is effectively a one-way corridor starting and ending at the breeding location. Mathematically, the flyway is a curve parametrised by arc-length. Flight speed depends on position along the flyway, to take account of factors such as wind and the pausing of birds at various locations for wintering or stopovers. Per-capita mortality along the flyway is both position and age-dependent, allowing for increased risks at stopover locations due to predation, and increased risks to immature birds. We also model indirect transmission, via contact with viruses, of avian influenza in migratory and nonmigratory birds, taking into account age structure. Sufficient conditions are obtained for the local stability of the disease-free equilibrium (for a species without migration) and for the disease-free periodic solution (for a migratory species).

The effects of intransitive loops of competition on the stability of ecological communities

Daniel Maes
University of Michigan–Ann Arbor, USA
Annette Ostling

Intransitive loops of competition are akin to a game of Rock-Paper-Scissors where there is not one dominant competitor. A prevalent hypothesis for intransitive loop interactions is that even-lengthed loops of species are unstable whereas odd-lengthed ones are stable. While this claim is common throughout the literature, there has not been a clear analytical proof of this result in general. Here we use the Lotka-Volterra competition model to study intransitive loop dynamics. We employ analytical approaches applied to the case of a 3-species intransitive loop in which the competition strengths are constant across species pairs in the loop and extend these approaches to arbitrarily many species. We also use numerical analyses to test the robustness of these results to variation in competition strengths across species pairs. We show analytically in the symmetric case that the coexistence equilibrium point of the Lotka-Volterra competition model with even-lengthed loops is indeed locally unstable, while for odd-lengthed loops it is locally stable. With numerical simulations, we can also understand the impacts of competition variability and correlation on the stability of such systems. All these results solidify a long-standing but unproved assertion that odd-lengthed intransitive loops can lead to stable coexistence.

Front propagation for the bistable reaction-diffusion equation on unbounded metric graphs

Yoshihisa Morita
Ryukoku University, Japan

We deal with the bistable reaction-diffusion equation arising in the population biology and consider it on unbounded metric graphs which are created by joining half-lines (branches) at some points (junctions). Since the equation allows a traveling front wave in the whole line, we can observe the front propagation far from the junctions. We show that there exists an entire solution which converges to the front-like profile as time goes to minus infinity on arbitrarily fixed branches. Moreover, in specific cases we give conditions under which the front propagation is blocked. This talk is based on the joint works by Jimbo-M (2019, 2021, 2023) and Iwasaki-Jimbo-M (2022).

Maximizing metapopulation growth rate and biomass in stream networks

Tung Nguyen
Texas A & M University, USA
Y. Wu, A. Veprauskas, T. Tang, Y. Zhou, C. Beckford, B. Chau, X. Chen, B. D. Rouhani, Y. Wu, Y. Yang, Z. Shuai

We consider the logistic metapopulation model over a stream network and use the metapopulation growth rate and the total biomass of the positive equilibrium as measures for population persistence. Our objective is to find distributions of resources that maximize these persistence measures. We begin our study by considering stream networks consisting of three nodes and prove that the strategy to maximize the total biomass is to concentrate all the resources in the most upstream locations. In contrast, when the diffusion rates are sufficiently small, the metapopulation growth rate is maximized when all resources are concentrated in one of the most downstream locations. These two main results are generalized to stream networks with any number of patches.

Competitive coexistence of populations with hierarchical size structure

Annette Ostling
University of Texas Austin, USA
Ursula Trigos-Rczkowski

Size-structure is thought to be a critical feature of biological populations but its incorporation into ecological theory of competition is so far limited. Growing taller, especially for a tree, can mean access to resources otherwise blocked, and hence strategy variation related to how quickly an individual may grow, or the maximum size it may reach, or at what height or how much it reproduces, has the potential to create coexistence opportunities. Existing studies have only explored competitive coexistence of population structured by such hierarchical size effects to a limited degree. Particularly pressing is delineating which aspects of demography must negatively depend on the density of taller individuals to create niche differentiation opportunities, and which specific life history variation across species is involved in those niche opportunities. Here we investigate a variety of cases of a simple partial differential equation model of competition with continuous size structure to gain insight into these. We find that a negative dependence of the production of offspring on the density of taller individuals creates the opportunity for competitive coexistence, while negative density-dependence of recruitment, which acts equally on all recruiting individuals (who are smaller than all established individuals) does not. Work on density-dependent mortality is still in progress.

Modelling habitat loss with partial differential equations: Degradation, destruction and fragmentation

Yurij Salmaniw
University of Alberta, Canada
H. Wang, Z. Shen

Habitat loss is the key component driving decreases in biodiversity and the extinction of species worldwide. Of particular concern is human-driven habitat loss, primarily through agriculture, resource extraction, and urbanization. In this talk, I will share the key findings of my Ph.D. work focusing on understanding the effects of various forms of habitat loss through the framework of partial differential equations (PDEs). First, I will explore a habitat degradation model in a single- and two-species competition setting. Then, we connect this degradation model to a habitat destruction model in a limiting sense. We then conclude with a discussion of habitat fragmentation, what a PDE setting has to offer for this complex issue, and some ambitious goals for future extensions of this work. This is joint work with my current supervisors, H. Wang and Z. Shen.

Identifiability of compartment model for infectious diseases under both perfect and flawed data

Tingting Tang
San Diego State University, USA

Compartment modeling has been used extensively in epidemics to understand and predict infectious diseases. With the increasing data availability, mathematical models fit cumulative data are used to estimate disease key transmission parameters. During this process, one important question arises regarding the model identifiability which handles the question of whether parameters can be correctly and accurately recovered given available data. In this talk, I will demonstrate the problems in incidence data accuracy with Covid 19 cases in Imperial Vally. Then, I use both a simple SEIR model and a SI vector-host model to demonstrate the impact of data type, data resolution, and optimization tools used in parameter estimation in accessing models' identifiability.

Coexistence due to life history variation revisited in models with explicit patch aging

Ursula Trigos-Raczkowski
University of Michigan Ann Arbor, USA
Annette Ostling, Azmy S. Ackleh, Rainey Lyons, Mattias Delgadino

The question of how competing species coexist remains a major challenge in theoretical and mathematical ecology. Disturbance and the subsequent progression of competitive dynamics–or /textitaging of patches–have been posited as creating opportunities for coexistence. However, prior work studying this possibility has limitations. Most importantly it has not clearly delineated coexistence opportunities that arise from explicit consideration of the progression of competitive dynamics with patch aging. Here we study a nonlinear PDE model which accounts for patch aging while remaining analytically tractable in many cases. We consider two types of density-dependence under disturbance: 1) on reproduction and 2) on recruitment. We analyze when the models allow coexistence that is *feasible* (both species have positive populations) and *stable* (mutual positive long-term invasions). We confirm our results using a 2nd order flux limiter numerical method. Under density-dependent recruitment, our model does not permit feasible coexistence. Under density-dependent reproduction, a reproduction-survival trade-off allows for feasible and stable coexistence. One species must have both higher reproduction and higher mortality than the other. Intermediate relative reproduction and death rates–which could arise from intermediate values of disturbance–lead to a wider coexistence region.

Threshold dynamics of a nonlocal dispersal HIV/AIDS epidemic model with spatial heterogeneity and antiretroviral therapy

Xiunan Wang
University of Tennessee at Chattanooga, USA
Peng Wu, Hao Wang

Antiretroviral therapy and long-range diffusion of HIV-infected individuals in heterogeneous environments can greatly impact the transmission and distribution of HIV/AIDS in a population. In this talk, I will present a nonlocal dispersal HIV/AIDS epidemic model incorporating spatial heterogeneity as well as antiretroviral therapy to study the spatial and temporal dynamics of HIV/AIDS in China. We establish the well-posedness and the existence of the global attractor for the system. Then we derive the basic reproduction ratio R_0 and obtain the global dynamics of the system in terms of R_0. Our numerical simulations suggest that: (1) the dynamics of HIV/AIDS transmission largely depends on the nonlocal dispersal kernel function; (2) it is necessary to monitor and design/refine rules for the mobility of HIV-infected population between remote and metropolitan areas to control the HIV/AIDS transmission; (3) antiretroviral therapy benefits not only each treated individual but also the entire community, and increasing the therapy coverage rate is one of the most effective ways to prolong the longevity of HIV-infected population. Moreover, spatial dispersal is also an important factor that cannot be ignored for designing antiretroviral therapy strategies.

Multiscale models for cholera dynamics

Jin Wang
University of Tennessee at Chattanooga, USA

We present a few multiscale modeling studies for cholera, a severe waterborne infection. The models couple the disease transmission and spread at the population level with the pathogen-host interaction inside the human body. We discuss unidirectional and bidirectional links that bridge the two scales. We investigate the mutual impacts of the within-host and between-host dynamics through both mathematical analysis and numerical simulation.

Ideal free dispersal in integrodifference equation models

Ying Zhou
Lafayette College, USA
Robert Stephen Cantrell, Chris Cosner

The motivation of the work presented here is to find what dispersal strategies are evolutionarily stable strategies (ESS) in an integrodifference equation framework, and look at how dispersal can evolve towards such an evolutionarily stable strategy. I will present integrodifference equation models that are temporally static or temporally periodic in which pairwise invasion analysis have shown that dispersal strategies associated with ideal free distributions are evolutionarily stable. I will also present some simulation results for a model with stochastic fluctuations.

Special Session 16: Celestial Mechanics and Hamiltonian Systems

Manuele Santoprete, Wilfrid Laurier University, Canada
Cristina Stoica, Wilfrid Laurier University, Canada
Zhifu Xie, University of Southern Mississippi, USA
Marian Gidea, Yeshiva University, USA

This special session will concentrate on the latest developments in the field of celestial mechanics which laid the foundations for the birth of dynamical systems. The study of the N-body problem continues to attract researchers in a wide range of fields including dynamical systems, topology, variational methods, algebraic and symplectic geometry, numerical methods and KAM theory. This special session provides a marketplace for ideas, and helps identify trends and areas of research in the field. This session brings main specialists in the field as well as leaders of different areas in dynamical systems with young researchers together to build an active and stimulating framework suitable to attack some of the many unsolved aspects in Celestial Mechanics. Some specific topics to be covered include Hamiltonian systems, Ergodic theory, variational methods, central configurations, the N-body problem in spaces of constant curvature, discovery of new periodic solutions, regularization of collisions, index theory and symplectic invariants, stability of periodic solutions and finally spacecraft orbital design. If the schedule permits, we anticipate ending the session with a discussion on open problems.

Relative equilibria and periodic orbits in a binary asteroid model

Lennard Bakker
Brigham Young University, USA
Nick Freeman

Binary asteroids of nearly equal mass are rare, the fourth pair 2017 YE5 being discovered in 2017. A four-body problem we propose bridges the familiar restricted three-body problem, that is traditionally used to model the motion of single asteroids, and the more general four-body problem. This four-body problem allows the gravitational interaction of the asteroids in a binary pair, of any mass ratio, along with the gravitational interaction of two primaries. We show in the equal primary mass and in equal binary asteroid mass case the analytic existence of relative equilibria and periodic orbits, some near the Eulerian type relative equilibria, and some of Hill type where asteroid orbits a distinct primary. We also show in the case of equal primary mass and nearly equal binary asteroid masses the analytic existence of comet-like periodic orbits.

Coorbital chaotic and homoclinic phenomena in the Restricted 3 Body Problem

Inmaculada Baldoma
Universitat Politecnica de Catalunya, Spain
M. Giralt, M. Guardia

The Restricted 3-Body Problem models the motion of a body of negligible mass under the gravitational influence of two massive bodies, called the primaries. If the primaries perform circular motions and the massless body is coplanar with them, one has the so called Restricted Planar Circular 3-Body Problem (RPC3BP). In synodic coordinates, can be modeled by a two degrees of freedom Hamiltonian system with five critical points, $L_1, .., L_5$, called the Lagrange points.
The Lagrange point L_3 is a saddle-center critical point, which is collinear with the primaries and beyond the largest one. When the ratio between the masses of the primaries μ is small, the modulus of the hyperbolic eigenvalues are weaker, by a factor of order $\sqrt{\mu}$, than the elliptic ones. Due to the rapidly rotating dynamics, the 1-dimensional unstable and stable manifold of L_3 are exponentially close to each other with respect to $\sqrt{\mu}$. In previous works we provided an asymptotic formula for the distance between these invariant manifolds for small ratios of the mass parameter. This result relies on a Stokes constant which, using computer assisted proofs, we will prove that is non zero. In this paper, we study different chaotic and homoclinic phenomena occurring in a neighborhood of L_3 and its invariant manifolds.
The first result concerns the existence of 2-round homoclinic connections to L_3, i.e. homoclinic orbits that approach the critical point 2-times. More concretely, we prove the existence of 2-round homoclinic orbits for a specific sequence of mass ratio parameters. The second result studies the family of Lyapunov periodic orbits of L_3 with Hamiltonian energy level exponentially close to that of L_3. In particular, we show that there exists a set of periodic orbits whose unstable and stable manifolds intersect transversally. By the Smale-Birkhoff homoclinic theorem, this implies the existence of chaotic motions (Smale horseshoe) exponentially close to L_3 and its invariant manifolds. In addition, we also show the existence of a generic unfolding of a quadratic homoclinic tangency and, as a consequence, the existence of Newhouse domains for the RPC3BP at coorbital motions.

Applications of the McGehee regularization to astrophysics, cosmology and celestial mechanics

Edward Belbruno
Yeshiva Uni. and Princeton Uni., USA

The McGehee regularization is an interesting way to regularize the flow of a dynamical system near a singular state by blowing up the singularity. This has been recently applied in astrophysics and cosmology to the motion of a particle about a Schwarzschild black hole (2011) and to regularize the big bang singularity (2013, 2018), respectively. Most recently, it has been applied to motion about an asteroid (2023). These problems are described and compared.
[Refs: CQG, v28, 2011 (EB, FP); CMDA. v115, 2013, (EB); CQG, v35, 2018 (EB, BKX); CMDA, v135, 2023, (EB, MG, W-TL)]

Energy drift in randomly perturbed Hamiltonian systems

Anna Maria Cherubini
Università del Salento, Italy
Marian Gidea

It is well known that, generically, integrable Hamiltonian systems subjected to small, time-dependent perturbations, generate some orbits that experience significant energy growth. In this work, we study the effect of random time-dependent perturbations on integrable Hamiltonian systems, in order extend results on the *Arnold diffusion problem* to the context of random dynamical systems.

Arnold diffusion in the elliptic Hill four-body problem

Marian Gidea
Yeshiva University, USA
Jaime Burgos-Garcia, Claudio Sierpe

We consider the planar elliptic restricted four-body problem as a model for the motion of Hektor's moonlet Skamandrios relative to the Sun, Jupiter and the Trojan asteroid Hektor. We derive the corresponding elliptic Hill four-body problem, representing the limiting case when the mass of the asteroid tends to zero. The resulting system can be viewed as a small perturbation of the circular Hill four-body problem, with the perturbation parameter being the eccentricity of Jupiter's orbit. We show that the effect of the perturbation can accumulate and yield some orbits whose energy undergoes significant changes over time. Consequently, it is possible that the moonlet can be ejected or crash onto the asteroid.

Finiteness of central configurations

Marshall Hampton
University of Minnesota, USA

Central configurations in the N-body problem give rise to interesting classes of orbits and serve as landmarks that help extend our knowledge of these dynamical systems. This talk will survey some recent results on proving the finiteness of central configurations under specific constraints using a variety of approaches including tropical geometry and interval arithmetic.

A modified parameterization method for invariant Lagrangian tori for partially integrable Hamiltonian systems

Alex Haro
Universitat de Barcelona & CRM, Spain
Jordi-Lluis Figueras

In this talk we present an a-posteriori KAM theorem for the existence of Lagrangian invariant tori in Hamiltonian systems of n degrees of freedom with $(n-d)$ first independent integrals in involution that induce a Hamiltonian action of the $(n-d)$-dimensional torus. We present a (modified) quasi-Newton method for the invariance equation of the torus parameterization, whose convergence from an initial approximation, and under appropriate non-degeneracy conditions, leads to the proof of the result with optimal estimates. The approach is suitable for developing numerical algorithms and performing computer-assisted proofs.

Europa-induced overlapping of secondary resonances in the 4:3 Jupiter-Ganymede unstable resonant orbit family

Bhanu Kumar
Jet Propulsion Laboratory, California Institute of Technology, USA
Rodney L. Anderson, Rafael de la Llave

The phenomenon of mean-motion resonance overlapping is known to be crucial for the generation of large-scale instability in celestial systems, and is useful for low-energy space mission trajectory design. In most related prior work, the model used is the planar CRTBP (PCRTBP). However, when designing tours of multi-moon systems, it is necessary to transition between resonances contained in regions of the phase space which are strongly affected by two moons, requiring at least a restricted 4-body model.
In this work, we investigate the Jupiter-Ganymede unstable 4:3 interior resonant orbits in a concentric circular restricted 4-body problem (CCR4BP) modeling the Jupiter-Europa-Ganymede system. In the PCRTBP, where Europa's mass is neglected, this is a one-parameter family of unstable periodic orbits,

where eccentricity can be used as the parameter. However, the more eccentric PCRTBP orbits do not continue successfully into the CCR4BP as tori. In this study, we show that this is due to the overlapping of secondary resonances with Europa inside the Jupiter-Ganymede resonant orbit family. The most prominent secondary resonances are 11/34, 12/37, 23/71, and 25/77. We also discuss the new objects which are expected to appear inside the secondary resonances and their potential applications.

Chaos and oscillatory orbits in the three body problem

Tere M-Seara
Universitat Politecnica de Catalunya, Spain
M. Guardia, P. Martin, J. Paradela

Consider the planar three body problem with masses $m_0 > 0$, $m_1 > 0$, $m_2 > 0$, and assume that all three are not equal. We consider a Poincaré map defined on a section of the phase space. After reduction of the problem by its first integrals this is a 4-dimensional map. We construct a hyperbolic invariant set of the Poincaré map where its dynamics is conjugated to the (infinite symbols) Bernouilli shift. As a consequence we prove the existence of chaotic motions and positive topological entropy for the planar three body problem. The chaotic behaviour also provides the existence of oscillatory motions for the planar three body problem.

Invariant manifolds of degenerate tori and double parabolic orbits to infinity in the $(n+2)$-body problem

Pau Martin
Universitat Politecnica de Catalunya, Spain
Inmaculada Baldoma, Ernest Fontich

Non hyperbolic fixed points and non hyperbolic periodic orbits play and important role in the dynamics of several well known problems. In the context of celestial mechanics, quite often objects at infinity are invariant but non hyperbolic. However, it has been proven with several degrees of generality that these objects, although degenerate, may have invariant stable and unstable manifolds that govern the dynamics of the system at points close to them. These type of objects also appear in problems of chemistry and economics.

In this work we consider *parabolic tori*, that is, tori such that the vector field vanishes in the normal directions up to certain order, and give conditions under which these tori have invariant stable and unstable manifolds with more than one stable or unstable directions. We require the dynamics on the tori to be conjugated to a rigid rotation with Diophantine frequency vector. We find the manifolds using the parametrization method after finding suitable approximate solutions of the corresponding invariance equation.

We apply the abstract theorem to the case of the planar $(n+2)$-body problem. We prove that, for any KAM tori of the n-body problem, if the masses of the last two bodies are small enough, there exist solutions defined for all $t > 0$ such that the first n bodies tend to the prescribed KAM tori while the last two go to infinity and arrive there with zero velocity (that is, they have a parabolic motion). The set of solutions satisfying these properties is a manifold whose dimension depends on the final configuration.

On the uniqueness of co-circular four body central configurations

Manuele Santoprete
Wilfrid Laurier University, Canada

In this talk we consider central configurations lying on a common circle in the Newtonian four-body problem. Using a topological argument we show that there is at most one co-circular central configuration for each cyclic ordering of the masses on the circle.

A pair of collision-based periodic orbits in three dimensions

Skyler Simmons
Utah Valley University, USA

I will present two collision-based periodic orbits of equal masses in three dimensions. The first orbit features eight bodies whose positions lie at the vertices of a rectangular prism at all times, in which each body collides in turn with its nearest three neighbors. The second orbit features six bodies which lie at the vertices of an octahedron at all times, with collisions occurring between opposite vertices at the origin. A construction of both orbits, regularization, and determination of the initial conditions will be given. Results relating to stability will also be discussed.

On the uniqueness of convex central configurations in the planar 4-body problem

Zhifu Xie
University of Southern Mississippi, USA
Shanzhong Sun, Peng You

In this talk, we present a rigorous computer-assisted proof (CAP) of the conjecture that there exists a unique convex central configuration for any four fixed nonnegative masses in a given order except those near zero. The proof employs the Krawczyk operator and interval analysis, as well as the implicit function theorem with the estimation on the size of the region where the implicit function exists.

Special Session 17: Nonlinear Models in Kinetic Theory, Collective Behavior, and Fluid Dynamics

Christopher Henderson, University of Chicago, USA
Stanley Snelson, Florida Institute of Technology, USA
Andrei Tarfulea, Louisiana State University, USA

This special session will address the modeling aspects and mathematical theory of physical and biological phenomena described by mesoscopic limits of small-scale systems. In particular, the focus is on the interrelated classes of kinetic and fluid dynamics equations and models for collective behavior. These models often feature long-range interactions, leading to nonlocal terms in the corresponding equations. Examples arise in subsonic hydrodynamics (pressure "communicates" much faster than fluid motion), biology (population and genetic drift arising from jump processes, as well as "flocking" in swarm dynamics) and plasma physics (characterized by high-velocity, strongly interacting ion motion). Such models involve novel mathematical challenges in deriving the limiting equations and determining their behavior quantitatively.

Moment estimates and global well-posedness for the binary-ternary Boltzmann equation

Ioakeim Ampatzoglou
New York University, USA
Irene M. Gamba, Natasa Pavlovic, Maja Taskovic

We will discuss the generation and propagation of polynomial and exponential moments, as well as the global well-posedness of the homogeneous binary-ternary Boltzmann equation. We will indicate that the co-existence of binary and ternary collisions yields better generation properties and time asymptotics, than when only binary or ternary collisions are considered. To address these questions, we develop for the first time angular averaging estimates for ternary interactions.

Critical local-wellposedness for the fully nonlinear Peskin problem

Stephen Cameron
Courant Institute, USA
Robert Strain

The Peskin problem describes the time evolution of a one dimensional elastic string immersed in a 2d steady Stokes fluid. It was first introduced by Peskin as a simplified model of a heart valve, and lead to the creation of the immersed boundary method. As one of the simplest models of a fluid-structure interaction, it has been extensively studied numerically but until recently very little was known analytically about it. Using a new formulation of the problem, we prove that it is locally well-posed in a scaling critical Besov space for an arbitrary tension law.

Global weak solutions to a hybrid Vlasov-MHD model for plasma dynamics

Bin Cheng
University of Surrey, England
Endre Suli, Cesare Tronci

We prove the global-in-time existence of large-data finite-energy weak solutions to an incompressible hybrid Vlasov-magnetohydrodynamic model in three space dimensions. The model couples three essential ingredients of magnetized plasmas: a transport equation for the probability density function, which models energetic rarefied particles of one species; the incompressible Navier-Stokes system for the bulk fluid; and a parabolic evolution equation, involving magnetic diffusivity, for the magnetic field. The physical derivation of our model is given. The key technical challenges in the analysis of the mathematical model are the nondissipative nature of the Vlasov-type particle equation and passage to the weak limits in the multilinear coupling terms. This is joint work with Endre Suli (Oxford) and Cesare Tronci (Surrey).

Global-in-time well-posedness of the homogeneous Landau-Coulomb equation for small L^p initial data

William Golding
University of Texas Austin, USA
Maria Gualdani, Amelie Loher

The Landau-Coulomb equation is a fundamental model in plasma physics that describes the statistical behavior of particles in a collisional plasma. Despite its widespread usage, the mathematical understanding of this equation has been limited due to competition between the reaction term and nonlinear, nonlocal diffusion term. This talk will address the global-in-time well-posedness of solutions for the homogeneous equation with Coulomb potential, when the initial data that are close to equilibrium in an L^p sense. In particular, using short-time smoothing estimates, we are able to construct global-in-time classical solutions for such initial data.

Fractional hypocoercivity

Laurent Lafleche
Institut Camille Jordan, CNRS and University Lyon 1, France
Emeric Bouin, Jean Dolbeault, Christian Schmeiser

In this talk, we will talk about the large time behavior of kinetic equations without spatial confinement and with fat tailed local thermodynamic equilibria. It has been proved in most of the cases that such operators can have an anomalous diffusion limit, meaning that in the appropriate scaling, the macroscopic equation is the fractional heat equation. At the level of the kinetic equation, we develop an L^2 hypocoercivity approach to obtain decay rates towards 0. It requires to find the good expression for the entropy and new functional inequalities of Poincaré type. The method is applied to kinetic equations with various linear collision operators: the Fokker-Planck operator, the Linear Boltzmann operator and the fractional Fokker-Planck operator. The result let appear a competition between the micro-scale and the macro-scale behavior.

Regularity theory for kinetic integro-differential equations

Amelie Loher
University of Cambridge, England

We discuss regularity estimates for kinetic equations. Concretely, we derive Schauder estimates using ideas from Campanato's approach for a general class of integro-differential equations. The same method with simplified estimates also works in the non-fractional case for kinetic Fokker-Planck-type equations in either divergence or non-divergence form. In particular our results are applicable to the Boltzmann and Landau equation.

Continuity for nonlocal hypoelliptic kinetic equations

Logan Stokols
Duke University, USA

In this talk, we'll explore a family of nonlocal kinetic equations that generalize the well-known Fokker-Planck equation and linearize the non-cutoff Boltzmann equation. We use a combination of hypoelliptic properties and De Giorgi's method to obtain Hölder continuity. Since our approach is entirely energy based, we can account for unbounded source terms and a wide range of dissipative degrees.

The Boltzmann process: Existence and construction

Padmanabhan Sundar
Louisiana State University, USA
S. Albeverio, B. Ruediger

First, a brief review of the existence and uniqueness of the Boltzmann-Enskog process is presented.
To construct a Boltzmann process, the existence of a solution f of the Boltzmann equation for hard spheres is assumed. A stochastic differential equation driven by a Poisson random measure that depends on f is introduced. The marginal distributions (in time) of its solution solve a linearized Boltzmann equation in the weak form. Further, if the distributions admit a probability density, we establish, under suitable conditions, that the density at each time t coincides with f. The stochastic process is hence called a Boltzmann process.This is a joint work with S. Albeverio and B. Ruediger.

Collective behaviors in macroscopic swarming dynamics

Changhui Tan
University of South Carolina, USA

I will present some recent progress on the analytical aspects of a class of macroscopic swarming models. Different types of nonlocal interactions will be discussed, which lead to different global wellposedness results, as well as asymptotic collective behaviors.

Macroscopic limits of nonlocal kinetic descriptions of consensus and relaxation dynamics

Andrea Tosin
Politecnico di Torino, Italy

In this talk, we present the derivation of macroscopic models of nonlocal consensus and relaxation dynamics in interacting multi-agent systems as hydrodynamic limits of Povzner-Boltzmann-type kinetic equations. First, we show that relaxation dynamics are well described, at the macroscopic scale, by first-order conservation laws with nonlocal flux. Next, we prove that consensus dynamics can instead be approximated macroscopically by a class of second-order models reminiscent of the celebrated Aw-Rascle-Zhang system of conservation laws for vehicular traffic, at least when the nonlocality of the interactions is sufficiently small. We also visualise numerically the correspondence between the solutions to the stochastic particle models underlying the kinetic description and their macroscopic limits.

Well-posedness and Gevrey regularity of some electrodiffusion models

Weinan Wang
University of Arizona, USA
Elie Abdo, Fizay-Noah Lee

In this talk, we consider the Nernst-Planck equations describing the nonlinear time evolution of multiple ionic concentrations in a two-dimensional incompressible fluid. The velocity of the fluid evolves according to either the Euler or Darcy's equations, both forced nonlinearly by the electric forces generated by the presence of charged ions. We address the global well-posedness and Gevrey regularity of the resulting electrodiffusion models in the periodic setting.

Boundary regularity for kinetic equations

Yuzhe Zhu
University of Cambridge, England

We consider the kinetic Fokker-Planck equation confined in a bounded spatial domain. We will discuss the regularization property of the solutions under inflow or reflection boundary conditions.

Special Session 18: Advanced Methodologies in Mathematical Materials Science and Biology

Toyohiko Aiki, Japan Women's University, Japan
Adrian Muntean, Karlstad University, Sweden

Materials science as well as life science present nowadays increasingly complex settings which require a thorough fundamental understanding especially if a reliable forecast is targeted. Often the understanding has to conquer information at space and time scales that cannot be reached with current experimental means. The special session is focused on the development of advanced mathematical methodologies potentially applicable in this context. Multiscale mathematical modeling including theoretical aspects of partial differential equations and of stochastically-interacting particle systems (and their numerical approximations) will play a central role in the discussions.

Numerical results on some mathematical models for elastic materials on the plane

Toyohiko Aiki
Japan Women's University, Japan
Chiharu Kosugi

In this talk we propose some mathematical models describing expansion and contraction motion of rings made of elastic material. The model is given as an initial boundary value problem for the beam equation which is a wave equation with fourth derivative with respect to the space variable. The domain is a one-dimensional interval, and the unknown function takes values in a plane. The aim of this talk is to present our modeling process, and numerical results obtained by the structure preserving method.

Quasi-variational inequality for a plasticity model with hardening phenomena

Yoshiho Akagawa
National Institute of Technology (KOSEN), Gifu College, Japan
Risei Kano, Takeshi Fukao

In this talk, we discuss the well-posedness of a plasticity model with hardening phenomena described by a quasi-variational inequality.
The prototype model of perfect plasticity is introduced by Duvau-Lions. The essential idea of this model is widely used to various studies. It is characterized by the constraint for the deviatoric part of the stress tensor. The threshold of the constraint plays an important role. In the case when the threshold depends on time or some unknown, then the model represents more realistic phenomena. In this study, we consider the case when the threshold depends on the unknown strain, in other words, in the history dependent situation.
The system is treated by the evolution equation governed by the time-dependent subdifferential, close to the Moreau sweeping process. Applying the abstract theory of evolution equation, we can obtain a well-posedeness result.
This talk is based on joint work with Risei Kano (Kochi University), and Takeshi Fukao (Kyoto University of Education).

Homogenization asymptotics of a reaction-diffusion-convection problem with exploding nonlinear drift – a two-scale convergence

Ida de Bonis
Sapienza University of Rome, Italy
E.N.M. Cirillo, A. Muntean, V. Raveendran

We study the periodic homogenization of a reaction-diffusion problem with nonlinear drift posed in an unbounded perforated domain. We are interested in deriving rigorously the upscaled model equations and the corresponding effective coefficients for the case when the microscopic dynamics are linked to a particular choice of characteristic length and time scales that lead to a fast exploding drift. The main mathematical difficulty lies in proving the two-scale compactness results needed for the passage to the homogenization limit. Our setting is closely related to the works by Marusik-Paloka and Piatnitskii as well as by V. Raveendran.
The results are presented in a recent joint work with E.N.M. Cirillo, A. Muntean and V. Raveendran.

Effective heat transfer conditions between porous media and fluid layers

Michael Eden
Karlstad University, Sweden
Tom Freudenberg

The effective heat transfer at interfaces between porous media and surrounding fluid layers is an important factor in many applications in geoengineering and machining. We are investigating effective models including interface conditions in the context of mathematical homogenization using the tool of periodic two-scale convergence. We present results for two different cases:
(a) The solid part of the porous media is assumed to consist of disconnected inclusions. Here, we arrive at a one-temperature problem exhibiting a memory term. (b) The solid matrix is assumed to be connected. Here, the limit problem is given by two-phase mixture model.
We compare and discuss these two limit models with several simulation studies both with and without convection.

On a variational inequality of Bingham and Navier-Stokes type

Takeshi Fukao
Ryukoku University, Japan
Takahito Kashiwabara

In this talk, we discuss the well-posedness of the variational inequality for a fluid dynamics of Bingham and Navier–Stokes type in three dimension.
This kind of problem was treated by Naumann–Wulst (1979), Kato (1993) for the Bingham fluid, based on the result by Duvaut–Lions (1976).
All of them, the solution makes weak sense in H^1, because one could not get the enough H^2-regularity. The problem is formulated the evolution equation governed by the subdifferential.
By discussing the characterization of the subdifferential, more precisely, the H^2-regularity result of the solution for the variational inequality, the Barbu truncation method works well to prove the well-posedness.
This talk is based on the joint work with Takahito Kashiwabara, University of Tokyo.

Quasi-variational inequalities for superconductivity model

Nobuyuki Kenmochi
Chiba University, Japan
Maria Gokieli, Marek Niezgodka

In this talk we treat a quasi-variational model of superconductivity, which was introduced by L. Prigozhin in 1996, under unknown dependent constraints. Our model is a coupled system of a parabolic variational inequality with gradient constraint for a magnetic field and a heat equation, interacting each other. Our approach is based on the general theory on parabolic quasi-variational inequalities in abstract Banach spaces evolved by the author with the co-authors in the last eight years.
We give an existence result of weak solutions of the system in a variational sense. In the proof we make use the concept of time-derivative operators associated with unknown dependent constraints as well as a compactness theorem for parabolic variational inequalities.

The large time behavior of solutions for the initial and boundary value problem representing stretching and shrinking motions of elastic curves

Chiharu Kosugi
Yamaguchi University, Japan
Toyohiko Aiki

Our research aim is to construct a mathematical model of stretching and shrinking motions for compressible elastic curves, for instance, rubber bands. We consider initial and boundary value problems for the beam equation known as a mathematical model for elastic material. The unknown function represents a position on the plane. In our research, we discuss two problems. The first problem has no viscosity term, and the second problem has viscosity term. The feature of these problems is that the strain is nonlinear and non-smooth. To overcome this difficulty, we suppose that the stress function has a singularity point. Thanks to this, we can obtain the lower boundedness of the strain. By this estimate we can prove existence and uniqueness of not only weak solutions but also strong solutions. For the problem with the viscosity, we can also show the large time behavior of solutions. It means that the omega - limit set of the solution orbit is not empty and included to the set of steady solutions. The key in the proof is a priori estimate for center of mass globally in time. This is a joint work with Toyohiko AIKI (Japan Women's University), Japan.

Multiscale model describing the swelling phenomenon in porous materials

Kota Kumazaki
Kyoto University of Education, Japan
Adrian Muntean

In this talk, we propose a multiscale model describing the swelling phenomenon in porous materials. This model consists of a diffusion equation for the relative humidity distributed in materials and a free boundary problem describing the swelling process in microscopic pores. We consider each microscopic pore as a one-dimensional interval and correspond the interval to each point of materials. In our previous results, for given relative humidity we showed the well-posedness of the free boundary problem. In this talk, we impose a governing equation for the relative humidity and discuss the existence and uniqueness of a locally-in-time solution of this model.

Global existence of solutions to a singular nonlocal phase field system with inertial term

Shunsuke Kurima
Tokyo University of Science, Japan

This talk deals with a nonlocal phase field system with inertial term. Colli–Colturato (2018) have established existence of solutions to a phase field system related to the entropy balance. Also, Colli–Grasselli–Ito (2002) have proved existence of solutions to a parabolic-hyperbolic Penrose–Fife phase field system. However, singular nonlocal phase field systems with inertial term seem to be not studied yet.
The present work asserts that we can derive existence of solutions to a singular nonlocal phase field system with inertial term.

Convergence of solutions for a two-species chemotaxis model to those for the Lotka–Volterra competition model

Masaaki Mizukami
Kyoto University of Education, Japan

This work is concerned with the question "how far does small chemotactic interaction perturb the Lotka–Volterra competition dynamics?". A two-species chemotaxis-competition model was studied by e.g., Bai–Winkler (2016) and Lin–Mu–Wang (2015); however, there are still many open problems. On the other hand, the Lotka–Volterra competition model has been studied extensively. Thus the development of this work will enable us to see new properties of solutions for the chemotaxis system. The main result of this talk gives uniform-in-time convergence of solutions for the two-species chemotaxis-competition system to those for the Lotka–Volterra competition model.

Pseudo-parabolic energy dissipation system associated with grain boundary motion

Daiki Mizuno
Chiba University, Japan
Ken, Shirakawa

In this talk, we consider a system of pseudo-parabolic PDEs, which is derived as a dissipation system (gradient flow) of a non-smooth functional. In this context, the governing functional is often called Kobayashi–Warren–Carter type energy (KWC type energy), and it is known as a free-energy of planar grain boundary motion, proposed by [Kobayashi et al., Physica D, 140, 141–150 (2000)]. The mathematical characteristics of our system is in the point that the pseudo-parabolicity yields stronger regularity than that in parabolic gradient flow, while the governing free-energy is non-smooth. Therefore, it would be expected that our energy dissipation system would be a distinct mathematical model of grain boundary motion, and furthermore, it would provide a mathematical method of minimization process governed by non-smooth energy. The objective of this study is to obtain some theoretical estimate for this expectation. As a part of the estimate, the mathematical issues concerned with the existence, uniqueness, continuous dependence, large-time behavior, and so on, will be discussed in the Main Theorems of this talk.

Existence and uniqueness of solutions to the moisture transport model for porous materials

Akiko Morimura
Japan Women's University, Japan
Toyohiko Aiki

We consider the initial-boundary value problems for nonlinear parabolic equations describing moisture transport in a porous material occupying a one-dimensional interval. This study is strongly motivated by Fukui, Iba, Hokoi, and Ogura's article, in which, they only reported validation of the model by comparing experimental and numerical results. Therefore, we aim to analyze this model, mathematically. We note that their system consists of two equations corresponding to conservation law for air and liquid in the region, and one equation is type of elliptic-parabolic. As a first step in this research, we simplify the model such that the mass distribution in air is given. The unknown function of our model indicates the chemical potential of water. Also, we approximate the elliptic-parabolic equation. The purpose of this talk is to establish existence and uniqueness of solutions to the approximate problem by applying the evolution equation theory and the standard fixed-point argument. This is a joint work with Toyohiko AIKI (Japan Women's University) and EBARA Corporation, Japan.

Phase separation and morphology growth in interacting ternary mixtures under evaporation: Well-posedness and numerical simulation of a nonlocal evolution system

Adrian Muntean
Karlstad University, Sweden
Rainey Lyons, Stela Andrea Muntean, Emilio N. M. Cirillo

We study a nonlinear coupled parabolic system with nonlocal drift terms modeling at the continuum level the inter-species interaction within a ternary mixture that allows the evaporation of one of the species. In the absence of evaporation, the proposed system the evolution system coincides with the hydrodynamic limit of a stochastically interacting particle system of Blume–Capel–type driven by the Kawasaki dynamics. We investigate the well-posedness of the target system posed in 3D and present preliminary numerical simulation results for a 2D scenario. We employ an approximation scheme based on finite difference to illustrate the effect of changing the characteristic time scale of the evaporation rate on the shape and connectivity of the evolving-in-time morphologies. The precise structure of our evolution system is motivated by technological issues involved in the design of organic solar cells, however, similar structures of model equations arise in other materials science-related contexts thatb are conceptually related (e.g. in the design of the internal structure of thin adhesive bands).

On a hierarchy of effective models for the biomechanics of human compact bone tissue

Grigor Nika
Karlstad University, Sweden

We derive a hierarchy of effective models that can be used to model the biomechanics of human compact bone taking into account scale-size effects observed experimentally. The classification of the effective models depends on the hierarchy of four characteristic lengths: The size of the heterogeneities, two intrinsic lengths of the constituents, and the overall characteristic length of the domain. Depending on the different scale interactions between the size of the heterogeneities, the two intrinsic lengths of the constituents, and the characteristic length of the domain we obtain either an effective Cauchy continuum or an effective Cosserat continuum. The passage to the limit relies on suitable use of the periodic unfolding operator. Moreover, we perform numerical simulations to validate our results.

Space-time periodic homogenization problem for nonlinear diffusion equations

Tomoyuki Oka
University of Tokyo, Japan
Goro Akagi

In this talk, we shall discuss a space-time homogenization problem for nonlinear diffusion equations with periodically oscillating (in space and time) coefficients. Main results consist of a homogenization theorem, i.e., convergence of solutions as the period of coefficients goes to zero and characterization of homogenized equations and qualitative properties of homogenized matrices, which deeply depend on the singularity and degeneracy of nonlinear diffusion under a certain critical case in terms of the log-ratio of spatial and temporal periods of the coefficients.

A structure-preserving scheme for the Cahn-Hilliard equation with dynamical boundary conditions based on the discrete variational derivative method

Makoto Okumura
Konan University, Japan

Recently, the Cahn-Hilliard equation with new dynamical boundary conditions has been proposed by Liu and Wu. This model has characteristic conservation laws in that each mass of the interior of the domain and the boundary are conserved. In addition, the total energy dissipation law, which represents that the sum of energy in bulk and on the boundary decreases, holds in this model. From the perspective of numerical computation, the properties often lead us to stable computation. Hence, if the designed schemes retain the properties in a discrete sense, then the schemes are expected to be stable. In this study, In this study, we consider the Liu-Wu model in a two-dimensional rectangular domain with periodic boundary conditions on the left and right boundaries and dynamic boundary conditions on the upper and lower boundaries. Then, we propose a structure-preserving scheme for this model that retains the aforementioned conservation and dissipation laws in a discrete sense. Also, we will talk about the proof of the solvability of the proposed scheme and show the results of numerical computations in the presentation.

Coupled stochastic systems of Skorokhod type: Well-posedness of a mathematical model and its applications

Thoa Thieu
Wilfrid Laurier University, USA
Adrian Muntean, Roderick Melnik

Population dynamics with complex biological interactions, accounting for uncertainty quantification, are critical for many application areas. However, due to the complexity of biological systems, the mathematical formulation of the corresponding problems faces the challenge that the corresponding stochastic processes should, in most cases, be considered in bounded domains. We propose a model based on a coupled system of reflecting Skorokhod-type stochastic differential equations with jump-like exit from a boundary. The setting describes the population dynamics of active and passive populations. As main working techniques, we use compactness methods and Skorokhod's representation of solutions to SDEs posed in bounded domains to prove the well-posedness of the system. This functional setting is a new point of view in the field of modelling and simulation of population dynamics. We provide the details of the model, as well as representative numerical examples, and discuss the applications of a Wilson-Cowan-type system, modelling the dynamics of two interacting populations of excitatory and inhibitory neurons. Furthermore, the presence of random input current, reflecting factors together with Poisson jumps, increases firing activity in neuronal systems.

Nonlinear evolution equation associated with hypergraph Laplacian

Shun Uchida
Oita University, Japan
Masahiro Ikeda

Hypergraph is a generalization of the usual graph which represents the grouping or connection of multiple members. In order to investigate the structure of network represented by a hypergraph, Prof. Yuichi Yoshida (2022) introduced hypergraph Laplacian, which can be defined as a subdifferential operator of some convex function on a finite dimensional space. In this talk, we first explain the definition and some basic properties of this operator from a viewpoint of the nonlinear evolution equation theory. By using them, we next consider the Cauchy and the time-periodic problem of a multivalued ordinary differential equation governed by the hypergraph Laplacian.

Second order nonlinear quasi-variational evolution inclusions

Noriaki Yamazaki
Kanagawa University, Japan
Nobuyuki Kenmochi, Ken Shirakawa

We consider second order nonlinear quasi-variational evolution inclusions governed by time-dependent subdifferentials in V^*. Here, V is a uniformly convex Banach space such that V is dense in a real Hilbert space H and the injection from V into H is compact. We also suppose that the dual space V^* of V is uniformly convex, and $H = H^*$.
In this talk, we establish the abstract result on the existence of solutions to our problem by applying the abstract theory of the time-derivative operators and the fixed-point theorem of Schauder type. In addition, we give some applications to nonlinear PDEs with gradient constraint for time-derivatives.
This is a joint work with Nobuyuki Kenmochi (Chiba University, Chiba, Japan) and Ken Shirakawa (Chiba University, Chiba, Japan).

Special Session 19: Stochastic Partial Differential Equations

Benjamin Gess, MPI MIS Leipzig, Germany
Michael Röckner, University of Bielefeld, Germany

Stochastic Partial Differential Equations (SPDE) and their applications is a relatively young field of mathematics. In the past two decades and it has, however, become one of the main research directions of Probability Theory, with rising activity across its entire spectrum. In particular, modern SPDE techniques and their combination with ideas from rough path theory led to Martin Hairer's theory of regularity structures for which he was awarded the Fields Medal in 2014. Since then this theory has been significantly extended, for example, by applications to the geometrical evolution of random loops on manifolds. The field of SPDE combines the classical area of partial differential equations (PDE) with modern branches of probability theory, in particular, stochastic analysis, and thus constitutes one of the most prominent contact points between analysis and stochastics. The aim of the session is to give an update on recent developments on SPDE and at the same time identify new frontiers with challenging open problems for the field, with emphasis on both theory and applications.

Numerical schemes for various stochastic models in hydrodynamic

Hakima Bessaih
Florida International University, USA
Annie Millet

Space-time numerical schemes are introduced for various stochastic models in fluid mechanics. This include the 2D stochastic Navier-Stokes equations, 2D Boussinesq equations and the 3D stochastic Brinkman-Forchheimer equations. Various rate of convergences are investigated and proved.

A unified forward-backward Lévy process driven SPDE and stochastic Differential Games

Wanyang Dai
Nanjing University, Peoples Republic of China

We establish a relationship between a unified forward-backward Lévy process driven stochastic partial differential equation (SPDE) and stochastic differential games (SDGs). The SDGs are with q players and are driven by a general-dimensional vector Lévy process. The unified forward-backward coupled SPDE is in both general dimensional vector-form and forward-backward coupling manner. The partial differential operators in its drift, diffusion, and jump coefficients are in time-variable and position-parameters over a space domain. Since the unified SPDE is with general nonlinearity and general high-order, we extend our recently developed approach from the existing Brownian driven case to a general Lévy driven case by constructing a new supporting topological space with the target to prove the unique existence of an adapted 4-tuple strong solution to the unified forward-backward SPDE under general local linear growth and Lipschitz conditions.

Global well-posedness of the 2D nonlinear Schrödinger equation with multiplicative spatial white noise on the full space

Arnaud Debussche
École Normale Supérieure de Rennes, France
Ruoyuan Liu, Nikolay Tzvetkov, Nicola Visciglia

We consider the nonlinear Schrödinger equation with multiplicative spatial white noise and an arbitrary polynomial nonlinearity on the two-dimensional full space domain. We prove global well-posedness by using a gauge-transform introduced by Hairer and Labbé (2015) and constructing the solution as a limit of solutions to a family of approximating equations. This paper extends a previous result by Debussche and Martin (2019) with a sub-quadratic nonlinearity.

The nonlinear stochastic heat equation in the critical dimension

Alexander Dunlap
New York University Courant Insitute, USA
Cole Graham, Yu Gu

I will discuss a two-dimensional stochastic heat equation with a nonlinear noise strength, and consider a limit in which the correlation length of the noise is taken to 0 but the noise is attenuated by a logarithmic factor. The limiting pointwise statistics can be related to a stochastic differential equation in which the diffusivity solves a PDE somewhat reminiscent of the porous medium equation. This relationship is established through the theory of forward-backward SDEs. I will also explain several cases in which the PDE can be solved explicitly, some of which correspond to known probabilistic models. This talk will be based on current joint work with Cole Graham and earlier joint work with Yu Gu.

Martingale solutions to the stochastic thin film equation

Manuel Gnann
TU Delft, Netherlands
Konstantinos Dareiotis, Benjamin Gess, Guenther Gruen, Max Sauerbrey

We prove existence of non-negative martingale solutions to a class of stochastic degenerate-parabolic fourth-order PDEs arising in surface-tension driven thin-film flow influenced by thermal noise. The construction applies to a range of mobilites including the cubic one which occurs under the assumption of a no-slip condition at the liquid-solid interface. Since their introduction more than 15 years ago, by Davidovitch, Moro, and Stone and by Gruen, Mecke, and Rauscher, the existence of solutions to stochastic thin-film equations for cubic mobilities has been an open problem, even in the case of sufficiently regular noise. Our proof of global-in-time solutions relies on a careful combination of entropy and energy estimates in conjunction with a tailor-made approximation procedure to control the formation of shocks caused by the nonlinear stochastic scalar conservation law structure of the noise. The construction of solutions with non-full support for the initial data using alpha-entropies shall be discussed.

Strong solutions to McKean-Vlasov SDEs with coefficients of Nemytskii-type

Sebastian Grube
Bielefeld University, Germany

We study a large class of McKean–Vlasov SDEs with drift and diffusion coefficient depending on the density of the solution's time marginal laws in a Nemytskii-type of way. A McKean–Vlasov SDE of this kind arises from the study of the associated nonlinear FPKE, for which is known that there exists a bounded Sobolev-regular Schwartz-distributional solution u. Via the superposition principle, it is already known that there exists a weak solution to the McKean–Vlasov SDE with time marginal densities u. We show that there exists a strong solution the McKean–Vlasov SDE, which is unique among weak solutions with time marginal densities u. The main tool is a restricted Yamada–Watanabe theorem for SDEs, which is obtained by an observation in the proof of the classical Yamada–Watanabe theorem.

Brownian fluctuations of flame fronts with small random advection

Christopher Henderson
University of Arizona, USA
Panagiotis Souganidis

We discuss the effect of small random advection in two models of front propagation in turbulent combustion. We (i) identify the order of the fluctuations of the front with respect to the size of the advection, and (ii) characterize them by the solution of a Hamilton-Jacobi equation forced by white noise. In the simplest case, the result yields, for both models, a front with Brownian fluctuations of the same scale as the size of the advection. That the fluctuations are the same for both models is somewhat surprising, in view of known differences between the two models in the large advection regime. We also mention connections to weakly inhomogeneous ASEP.

The porous medium equation: Rescaled zero-range process, large deviations and gradient flow

Daniel Heydecker
Max Planck Institute for Mathematics in the Sciences, Germany

We study a rescaling of the zero-range process with homogenous jump rates $g(k) = k^\alpha$ with arbitrary $\alpha \geq 1$. With a simultaneous rescaling of space, time and particle size, we identify the dynamical large deviations from the porous medium equation, using pathwise discretised regularity estimates to prove a version of the superexponential estimate in the spirit of the Aubin-Lions-Simons lemma. Finally, we use the large deviation principle to give an expression of the porous medium equation as the gradient flow of the Boltzmann entropy with respect to a tailor-made Wasserstein-type distance.

Singular limit for a stochastic Allen-Cahn equation with nonlinear diffusion

Danielle Hilhorst
CNRS and University Paris-Sud, France
Perla El Kettani, Yong Jung Kim, Hyunjoon Park

We study a stochastic Allen-Cahn equation with nonlinear diffusion and a mild random noise on a bounded domain in Rn. We consider initial data that satisfy some non-degeneracy conditions, and prove that steep transition layers develop within a very short time. We then study the motion of these transition layers and derive a stochastic motion law for the sharp interface limit.

On the Boussinesq hypothesis for a stochastic Proudman-Taylor model

Dejun Luo
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Republic of China
Franco Flandoli

We introduce a stochastic version of Proudman-Taylor model, a 2D-3C fluid approximation of the 3D Navier-Stokes equations, with the small scale turbulence modeled by a transport-stretching noise. For

this model we may rigorously take a scaling limit leading to a deterministic model with additional viscosity on large scales. In certain noise examples without mirror smmetry, we identify an AKA effect.

On nonlinear Markov Processes in the sense of McKean

Marco Rehmeier
Bielefeld University, Germany
Michael Röckner

We develop a theory of nonlinear Markov processes in the sense of McKean's seminal work from 1966 and provide a general construction of such processes with one-dimensional time marginals given by solutions to nonlinear Fokker–Planck–Kolmogorov equations. These processes consist of path laws of weak solutions to the corresponding McKean–Vlasov stochastic differential equation. We stress that neither of these equations needs to be well-posed. Our theory applies to a large new class of examples, e.g. the porous media equation (PME) on $\mathbb{R}^d$, $d \geq 1$, with general diffusivity and transport-type drift, which includes the classical PME with its Barenblatt solutions as the one-dimensional time marginal densities of the corresponding nonlinear Markov process.

Convex integration techniques in stochastic fluid dynamics

Andre Schenke
Courant Institute of Mathematical Sciences, USA

Convex integration is a technique that was pioneered by John Nash almost 70 years ago in his study of the isometric embedding problem. More recently, in about the last 15 years, the technique has seen spectacular success when applied by Buckmaster, De Lellis, Isett, Székelyhidi, Vicol and others to various equations of fluid dynamics (in particular Euler and Navier–Stokes equations).
More recently, in the last 5 years, it has been applied by an increasing number of people (following pioneering works of Hofmanovà, Zhu and Zhu) to the equations of stochastic fluid dynamics. In this talk I will report recent convex integration results for stochastic fluid dynamical equations yielding nonuniqueness in law for those equations.

Mean field game systems with common noise and degenerate idiosyncratic noise

Benjamin Seeger
University of Texas Austin, USA
P. Cardaliaguet, P. Souganidis

I will describe the forward-backward system of stochastic partial differential equations describing a mean field game for a large population of small players subject to both idiosyncratic and common noise. The unique feature of the problem is that the idiosyncratic noise coefficient may be degenerate, so that the system does not admit smooth solutions in general. A new notion of weak solutions for backward stochastic Hamilton-Jacobi-Bellman equations must be developed, and this is used to build probabilistically weak solutions of the mean field game system. The uniqueness of a strong solution can be proved under additional structural assumptions.

Invariant measure of the 2D Yang-Mills Langevin dynamic

Hao Shen
University of Wisconsin Madison, USA

In an earlier work by Chandra, Chevyrev, Hairer and the speaker [CCHS] in 2020, we constructed an orbit space of the 2D Yang-Mills as well as a Markov process on the orbit space. In this talk, we prove that there exists a unique probability measure on the orbit space such that its finite dimensional distributions, seen as a process indexed by loops, agree with those of the YM measure in the sense of Lévy's earlier work in 2016. Moreover, this is the unique invariant probability measure on the orbit space for the Markov process in [CCHS]. Based on joint work with Ilya Chevyrev.

Approximation of optimal feedback controls for stochastic reaction-diffusion equations

Alexander Vogler
TU Berlin, Germany
Stannat, Wilhelm

We approximate optimal feedback controls for stochastic reaction-diffusion equations with additive noise by first reducing the problem to controls of feedback form and then approximating the feedback function using finitely based approximations. Using structural assumptions on the finitely based approximations, rates for the approximation error of the cost can be obtained. Based on our approximation results we introduce a numerical algorithm that significantly reduces the computational complexity of finding controls with asymptotically optimal cost. Numerical experiments using artificial neural networks as well as radial basis function networks illustrate the performance of our algorithm. Our approach can also be applied to stochastic control problems for high dimensional stochastic differential equations and more general stochastic partial differential equations.

Multi solitary waves to stochastic nonlinear Schrödinger equations

Deng Zhang
Shanghai Jiao Tong University, Peoples Republic of China
Michael Röckner, Yiming Su

In this talk we will present the recent work on the multi solitary waves to stochastic nonlinear Schrödinger equations driven by linear multiplicative noise, in both the mass-critical and subcritical cases. Unlike in the deterministic case, the existence of stochastic multi-solitons cannot be obtained from that of stochastic multi-bubble blowup solutions, due to the absence of pseudo-conformal invariance. We present a constructive proof by utilizing the rescaling approach and the modulation method. The constructed multi-solitons behave asymptotically as a sum of finitely many solitary waves, and the convergence rate of the remainders can be of either exponential or polynomial type, which reflects the effects of noise on the asymptotical behavior of solutions.

Special Session 20: Control and Optimization: New Developments and Applications

Monica Motta, University of Padova, Italy
Ellina Grigorieva, Texas Woman's University, USA

This special session on new developments in control and optimization and their applications will bring together a selected group of experts in these areas. The growing importance of control and optimization has been realized in recent years. This is due not only to theoretical developments, but also because of numerous applications to engineering, economics and life sciences. The topics which will be discussed include turnpike phenomenon, averaging in optimal control, impulsive optimal control problems, necessary and sufficient optimality conditions, qualitative and quantitative aspects of optimal control, control and stabilization of PDEs.

Optimization of the alternation of drug intake and rest intervals in adaptive melanoma treatment protocols

Ellina Grigorieva
Texas Woman's University, USA
Evgenii Khailov

Current targeted treatments for melanoma are based on the continuous use of the maximum tolerated dose by a patient. At the same time, they quickly eliminate drug-sensitive cancer cells. As a result, such treatment changes the competition between drug-sensitive and drug-resistant cancer cells in favor of the latter. Therefore, drug-resistant cancer cells begin to dominate in the patient's body, and the applied treatment may be ineffective.
A new direction in the treatment of melanoma is adaptive therapy. It allows a significant number of drug-sensitive cancer cells to survive through the use of minimally effective doses of drugs or temporary interruptions in their intake. As a result, these cells inhibit the proliferation of drug-resistant cancer cells by competing for shared limited resources. For successful results of adaptive therapy, it is extremely important to find the optimal moments of switching from the stage of its active implementation to the stage of its absence (rest intervals for a patient) and vice versa, taking into account the characteristics of the patient.
In this report, for a given time interval, which is a general period of melanoma treatment, the Lotka-Volterra mathematical model, given by a system of ordinary differential equations, is considered, which describes the competition between drug-sensitive and drug-resistant cancer cells during adaptive therapy. This model also contains a control function of time responsible for the transition from the stage of active implementation of adaptive therapy to the stage of its absence and vice versa. To find the optimal moments of switching between these stages, the task is to minimize the cancer cell load both during the entire general period of melanoma treatment and at its final moment. An analytical study of such a minimization problem is carried out using the Pontryagin maximum principle. The results of the study are confirmed by numerical calculations performed in the BOCOP-2.0.5 environment for the values of the parameters of the original Lotka-Volterra model and its initial conditions taken from real clinical practice data.

Non-smooth Frobenius' theorem

Yuhki Hosoya
Faculty of Economics, Chuo University, Japan

This paper studies a extension of Frobenius theorem with not differentiable but Lipschitz environment. We use Rademacher's theorem and extended existence theorem of a partial differential equation to construct contour hypersurfaces, and recover the solution function by using these structure.

Application of a stochastic approximation method for the elastography inverse problem

Baasansuren Jadamba
Rochester Institute of Technology, USA
Rachel Hawks

This talk will focus on an adaptive finite element solution framework for the problem of identifying a distributed parameter in a linear elasticity system. We formulate the problem of estimating the tissue stiffness parameter in the system from measurements of displacement as an optimization problem. A stochastic approximation method with finite element discretization for the solution of the problem is introduced and adaptive mesh refinement framework that provides the resolution needed for the recovery of the parameter is discussed.

Intrinsic and apparent singularities in differentially flat systems

Yirmeyahu Kaminski
Holon Institute of Technology, Israel
Jean Levine, Francois Ollivier

We study the singularities of differentially flat systems, in the perspective of providing global or semi-global motion planning solutions for such systems: flat outputs may fail to be globally defined, thus potentially preventing from planning trajectories leaving their domain of definition, the complement of which we call singular. Such singular subsets are classified into two types: apparent and intrinsic. A rigorous definition of these singularities is introduced in terms of atlas and local charts in the framework of the differential geometry of jets of infinite order and Lie-Backlund isomorphisms. We then give an inclusion result allowing to effectively compute all or part of the intrinsic singularities. Finally, we show how our results apply to the case of n-dimensional affine systems with n-1 commands.

A new regularized stochastic approximation framework for stochastic inverse problems

Akhtar Khan
Rochester Institute of Technology, USA
J. Dippon, J. Gwinner, M. Sama

This talk will focus on the nonlinear inverse problem of estimating stochastic parameters in the fourth-order partial differential equation with random data. The primary focus will be on developing a new regularized stochastic extragradient framework for a nonlinear variational inequality, which subsumes the optimality conditions for the optimization formulation of the inverse problem. Numerical results will be presented.

On affine equivalence of sub-riemannian metrics on step 2 distributions

Zaifeng Lin
Texas A & M University, USA
Igor Zelenko

The classical result of Eisenhart states that if a Riemannian metric g admits a Riemannian metric that is not constantly proportional to g and has the same (parameterized) geodesics as g in a neighborhood of a given point, then g is a direct product of two Riemannian metrics in this neighborhood. The talk is devoted to describing the extension of this result to sub-Riemannian metrics on a class of step 2 distributions.

Stochastic differential inclusions and set-valued stochastic differential equations

Mariusz Michta
University of Zielona Gora, Poland

In the talk, we establish properties of solutions to stochastic differential inclusions and set-valued stochastic differential equations with respect to semimartingale integrators. We present some connections between their solutions. In particular, we show that attainable sets of solutions to stochastic differential inclusions are subsets of values of multivalued solutions of certain set-valued differential stochastic equations. We also show that every solution to stochastic inclusion is a continuous selection of a multivalued solution of an associated set-valued stochastic equation. The results obtained in the paper generalize results dealing with this topic known both in deterministic and stochastic cases.

High order Lyapunov functions and Lie-bracket-based feedback stabilizability with cost regulation

Monica Motta
University of Padua, Italy
Giovanni Fusco, Franco Rampazzo

We consider an optimal control problem where the state has to approach asymptotically a closed target, while paying an integral cost with a nonnegative Lagrangian. We generalize the partial diferential inequality that usually defines a Control Lyapunov Function by introducing a new, weaker differential inequality, which involves both the Lagrangian and higher order dynamics' directions expressed in form of iterated Lie brackets up to a certain degree k, possibly greater than 1. In particular, we show that the existence of a solution U of the resulting extended relation allows us to construct explicitly a Lie-bracket-based feedback which sample stabilizes the system to the target and, at the same time, provides a bound for the cost, given by a U-dependent function. This means that, with a sample and hold technique, we construct trajectories which take into consideration not only the dynamics' vector fields but also their iterated Lie brackets. This is achieved by prescribing, for every small interval of a time partition, a state-dependent control strategy that makes the corresponding trajectory approximate the direction of an iterated Lie bracket. Furthermore, while the state approaches the target, the feedback regulates the cost as well, i.e. it guarantees a bound on the cost. We call such a solution U a degree-k Minimum Restraint Function.

On the structure of the value function of optimal exit time problems

Tien Khai Nguyen
North Carolina State University, USA
Piermarco Cannarsa, Marco Mazzola

Consider an optimal exit time problem with general running and terminal costs, and a target having an inner ball property for a nonlinear control system that satisfies mild controllability assumptions. In particular, the Petrov's condition at the boundary at the target is not required and the value function fails to be locally Lipschitz. In this talk, we shall establish a representation formula of proximal (horizontal) supergradients of the value function and derive its regularity properties.

Genericity of projectively and affinely rigid sub-Riemannian Metrics

Igor Zelenko
Texas A & M University, USA
Frederic Jean, Sofya Maslovskaya

Sub-Riemannian metrics on a manifold are defined by a distribution (a subbundle of the tangent bundle) together with a Euclidean structure on each fiber. The Riemannian metrics correspond to the case when the distribution is the whole tangent bundle. Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case, both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. separation of variables (the de Rham decomposition) occur, while for the analogous property in the projective equivalence case, a more involved ("twisted") product structure is necessary. The latter is also related to the existence of commuting nontrivial integrals quadratic with respect to velocities for the corresponding geodesic flow. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics. In particular, we will discuss the genericity of metrics that do not admit non-constantly proportional affinely/projectively equivalent metrics on a given distribution (such metrics are called projectively/ affinely rigid) and the genericity of distributions for which all sub-Riemannian metrics are projective/affinely rigid.

Special Session 21: Evolution Equations and Integrable Systems

Alex Himonas, University of Notre Dame, USA
Curtis Holliman, The Catholic University of America, USA
Fangchi Yan, Virginia Tech, USA

The theme of this session is nonlinear evolution equations and integrable systems including the NLS equation, the KdV equation, the Camassa-Holm equation, and the Euler equations of hydrodynamics. Topics studied for these equations include, among others, traveling waves, initial and initial-boundary value problems, local and global well-posedness, inverse scattering, stability, and integrability.

Periodic Hölder waves in a class of negative-order dispersive equations

Fredrik Hildrum
Norwegian University of Science and Technology, Norway
Jun Xue

We prove the existence of highest, cusped, periodic travelling-wave solutions with exact and optimal α-Hölder continuity in a class of fractional negative-order dispersive equations of the form

$$u_t + (|\mathrm{D}|^{-\alpha} u + n(u))_x = 0$$

for every $\alpha \in (0,1)$ with homogeneous Fourier multiplier $|\mathrm{D}|^{-\alpha}$. We tackle nonlinearities $n(u)$ of the type $|u|^p$ or $u|u|^{p-1}$ for all real $p > 1$, and show that when n is odd, the waves also feature antisymmetry and thus contain inverted cusps. Tools involve detailed pointwise estimates in tandem with analytic global bifurcation, where we resolve the issue with nonsmooth n by means of regularisation. We believe that both the construction of highest antisymmetric waves and the regularisation of nonsmooth terms to an analytic bifurcation setting are new in this context, with direct applicability also to generalised versions of the Whitham, the Burgers–Poisson, the Burgers–Hilbert, the Degasperis–Procesi, the reduced Ostrovsky, and the bidirectional Whitham equations.

Phase reduction and response of a photonic oscillator

Georgia Himona
National Technical University of Athens, Greece
Yannis Kominis, Vassilios Kovanis

The concepts of isochrons and asymptotic phase, originally introduced in the context of mathematical biology, refer to the timing properties of robust self-sustained oscillations (limit cycles) occurring in any physical or man-made system. In the context of a tunable photonic oscillator, specifically an optically injected semiconductor laser where two semiconductor lasers are coupled in a controller-follower configuration, these notions can be mathematically established through the Fourier averages of an observable quantity such as the intensity of the system. The introduction of these concepts into the study of the original system when subject to periodic external perturbation yields an efficient reduction of the three-dimensional, or in general the multidimensional, system to a one-dimensional circle map. This reduction in dimensionality enables easier analysis and interpretation of the system's behavior. In this work, the dynamical behavior of the circle map—determined by the phase response of the limit-cycle oscillation—corresponding to different kinds of input stimuli is numerically investigated. Thus, conditions for resonant synchronization resulting in desirable outputs of the original nonlinear system of ordinary differential equations can be obtained towards potential practical applications related to photonic signal-processing units.

Initial-boundary value problems for evolution equations via the Fokas method

Alex Himonas
University of Notre Dame, USA

The Fokas unified transform method provides a novel approach for solving initial-boundary value problems (ibvp's) for linear and integrable nonlinear partial differential equations. In particular, it gives solution formulas for forced linear ibvp's. Using these formulas Fokas and collaborators initiated a new approach for studying the well-posedness in Sobolev spaces of ibvp's for nonlinear evolution equations, which is analogous to the way well-posedness of initial value problems (ivp's) are studied based on the Fourier method. Utilizing the Fokas solution formulas linear estimates are derived, and then using the multilinear estimates suggested by the nonlinearity it is shown that the iteration map defined by this formula is a contraction in appropriate solution spaces. In this talk we will present key points of this approach for the Korteweg-de Vries, the nonlinear Schrödinger and related equations and systems. The talk is based on collaborative work with A. Fokas, D. Mantzavinos and F. Yan.

Blow up conditions for generalizations of the Camassa-Holm equation

Curtis Holliman
The Catholic University of America, USA
Alex Himonas

We examine some new blowup results for generalizations of the Camassa-Holm equation. Additionally, we look at some of the consequences of these results on well-posedness of these equations in rough spaces.

Continuity of the data-to-solution map for conservation laws

John Holmes
The Ohio State University, USA
Barbara Keyfitz

In 1975, Kato proved the well-posedness (existence, uniqueness, and continuous dependence of the solution on the initial data) for the Cauchy problem corresponding to a general system of symmetric hyperbolic equations. At the end of the paper, he showed via the Burgers equation, that one should not expect that the data-to-solution map be Hölder continuous for any Hölder exponent. Since his result, many mathematicians have refined his ideas while investigating the continuity of the data-to-solution map for similar equations (Euler, Camassa-Holm, others) and proved that for these problems the data-to-solution map is not even uniformly continuous. In this talk we will explore the development of these results, and we will use the ideas introduced by Kato and other mathematicians to prove that, for a general class of hyperbolic conservation laws, the data-to-solution map is not uniformly continuous.

Critical well-posedness for the derivative nonlinear Schrödinger equation on the line

Maria Ntekoume
Rice University, USA
Benjamin Harrop-Griffiths, Rowan Killip, Monica Visan

This talk focuses on the well-posedness of the derivative nonlinear Schrödinger equation on the line. This model is known to be completely integrable and L^2-critical with respect to scaling. However, until recently not much was known regarding the well-posendess of the equation below $H^{\frac{1}{2}}$. In this talk we prove that the problem is well-posed in the critical space L^2 on the line, highlighting several recent results that led to this resolution. This is joint work with Benjamin Harrop-Griffiths, Rowan Killip, and Monica Visan.

The Cauchy problem of a third-order dispersive Camassa-Holm equation with cubic nonlinearities

Gerson Petronilho
Universidade Federal de São Carlos, Brazil
Alex A. Himonas

The Cauchy problem of a third-order dispersive Camassa-Holm equation with cubic nonlinearities having initial data $\varphi(x)$ in analytic spaces is studied. First, local well-posedness in analytic Gevrey spaces, is established by using trilinear estimates in analytic Bourgain spaces. Then, using the fact that solution of this equation conserve their H^1-norm, an almost conservation law in the corresponding analytic Gevrey spaces is derived.

Unique continuation for the Benjamin-Ono equation and the Camassa-Holm equation

Gustavo Ponce
University of California Santa Barbara, USA
F. Linares, C. E. Kenig, L. Vega

We shall review unique continuation (UC) results for the Benjamin-Ono eq. and the Camassa-Holm equation. These UC's are of local and at infinity types. We shall present some differences and discuss some natural questions arising from them.

Finite time blowup for the nonlinear Schrödinger equation with a delta potential

Sarah Raynor
Wake Forest University, USA
B. Hauser, J. Holmes, E. O'Keefe, C. Yu

In this talk, we study the Cauchy problem for the nonlinear Schrödinger equation with a delta potential. We show that under certain conditions, the supreme norm of the solution tends to infinity in finite time. in order to prove this, we study the associated Lagrangian and Hamiltonian, and derive an estimate of the associated variance. We also derive several conservation laws which a classical solution of the Cauchy problem must also satisfy.

On the Cauchy problem of the modified b-family

Brian Reyes
University of Notre Dame, USA

We consider the Cauchy problem of the modified b-family of equations and study its well-posedness in Sobolev and analytic spaces. Also, we study the evolution of the uniform radius of analyticity for global solutions.

On elastic solids with strain-gradient elastic boundary surfaces

Casey Rodriguez
University of North Carolina Chapel Hill, USA

Since the study of surface tension by Gibbs, it has become clear that surface stresses must be accounted for when modeling elastic bodies at small length scales. In this talk we report on a recently proposed mathematical model of a bulk solid containing a boundary surface with strain-gradient surface elasticity. The partial differential equations governing

equilibrium states are the Euler-Lagrange equations associated to a Lagrangian energy functional with a novel surface energy density depending on the deformed surface's relative curvature, stretching, and stretching gradient. After considering its mathematical precursors and properties, we then discuss its promise for modeling fracture without the pathological singularities arising in more classical models.

Fokas diagonalization

Dave Smith
Yale-NUS College, Singapore

We describe a new form of diagonalization for linear two point constant coefficient differential operators with arbitrary linear boundary conditions. Although the diagonalization is in a weaker sense than that usually employed to solve initial boundary value problems (IBVP), we show that it is sufficient to solve IBVP whose spatial parts are described by such operators. We argue that the method described may be viewed as a reimplementation of the Fokas transform method for linear evolution equations on the finite interval. The results are extended to multipoint and interface operators, including operators defined on networks of finite intervals, in which the coefficients of the differential operator may vary between subintervals, and arbitrary interface and boundary conditions may be imposed; differential operators with piecewise constant coefficients are thus included.

Solutions of weakly dissipated KdV equations with variable coefficients on a periodic domain

Shu-Ming Sun
Virginia Tech, USA

The talk will discuss the solutions of the Cauchy problem for weakly dissipated KdV equation with variable coefficients posed on a periodic domain. The KdV type of equation has variable coefficients on the dissipative and dispersive terms. Under the condition that the integral of the coefficient for the dissipative term over a period keeps a correct sign, it is shown that the corresponding Cauchy problem admits a unique solution in appropriate Sobolev spaces and the solution possesses the sharp Kato smoothing property. Moreover, the nonlinear part of the solution has the strong Kato smoothing property and the sharp double Kato smoothing property. (This is a joint work with X. Yang. B.-Y. Zhang and N. Zhong)

Weak diffeomorphisms and solutions to conservation laws

Feride Tiglay
Ohio State University, USA
Barbara Keyfitz, John Holmes

Evolution equations which describe the changes in a velocity field over time have been classically studied within the Eulerian or Lagrangian frame of reference. Classically, these frameworks are equivalent descriptions of the same problem, and the equivalence can be demonstrated by constructing particle paths. For hyperbolic conservation laws, we extend the equivalence between these frameworks to weak solutions for a broad class of problems.

The well-posedness of a KdV system on the half-line

Fangchi Yan
Virginia Tech, USA
Alex Himonas

Using the Fokas unified transform method the well-posedness of the initial-boundary value problem of a system of KdV type equations on the half-line is studied for initial data in spatial Sobolev spaces and boundary data in the temporal Sobolev spaces suggested by the time regularity of the Cauchy problem for the corresponding linear system. First, linear estimates in Bourgain spaces are derived by utilizing the Fokas solution formula of the corresponding forced linear system. Then, using these and the needed bilinear estimates, it is shown that the iteration map defined by the Fokas solution formula is a contraction in an appropriate solution space. This talk is based on a joint work with Alex Himonas.

Global solutions of quasilinear mKdV equations

Qingtian Zhang
West Virginia University, USA
Fangchi Yan

We consider the quasi-linear Hamiltonian mKdV equation $\varphi_t + \partial_x^3\varphi + \partial_x(\varphi^3) + \partial_x[c(\varphi)\partial_x(c(\varphi)\partial_x\varphi)] = 0$. We prove that when the initial data is sufficiently smooth, localized and small, the solution exists global-in-time.

Special Session 22: Mathematical Modeling of Pandemics

Benedetto Piccoli, Rutgers University Camden, USA
Ryan Weightman, Rutgers University Camden, USA

This special session will focus on the field of epidemiological modeling of pandemics. The study of how best to model a pandemic exploded in 2019 as the COVID-19 pandemic took hold. New ideas and model adaptations have not slowed in the years since. Modelers in a wide range of backgrounds work together and work in parallel to develop frameworks focused on analyzing past trends, understanding our current relationship with the virus, and extrapolating best practices for the future. This sudden need for epidemiological modeling worldwide has produced many advances in modeling capabilities and applications. This special session serves to bring together many who have worked to answer the driving questions of the field as well as those interested in the topic in order to build a forum of rich conversation where both senior and young researchers may gain a better understanding of the recent contributions to the field and its future trajectories.

Sensitivity analysis and uncertainty quantification in inverse problems governed by epidemic models

Alen Alexanderian
North Carolina State University, USA

We consider inverse problems governed by systems of differential equations that contain uncertain parameters in addition to the parameters being estimated. For example, while the goal may be estimating some coefficients or coefficient functions, one might have uncertainties in initial conditions or source terms. In such problems, it is important to understand the sensitivity of the solution of the inverse problem to the uncertain model parameters. It is also of interest to understand the sensitivity of the inverse problem solution to different types of measurements or parameters describing the experimental setup. Hyper-differential sensitivity analysis (HDSA) is a sensitivity analysis approach that provides tools for such tasks. In this talk, we discuss HDSA for systems governed by ordinary differential equations with a focus on illustrative model problems in epidemiology. We will also discuss uncertainty analysis in the class of parameterized inverse problems under study.

A stochastic SEIRD model of COVID-19 and its intervention

Saroj Biswas
Temple University, USA
N.U. Ahmed

Most of the studies on COVID-19 assumes a constant transmission rate although it is not so since it depends on human mobility and behavior. In this research, we present a stochastic SEIRD epidemiological model of COVID-19 by considering the virus transmission rate and the patient recovery rate as random processes. For controlling the pandemic, we consider three control options: a) social contact mitigation and suppression, b) use of novel treatment modalities (during early stage of pandemic when vaccines were not available), and c) vaccination. Fundamentally the pandemic intervention problem can be viewed as a mathematical optimization problem as there are contradictory outcomes in terms of reduced infection and fatalities but with serious economic downturns. Concepts of stochastic optimal control theory are used to determine the optimal control (intervention) policy. Simulation results show the effectiveness of the control policy.

Using hybrid systems to develop epidemiological models on a global scale

Monique Chyba
University of Hawaii, USA
Richard S. Carney

Mathematical modeling of disease spread became more important than ever during the COVID-19 pandemic. Forecasting infections and hospitalizations to take appropriate mitigations measure has been key to contain the pandemic as well as possible. Mitigations measures were however taken locally, and a lack of coordination between nations allowed for new variants to emerge and travel around the globe. Understanding how travels restrictions impact the spread of the virus globally is the focus of our work. This can be described as a hybrid system that arise from interactions between continuous state dynamics and discrete state dynamics. The local continuous dynamics is captured by SEIR models while the global dynamic is represented by a graph. We introduce guard conditions to trigger the discrete events, define what an execution is in that framework and illustrate the concept using numerical simulations.

On the stochastic engine of transmittable diseases in exponentially growing populations

Torsten Lindström
Linnaeus University, Sweden

The purpose of this paper is to analyze the mechanism for the interplay of deterministic and stochastic models for transmittable diseases. Deterministic models for contagious diseases are prone to predict global stability. Small natural birth and death rates in comparison to disease parameters like the contact rate and the removal rate ensures that the globally stable endemic equilibrium corresponds to a tiny average proportion of infected individuals. Asymptotic equilibrium levels corresponding to low numbers of individuals invalidate the deterministic results.

Diffusion effects force frequency functions of the stochastic model to possess similar stability properties as the deterministic model. Particular simulations of the stochastic model predict, however, oscillatory patterns. Small and isolated populations show longer periods, more violent oscillations, and larger probabilities of extinction.
We prove that evolution maximizes the infectiousness of the disease as measured by the ability to increase the proportion of infected individuals. This holds provided the stochastic oscillations are moderate enough to keep the proportion of susceptible individuals near a deterministic equilibrium.
We close our paper with a discussion of the herd-immunity concept and stress its close relation to vaccination-programs.

Mathematical modeling of the spread of COVID-19 on a university campus

Kaitlyn Muller
Villanova University, USA
Peter Muller

We present a deterministic transmission dynamic compartmental model for the spread of the novel coronavirus on a college campus for the purpose of investigating strategies to mitigate an outbreak. It is the goal of this project to determine the utility of certain containment strategies including gateway testing, surveillance testing, and contact tracing. It is assumed that students and faculty/staff take part in standard public health practices to reduce the spread such as mask wearing, hand washing, and social distancing. These efforts are modeled by relative changes in the contact rates between populations. We modify a standard SEIR type model to reflect what is known about COVID-19, in particular the existence of asymptomatic carriers. We also modify the model to reflect the population present on a college campus, separating into students and faculty. We determine effectiveness of mitigation measures by looking at relative changes in the total number of cases as well as the effect a measure has on the estimated basic reproductive number. We present model fitting to data collected at various universities during Fall 2020 and Spring 2021 semesters. In addition we explore the effect of superspreader events as well as time-varying parameters.

Optimal control of the COVID-19 pandemic with non-pharmaceutical interventions

Alex Perkins
University of Notre Dame, USA

In the earliest months of the COVID-19 pandemic, societies across the world resorted to social distancing to slow the spread of the SARS-CoV-2 virus. Due to the economic impacts of social distancing, a desire to relax those measures quickly arose. To characterize a range of possible strategies for control and to understand the consequences of relaxing those measures in such a situation, we performed an optimal control analysis of a mathematical model of SARS-CoV-2 transmission. Given that the pandemic was already underway and controls had already been initiated, we calibrated our model to data from the USA as of May 2020 and focused our analysis on optimal controls through December 2021. We found that a major factor that differentiated strategies that prioritize lives saved versus reduced time under control is how quickly control is relaxed once social distancing restrictions expired in May 2020. Strategies that maintained control at a high level until at least summer 2020 allowed for tapering of control thereafter and minimal deaths, whereas strategies that relaxed control in the short term led to fewer options for control later and a higher likelihood of exceeding hospital capacity. Our results also highlight that the potential scope for controlling COVID-19 until a vaccine became available depended on epidemiological parameters about which there was considerable uncertainty at the time, including the basic reproduction number and the effectiveness of social distancing.

Spontaneous human behavioral change in epidemiological models

Stephen Schecter
North Carolina State University, USA

In epidemiological models such as the SIR model

$$\begin{aligned}\dot{S} &= -\beta S I,\\ \dot{I} &= \beta S I - \gamma I,\\ \dot{R} &= \gamma I,\end{aligned}$$

the effective contact rate β is treated as a constant. In fact we know from experience that β behaves more like a state variable: due to fear, contact rates often fall as infection rates rise. We shall discuss a simple extension of SIR due to Piero Poletti and collaborators in which a single equation models how response to a changing infection rate spreads through a population by contact, much like the epidemic itself. We show how the model can be analyzed using geometric singular perturbation theory. Unlike the usual SIR model, the extended model can produce successive "waves" of infection.

Special Session 23: Topological and Variational Methods for Differential Equations

John Graef, University of Tennessee at Chattanooga, USA
Lingju Kong, University of Tennessee at Chattanooga, USA
Min Wang, Kennesaw State University, USA

Topological and variational methods are important techniques in the analysis of initial and boundary value problems for differential equations. Both theoretical results and applications to real world problems are appropriate for inclusion in this special session.

Comparison of green's functions for two-point boundary value problems with fractional derivative boundary conditions

Paul Eloe
Univeristy of Dayton, USA
Jeffrey T. Neugebauer

In this paper, we compare Green's functions for two point fractional differential equations where both conditions at the right endpoint depend on a fractional derivative. This is accomplished by showing that the difference of two Green's functions is sufficiently smooth and satisfies a fractional boundary value problem. The sign of the solution of this boundary value problem depends on a fractional derivative of a Green's function evaluated at the right endpoint. These results answer an open question raised by the authors in a previous paper.

The linear and nonlinear parts of a semi-linear operator on fixed point index

Wenying Feng
Trent University, Canada
Shugui Kang, Yanlei Zhang

Semi-linear operators are common in the applications of Boundary Value Problems. A semi-linear operator can be written as the composition of a linear operator and a nonlinear map. We obtain results on fixed point index based on the associated properties between the linear and the nonlinear parts. The results are related to the theory of nonlinear and semi-linear spectrum. Applications of the theorems are given by examples of positive solutions for a second-order differential equation with separated boundary conditions.

A topological approach to nonlocal elliptic partial differential equations on an annulus

Christopher Goodrich
UNSW Sydney, Australia

I will consider the boundary value problem

$$-a\left(\int_{\Omega} |u|^q \, d\mathbf{s}\right) \Delta u(\mathbf{x}) = \lambda g\big(u(\mathbf{x})\big),\ \mathbf{x} \in \Omega,$$

where Ω is an annular region in $\mathbb{R}^n$ for $n \geq 3$.

Recent advances in double-phase problems with variable exponent

Yun-Ho Kim
Sangmyung University, Korea

This talk is devoted to the study of the L^∞-bound of solutions to the double-phase nonlinear problem with variable exponent by the case of a combined effect of concave-convex nonlinearities. The main tools are the De Giorgi iteration method and a truncated energy technique. Applying this and variational methods, we give the existence of nontrivial solutions belonging to L^∞-space when the condition on a nonlinear convex term does not assume the Ambrosetti-Rabinowitz condition in general. Also we introduce the recent works related to the double-phase nonlinear problems. In particular, on a new class of nonlinear terms we give the existence result of small energy solutions via applying the dual fountain theorem.

On a new class of Kirchhoff equations involving the $p(x)$-Laplacian

Seol Vin Kim
Seoul National University, Korea
Yun-Ho Kim

In this talk, we are concerned with the existence result of a sequence of infinitely many small energy solutions to the fractional p-Laplacian equations of Kirchhoff-Schrödinger type with concave–convex nonlinearities when the convex term does not require the Ambrosetti-Rabinowitz condition. The main aim of the present talk, on a new class of the Kirchhoff term, is to discuss the multiplicity result of nontrivial solutions by using the dual fountain theorem as the main tool.

On the spectrum of biharmonic systems

Lingju Kong
University of Tennessee at Chattanooga, USA
Roger Nichols, Min Wang

We study the spectrum of a (p, q)-biharmonic system on a bounded domain. Under appropriate conditions, we prove that the system either has at least one nondecreasing sequence of positive eigenvalues, or has at least one noninceasing sequence of negative eigenvalues or has both at least one nondecreasing sequence

of positive eigenvalues and at least one nonincreasing sequence of negative eigenvalues. Our results apply respectively to the cases when the weight functions in the system are positive definite, negative definite, or indefinite.

Equivalence of linear fractional differential and integral equations

Kunquan Lan
Toronto Metropolitan University, Canada

In this presentation, I shall present recent advances on equivalence of linear first-order or higher-order fractional differential and integral equations.

Two point fractional boundary value problems with a fractional boundary condition

Jeffrey Lyons
The Citadel, USA
Jeffrey T. Neugebauer

In this talk, we employ Krasnoselskii's fixed point theorem to show the existence of positive solutions to three different two point Riemann-Liouville fractional boundary value problems with Riemann-Liouville fractional boundary conditions. Also, nonexistence results are given.

Weakly nonlinear boundary value problems for various partial differential equations

Dan Maroncelli
College of Charleston, USA
Mauricio Rivas

In this work, we study the existence of solutions to the following generalized nonlinear two-parameter problem

$$a(u, v) = \lambda b(u, v) + \mu m(u, v) + \varepsilon F(u, v), \qquad (1)$$

for a triple (a, b, m) of continuous, symmetric bilinear forms on a real separable Hilbert space V and nonlinear form F. This problem is a natural abstraction of nonlinear problems that occur for a large class of differential operators, various elliptic pde's with nonlinearities in either the differential equation and/or the boundary conditions being a special subclass.
We will start by discussing a Fredholm alternative for the associated linear two-parameter eigenvalue problem and then we will use this characterization to construct solutions to various nonlinear problems.

Green's functions for a two-term boundary value problem

Jeffrey Neugebauer
Eastern Kentucky University, USA
Justin Bowman, Paul Eloe

In this talk, we study the Green's function corresponding to the boundary value problem $y'' - p(t)y = h(t)$, $y(0) - y(1) = 0$, $ay'(0) + y'(1) = 0$, where $a0$ on $[0, 1]$, where p is chosen so the boundary value problem is not at resonance. We obtain sign properties and comparison results for the Green's function without constructing the Green's function.

A variational framework for second order backward discrete boundary value problems

Min Wang
Kennesaw State University, USA
Liancheng Wang, James Zhang

In this talk, we study two types of second order discrete boundary value problems involving backward difference operators. A new variational framework is developed and used to prove the existence of solutions to the boundary value problems by the critical point theory. An example is then provided to demonstrate the application of our theoretical results

Positive solutions to a third order boundary value problem with a parameter

Miroslawa Zima
Institute of Mathematics, University of Rzeszow, Poland
Gabriela Szajnowska

We discuss the existence of positive solutions to the equation

$$-u''' + m^2 u' = f(t, u, u')$$

under nonlocal boundary conditions $u(0) = 0$, $u'(0) = \alpha[u]$, $u'(1) = \beta[u]$, where m is a positive parameter, and α and β are the functionals acting on the space $C^1[0, 1]$. Our approach is based on the Krasnosel'skiĭ-Guo fixed point fixed point theorem in cones and the properties of the Green's function corresponding to the BVP under study.

Special Session 24: Geometric Methods in Spectral Theory of Traveling Waves and Patterns

Graham Cox, Memorial University of Newfoundland, Canada
Yuri Latushkin, University of Missouri, USA
Alim Sukhtayev, Miami University, USA

We will bring together researchers working on various stability issues for such special solutions of partial differential equations as periodic and solitary waves. All aspects of stability/instability will be discussed, with a special emphasis on methods of spectral theory that lie at the heart of stability analyses. It will be expected of speakers that they will spend some time explaining the perspective underlying their work in order to stimulate further discussion and collaboration in the field.
A particular main theme will be applications of infinite dimensional symplectic geometry in the spectral theory of the operators obtained by linearizing the partial differential equation about the traveling wave or other special solution. Specifically, we expect a number of talks to be concerned with the relation of the Maslov index, a topological invariant defined as the signed number of intersections of a path formed by Lagrangian subspaces with a train of a fixed subspace, and the Morse index counting the number of unstable eigenvalues of the linearization. Recently this topic has been the focus of attention of a large group of researchers, and a special session with this emphasis will foster further collaborations in this area.

Turing instability via the generalized Maslov index

Graham Cox
Memorial University of Newfoundland, Canada
Thomas Baird, Paul Cornwell, Christopher Jones, Robert Marangell

The Maslov index is a topological invariant naturally associated to a Hamiltonian system. This is a useful tool for linear stability analysis, provided the eigenvalue equation can be rewritten in a Hamiltonian form. In this talk I will describe a recent generalization of the Maslov index to non-Hamiltonian systems. This generalization allows one to study reaction-diffusion system of activator-inhibitor type. As an application, I will show how this new index can be used to characterize the Turing instability.

Hamiltonian spectral flows, the Maslov index, and the stability of NLS standing waves

Mitchell Curran
University of Sydney, Australia
Graham Cox, Yuri Latushkin, Robert Marangell

We give a lower bound for the number of positive, real eigenvalues of the Hamiltonian differential operator

$$\begin{pmatrix} 0 & -L_- \\ L_+ & 0 \end{pmatrix}$$

on a compact interval with Dirichlet boundary conditions, where $L_\pm$ are arbitrary scalar-valued Schrödinger operators. Such an operator arises, for example, when linearising about a standing wave in the nonlinear Schrödinger (NLS) equation. Our lower bound follows from a straightforward application of the "Maslov box", and includes a contribution to the Maslov index from a degenerate crossing. We compute this contribution via a homotopy argument, analysing the local behaviour of the eigenvalue curves – which represent the evolution of the eigenvalues as the domain is shrunk or expanded – to do so. Applying our theory to standing wave solutions of the NLS equation leads to compact interval analogues of the Jones-Grillakis instability theorem and the Vakhitov-Kolokolov criterion. Comparison with existing lower bounds, which make use of constrained eigenvalue counts for L_+ and L_-, leads to some interesting connections between the objects appearing therein and the contribution from the degenerate crossing.

On the interplay of transient dynamics and noise-induced tipping

Emmanuel Fleurantin
George Mason University, University of North Carolina Chapel Hill, USA
Katherine Slyman, Blake Barker, Christopher K.R.T. Jones

In this talk, we will describe recent analytical and computational tools to identify Most Probable Escape Paths (MPEPs) in stochastic dynamical systems (SDEs). The study of noise-induced tipping has been dominated by Freidlin-Wentzell (FW) theory. A downside of the FW theory is that it necessitates vanishingly small noise. However, for several applications of interest, particularly in environmental, biological or social contexts, intermediate noise is of more relevance. In focusing on the intermediate noise regime, the transient behavior of the underlying deterministic system will play a key role. We will thus use a dynamical system approach to identify MPEPs in SDEs. The Maslov index will help us distinguish which critical points of the FW functional are minimizers and help explain the effect of the interaction of noise and transient dynamics. The Onsager-Machlup (OM) functional, which is treated as a perturbation of the FW functional, will provide a selection mechanism to pick out a specific MPEP. Our computations will then be compared with Monte Carlo simulations in order to verify theoretical predictions. We will use a 2-dimensional autonomous system with a stable equilibrium solution coexisting with an unstable periodic orbit as a focal point for our methodology.

Oscillation theory and instability of nonlinear waves

Peter Howard
Texas A & M University, USA

In recent work, Baird et al. have introduced a generalized Maslov index which allows oscillation techniques that have previously been restricted to eigenvalue problems with underlying Hamiltonian structure to be extended to the non-Hamiltonian setting [T. J. Baird, P. Cornwell, G. Cox, C. Jones, and R. Marangell, Generalized Maslov indices for non-Hamiltonian systems, SIAM J. Math. Anal. 54 (2022) 1623-1668].
I will discuss how this approach can be implemented in the analysis of spectral instability for nonlinear waves, taking as my setting a class of equations previously investigated by Pego and Weinstein via the Evans function [R. L. Pego and M. I. Weinstein, Eigenvalues, and instabilities of solitary waves, Phil. Trans. R. Soc. Lond. A 340 (1992) 47-94].

Spectral and linear stability of peakon solutions

Stephane Lafortune
College of Charleston, USA
Stathis Charalampidis, Panos Kevrekidis, Ross Parker, Dmitry Pelinovsky

The Camassa-Holm equation with linear dispersion was originally derived as an asymptotic equation in shallow water wave theory. Among its many interesting mathematical properties, perhaps the most striking is the fact that it admits weak multi-soliton solutions - 'peakons'- with a peaked shape corresponding to a discontinuous first derivative. There exists a one-parameter family of generalized Camassa-Holm equations, most of which are not integrable, but which all admit peakon solutions.
In this talk, we establish information about the spectral stability/instability of those solutions. By spectral stability analysis, we mean the analysis of the spectrum of the operator arising from the linearised equation. Furthermore, we extend our analysis to the peakon solutions of the Novikov equation.
The computation of the spectrum differs from similar problems due to the following two facts: the solutions to the equations are not analytical, and the linear operator is nonlocal as it contains integral terms.

Maslov index and the spectrum of differential operators

Yuri Latushkin
University of Missouri, USA
G. Cox, A. Sukhtayev, S. Sukhtaiv

We will give a review of recent results on applications of the Maslov index in spectral theory of differential operators

Multi-scale reduction of modified Holling-Tanner model with an Allee effect

Vahagn Manukian
Miami University, USA

Through multi-scale analysis of a modified Holling-Tanner model with an Allee effect, we show that there is a parameter regime in the system where the dynamics can be captured through the KPP equation. Additionally, we show that within a certain parameter regime, the dynamics can be reduced to the bistable equation. We will show the existence of front solutions within these regimes and discuss their stability.

Spectrum of non-planar traveling waves

Alin Pogan
Miami University, USA
Yuri Latushkin, Anna Ghazaryan

We prove that a class of non self-adjoint second order differential operators acting in cylinders have only real discrete spectrum located to the right of the right most point of the essential spectrum. We describe the essential spectrum using the limiting properties of the potential. To track the discrete spectrum we use spatial dynamics and bi-semigroups of linear operators to estimate the decay rate of eigenfunctions associated to isolated eigenvalues.

On the stability of the periodic waves for the Benney and Zakharov systems

Milena Stanislavova
University of Alabama Birmingham, USA
Sevdzhan Hakkaev, Atanas Stefanov

We analyze the Benney model for interaction of short and long waves in resonant water wave interactions. Our particular interest is in the periodic traveling waves, which we construct and study in detail. The main results are that, for all natural values of the parameters, the periodic dnoidal waves are spectrally stable with respect to perturbations of the same period. For another natural set of parameters, we construct the snoidal waves, which exhibit instabilities, in the same setup. For the periodic travelling waves of the Zakharov system, we show that, for all natural values of the parameters, the dnoidal waves are spectrally stable with respect to perturbations of the same period.

Arnold conjecture on CP^n as a bifurcation problem

Maciej Starostka
Martin Luther University Halle-Wittenberg, Germany

We discuss how the Arnold conjecture on CP^n is equivalent to a certain bifurcation problem. Namely, we end up with a family $F_\lambda = L + K_\lambda : H \to H$ such that L is a fixed Fredholm map, K_λ is a nonlinear compact perturbation with $K_\lambda(0) = 0$ and H is a separable Hilbert space. Since K_λ may not be differentiable at 0 we cannot use spectral flow (or comparison of Morse indicies) arguments to show existence of bifurcation. Therefore Conley index together with its relative cup-length will be briefly discussed.

Solitary waves in the NLS system of the third-harmonic generation

Atanas Stefanov
University of Alabama Birmingham, USA
Abba Ramadan

The NLS system of the third-harmonic generation is analyzed. Our interest is in solitary wave solutions and their stability properties. The recent work of Oliveira and Pastor, discusses global well-posedness vs. finite time blow up, as well as other aspects of the dynamics. They have also established some stability/instability results for these waves.
In this work, we systematically build and study solitary waves for this important model. We construct the waves in the largest possible parameter space, and we provide a complete classification of their stability. In dimension one, we show stability, whereas in dimensions two and three, they are generally spectrally unstable, except for a small region, where they do enjoy an extra pseudo-conformal symmetry.
Finally, we discuss the instability by blowup. In the three dimensional case and for somewhat restricted set of parameters, we use virial identities methods to derive the strong instability. In the two dimensional case, the virial identities reduce matters, via conservation of mass and energy, directly to the initial data. Blow up is confirmed for all data, sufficiently close to the (unstable) soliton, with strictly larger mass.

Resolvent expansions for self-adjoint operators via boundary triplets

Selim Sukhtaiev
Auburn University, USA
Yuri Latushkin

In this talk we will discuss asymptotic perturbation theory for varying self-adjoint extensions of symmetric operators. Specifically, we derive a Riccati-type differential equation and second order asymptotic expansion for resolvents of self-adjoint extensions determined by smooth one-parameter families of Lagrangian planes. This asymptotic perturbation theory yields a symplectic version of the abstract Kato selection theorem and Hadamard-Rellich-type variational formula for slopes of multiple eigenvalue curves bifurcating from an eigenvalue of the unperturbed operator. Applications are given to quantum graphs, periodic Kronig-Penney model, elliptic second order partial differential operators with Robin boundary conditions, and physically relevant heat equations with thermal conductivity.

Fredholm determinants, Evans functions and Maslov indices

Alim Sukhtayev
Miami University, USA
Graham Cox, Yuri Latushkin

The Evans function is a well known tool for locating spectra of differential operators in one spatial dimension. We construct a multidimensional analogue as the modified Fredholm determinant of a ratio of Dirichlet-to-Robin operators on the boundary. This gives a tool for studying the eigenvalue counting functions of second-order elliptic operators that need not be self-adjoint. In the self-adjoint case we relate our construction to the Maslov index, another well known tool in the spectral theory of differential operators.

K-theoretic methods in the computation of the Maslov-index

Nils Waterstraat
Martin Luther University Halle-Wittenberg, Germany

We review some recent results on a connection between the spectral flow, topological K-theory and the Maslov index, that have yielded applications to homoclinics of Hamiltonian systems and geodesics in semi-Riemannian manifolds.

Special Session 25: Mathematical Modeling and Quantitative System Pharmacology

Christopher Denaro, Rutgers University Camden, USA
Benedetto Piccoli, Rutgers University Camden, USA

This special session will focus on the growing field of quantitative systems pharmacology. In recent years, clinical trials addressing neurodegenerative and other rare diseases have faced complications with disease pathophysiology and drug-target engagement. The field of quantitative systems pharmacology aims to address these issues in a data-driven, mechanistic way by using mathematical models to estimate human-disease interaction. QSP model development strikes a difficult balance between complexity and simplicity, as model priors must be informed by clinical or experimental data. This special session serves to bring together modelers from mathematical, systems biological, and regulatory background to address common challenges and identify best practices when developing quantitative systems pharmacology models.

Quantitative systems pharmacology: More of the same or something?

Ioannis Androulakis
Rutgers University, USA

Quantitative Systems Pharmacology (QSP) has emerged as a powerful ensemble of approaches aiming to develop integrated mathematical and computational models that elucidate the complex interactions between pharmacology, physiology, and disease. As the field grows and matures, its applications expand beyond the boundaries of research and development and slowly enter the decision-making and regulatory arenas. QSP aims to provide an integrated understanding of pathology in the context of therapeutic interventions. Because of its ambitious nature and the fact that QSP emerged in an uncoordinated manner due to activities distributed across organizations and academic institutions, high entropy characterizes the tools, methods, and computational methodologies and approaches used. The eventual acceptance of QSP model predictions as supporting material for an application to a regulatory agency will require that two key aspects are considered: 1) increase confidence in the QSP framework, which drives standardization and assessment, and 2) careful articulation of the expectations. In this talk, we will summarize how the field emerged and evolved, and through examples, we will highlight some of the challenges and opportunities that are emerging.

Crowdsourcing patient engagement in Parkinson's disease: A patient swarm approach to refine the development of a quantitative systems pharmacology (QSP) model via a digital research environment (DRE)

Jeff Barrett
Aridhia Bioinformatics, USA

QSP models take significant time and effort to create when compared to statistical or classical empirically driven models; mostly because of the length of time to identify, evaluate and populate credible model priors. We are constructing a digital research environment (DRE) that receives data from a community-centric crowdsourcing approach that includes patients (patient swarm) so that model priors can be more efficiently catalogued and evaluated by data curators and QSP modelers. Crowdsourcing efforts will be evaluated as the model is being constructed and the patient swarm will be engaged to comment on both the structure and its predictive potential to explain disease progression and evaluate historical and current development candidates in real-time. In addition, patient-generated disease trajectories will be used as a real-world data source to validate the model. Upon completion these will serve to verify that the model is able to generate synthetic data that more closely mimics the heterogeneity of the family of disease etiologies currently classified as Parkinson's Disease. Patient research participants will describe their illness in quantitative terms with the help of an experienced QSP modeling team, some of whom will construct a model based on priors collected from all available sources (public and private sector) using an AI/ML driven text mining approach to identify source data from the literature.

Mechanistic pharmacodynamic modeling of single mammalian cells

Marc Birtwistle
Clemson University, USA

Modeling is routinely used across multiple disciplines to priortize designs before testing and implementation, but this is seldom done in the medical and pharmaceutical industries because reliable models are difficult to build and the systems are extremely complex and incompletly understood. My lab is building models and approaches for capturing mechanisms that drive behavior of single mammalian cells as a key foundational component of such efforts. In particular, we have been building hybrid models comprising large-scale ordinary differential equation systems with stochastic components to capture low molecule number fluctuations with Poisson-like descriptions. A major simulation bottleneck in such large-scale cellular modeling is often not solving the equations themselves, but rather communication between submodels with different formalisms. In this talk, I'll present some of our latest work in these areas, highlighting some of the interesting biological findings arising from such modeling, as well as what I believe are key numerical and computational method barriers that if solved would catalyze growth in the field.

Understanding the antiviral effects of RNAi-based therapy on chronic hepatitis B infection

Stanca Ciupe
Virginia Tech, USA
Sarah Kadelka, Harel Dahari

Current treatment options focus on removing circulating HBV DNA but are suboptimal in removing hepatitis B s- and e-antigens. ARC-520, a RNA interference drug, had induced substantial hepatitis B s- and e- antigen reductions in animals and patients receiving therapy. We study the effect of ARC-520 on hepatitis B s- and e-antigen decline by developing mathematical models for the dynamics of intracellular and serum viral replication, and compare it to patient HBV DNA, hepatitis B s- and e-antigen data from a clinical trial with one ARC-520 injection and daily nucleoside analogue therapy. We examine biological parameters describing the different phases of HBV DNA, s-antigen and e-antigen decline and rebound after treatment initiation, and estimate treatment effectiveness.

Simulating combination therapies for the treatment of tuberculosis using linear in flux expressions

Christopher Denaro
Rutgers University Camden, USA
Sean McQuade, Benedetto Piccoli

Tuberculosis (TB) is a mainly pulmonary disease caused by infection of Mycobacterium Tuberculosis. Treatment of TB can become difficult, as the bacteria can form granuloma as a defense against antibiotic treatment. As a result, treatment of TB can require a combination therapy of three to four antibiotics for up to six months. Because there are many antibiotics to choose from, the treatment space of potential therapies grows rapidly. In this work, we describe a pipeline for estimating the effect of a combination therapy. We simulate the therapy's impact on Mycobacterium Tuberculosis metabolic networks based on gene differential expression data from single treatment microarray data. The simulations are built on a novel approach - Linear in Flux Expressions (LIFE) methodology describing a system of ordinary differential equations. This system of equations encodes a metabolic network - describing the mass flow through the system. The computation pipeline we describe provides a framework for estimating the efficacy of a combination therapy based on the perturbation of metabolites. This framework may be used to narrow the search for new combination therapies.

Maximizing efficacy and minimizing toxicity in HER2+ breast cancer treatment: An optimal control study

Ernesto Lima
University of Texas Austin, USA
Reid A. F. Wyde, Anna G. Sorace, Thomas E. Yankeelov

Human epidermal growth factor receptor 2 positive (HER2+) breast cancer is often treated with drugs targeting the HER2 receptor and chemotherapy, but identifying the optimal regimen is challenging. Using data from a murine model of HER2+ breast cancer, treated with trastuzumab and doxorubicin, we developed a framework for model development, calibration, selection, and treatment optimization. We proposed ten different models to characterize the dynamic relationship between tumor volume and drug availability, as well as the drug-drug interaction, and used a Bayesian framework to calibrate each model. We selected the model with the highest Bayesian information criterion weight to represent the biological system. Applying optimal control theory to this model, we identified two optimal treatment protocols. In the first protocol, using the same experimental doses for both drugs, the model predicts a 45% reduction in tumor burden compared to the experimentally delivered regimen. In the second protocol, using the same experimental trastuzumab dose but only 43% of the doxorubicin dose used experimentally, the model predicts the same tumor control as achieved experimentally. Our results suggest that mathematical modeling and optimal control theory can be effective tools for identifying therapeutic regimens that maximize efficacy and minimize toxicity in HER2+ breast cancer treatment.

Reconceptualizing antimicrobial therapy in the context of the host-pathogen interaction

Gauri Rao
University of North Carolina, USA

The global antimicrobial resistance crisis is a slow-motion tsunami. The lack of novel treatment options and the mounting resistance to currently approved agents are making a bad situation worse. Infections caused by Gram-negative bacteria are especially challenging to treat. Hence, refocusing efforts on combining in silico mathematical modeling strategies using both preclinical (in vitro and in vivo) and clinical pharmacokinetic/pharmacodynamic time course data to design and optimize antimicrobial treatment taking the interactions between host and pathogen into consideration is an attractive strategy. Using a systems based approach to model progression of bacterial infection taking the host immune response into consideration model enables us to understand the contribution of the host relative to drug treatment. Similarly, understanding immunodynamics in addition to viral kinetics within the host following influenza, an acute respiratory tract infection will enable us to determine the optimal time as well as dose and frequency of dosing antiviral treatment.

Predicting efficacy of cancer immunotherapy using a quantitative systems pharmacology model

Hanwen Wang
Johns Hopkins University, USA

Cancer is a leading cause of death worldwide with over 19 million estimated new cases and nearly 10 million deaths in 2020. While the number of clinical trials is increasing over the past decades, the success rate of oncology trials remains the lowest among all therapeutic areas. This challenge necessitates the development of computational tools to predict the effectiveness of drugs of interest and identify predictive biomarkers for various treatment combinations. In model-informed drug discovery (MIDD) and clinical trial design, quantitative systems pharmacology (QSP) models have begun to play crucial roles due to their ability to integrate mechanistic knowledge from cancer biology and pharmacology into a quantitative framework. I will present a modular QSP platform for immuno-oncology (QSP-IO) that describes the cancer-immunity cycle, which allows for varying degrees of complexity based on our research goals. I will introduce the model structure, discuss methods for creating a virtual patient population for in silico clinical trial simulation, and present the results from an in silico clinical trial of PD-L1 inhibition in advanced non-small cell lung cancer. I aim to demonstrate the potential of QSP models to discover predictive biomarkers, make efficacy predictions for drugs of interest, and guide future clinical trials.

Combing multidimensional data to guide personalized diagnosis and treatment

Tongli Zhang
University of Cincinnati, USA

Most diseases are characterized by complex dynamical features that emerge from interaction between multi-scale components, including biomolecules, cells, organs, and even patients. Furthermore, current clinical and experimental approaches often only provide limited data that spin between different scales and differ between individuals. Novel theoretical methods are needed to effectively utilize these data to design personalized diagnosis and treatment. In this work, we illustrate how a novel methodology that combines the power of mechanistic modeling, machine learning (ML) and artificial intelligence (AI) could overcome the limitation of each individual tool and extract promising biomarkers that can guide personalized diagnosis and treatment. We expect that the board application of this general methodology could help improve the diagnosis and facilitate the development of novel treatment of many complex diseases.

Special Session 27: Recent Trends in Navier-Stokes Equations, Euler Equations, and Related Problems

Sarka Necasova, Mathematical Institute ASCR, Czech Republic
Reimund Rautmann, University of Paderborn, Germany
Werner Varnhorn, University of Kassel, Germany

For many decades the basic equations of fluid dynamics have been the focus of great interest and active research worldwide. Recently developed approaches and analytical tools helped to establish new criteria of uniqueness, stability, and regularity of weak and mild solutions, depending on different conditions of the data, and on results concerning multi-component flows. The same applies to the reaction between fluids and immersed bodies such as boundary control of self-propelled objects. Moreover, we dispose of new results concerning the asymptotic behaviour of suitable unsteady weak solutions with increasing time or with vanishing viscosity, and also concerning the general question whether or not the unsteady Navier-Stokes equations admit global attractors. Therefore, our special session aims at bringing together researchers from all over the world working in all of these or related fields in order to initiate fruitful discussions and cooperations.

Steady state solution of Shliomis model

Cherif Amrouche
Université de Pau et des Pays de l'Adour, France
Saliha Boukassa

In this work we study a model of differential system introduced by Shliomis to describe the stationary flow of an incompressible ferrofluid submitted to the action of an external magnetic field. The system is a combination of the Navier-Stokes equations, the magnetization equation. We prove the existence of weak solution of this problem and we give regularity results in L^p−theory.

Existence analysis of a stationary compressible fluid model for heat-conducting and chemically reacting mixtures

Miroslav Bulìček
Charles University, Czech Republic

We consider a steady flow of heat conducting chemically reacting mixtures in dimension two and three. We focus on both compressible and incompressible setting whose thermodynamics is described by general free energies are considered satisfying some fundamental structural assumptions. We discuss the conditions leading to the existence of weak solution. It is noticeable that the considered models are thermodynamically consistent on one hand and are able to cover the classical models like the Maxwell-Stefan cross-diffusion equations in the Fick-Onsager form as a special case on the other hand. Compared to previous works, a very general model class is analysed, including cross-diffusion effects, temperature gradients, compressible fluids, and different molar masses (in case of compressible fluid). In addition, the technique leading to the compactness of the pressure is a nontrivial generalisation of the use Feireisl's oscillation defect measure and the newly developed technique relies heavily on the convexity of the free energy and the strong convergence of the relative chemical potentials.

On the high compressible limit for the Navier-Stokes-Korteweg model with density dependent viscosity

Matteo Caggio
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Donatella Donatelli

The talk is devoted to the regime of high Mach number flows for compressible barotropic fluids with density dependent viscosity. The Korteweg model as an isothermal model of capillary and quantum compressible fluids is considered. A weak-strong uniqueness analysis is also discussed.

Near resonance approximation of rotating Navier-Stokes equations

Bin Cheng
University of Surrey, England
Zisis N. Sakellaris

We formalise the concept of near resonance for the rotating Navier-Stokes equations, based on which we propose a novel way to approximate the original PDE. The spatial domain is a three-dimensional flat torus of arbitrary aspect ratios. We prove that the family of proposed PDEs are globally well-posed for any rotation rate and initial datum of any size in any H^s space with $s \geq 0$. Such approximations retain much more 3-mode interactions, thus more accurate, than the conventional exact resonance approach. Our approach is free from any limiting argument that requires physical parameters to tend to zero or infinity, and is free from any small divisor argument (so estimates depend smoothly on the torus' aspect ratios). The key estimate hinges on counting of integer solutions of Diophantine inequalities rather than Diophantine equations. Using a range of novel ideas, we handle rigorously and optimally challenges arising from the non-trivial irrational functions in these inequalities. The main results and ingredients of the proofs can form part of the mathematical foundation of a non-asymptotic approach to nonlinear oscillatory dynamics in real-world applications.

Singularity formation for fluid equations and models

Mimi Dai
University of Illinois Chicago, USA

We will discuss the possible scenarios of finite time singularity formation for some fluid equations and models.

On the motion of several small rigid bodies in a viscous incompressible fluid

Eduard Feireisl
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Arnab Roy, Arghir Zarnescu

We consider the motion of a finite numeber of rigid bodies immersed in a viscous incompressible fluid contained in a bounded domain in the Euclidean space. We show the fluid flow is not influenced by the presence of the bodies in the asymptotic limit of small radius of the bodies.The result depends solely on the geometry of the bodies and is independent of their mass densities. Collisions are allowed and the initial data are arbitrary with finite energy.

Existence of weak solutions for a compressible multi-component fluid structure interaction problem

Martin Kalousek
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Sourav Mitra, Šàrka Nečasovà

We address the question of global in time existence of weak solutions of a system of PDEs governing the interaction between two compressible mutually noninteracting fluids and a shell of Koiter type encompassing a time dependent 3D domain filled by the fluids. The dynamics of the fluids is modelled by compressible Navier-Stokes equations with a physically realistic pressure depending on densities of both the fluids. The shell constitutes the boundary of the fluid domain and it possesses a non-linear, non-convex Koiter energy.

Uniqueness and regularity of flows of non-Newtonian fluids

Petr Kaplicky
Charles University, Czech Republic
M. Bulìček, F. Ettwein, D. Pražàk

We give overview of results on non/uniqueness of flows of generalized Newtonian fluids. The prototypical example of the Cauchy stress tensor has the form $\mathbb{T}(\mathbb{D}) = (1+|\mathbb{D}|^2)^{(p-2)/2}\mathbb{D}$ for some growth parameter $p > 1$. For $p \geq 11/5$ we show sufficient regularity of any solution to obtain its uniqueness provided data of the problem are sufficiently smooth. We remark that the bound $p \geq 11/5$ corresponds to the situation when we can test the weak formulation of the equation with the solution under its natural regularity.

Flow of a heat conducting fluid in a time-dependent domain

Ondrej Kreml
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Vaclav Macha, Sarka Necasova, Aneta Wroblewska-Kaminska

We consider a flow of heat conducting fluid inside a moving domain which changes in time in a prescribed way. The flow is governed by the Navier-Stokes-Fourier system consisting of equation of continuity, momentum balance, entropy balance and energy equality. We show the existence of a variational solution to this problem using a penalization approach and several limiting processes.

Regularity criteria for solutions of the Navier-Stokes equations in terms of the derivatives of several fundamental quantities along the streamlines

Petr Kucera
Czech Technical University, Czech Republic

In this talk, we deal with the conditional regularity of the weak solutions of the Navier-Stokes equations on a bounded domain. In the first part of the talk, we study an optimal regularity criterion based on the control of the energy flow. In the next part, we study the regularity criteria which are based on the directional derivatives of several fundamental quantities along the streamlines, the pressure, the velocity field, and the Bernoulli pressure.

Local-in-time existence of strong solutions to a class of the compressible non-Newtonian Navier-Stokes equations

Vaclav Macha
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Martin Kalousek, Šàrka Nečasovà

We show the local-in-time existence of a strong solution to the generalized compressible Navier-Stokes equations for arbitrarily large initial data. The goal is reached by L^p-theory for linearized equations which are obtained with help of the Weis multiplier theorem and can be seen as a generalization of the work of Shibata and Enomoto (devoted to compressible fluids) to compressible non-Newtonian fluids.

Stochastic three-Dimensional Navier-Stokes equations and waves: Averaging, convergence, regularity and nonlinear dynamics

Alex Mahalov
Arizona State University, USA

We consider stochastic three-dimensional Navier-Stokes equations + Waves. Regularity results are established by bootstrapping from global regularity of the averaged stochastic resonant equations and convergence theorems. The averaged covariance operator couples stochastic and wave effects. The regularization time horizon is long. Infinite time regularity is proven for the deterministic case. Regularization is the consequence of precise mechanisms of relevant three-dimensional nonlinear interactions. We establish multi-scale stochastic averaging, convergence and regularity theorems in a general framework of three-dimensional nonlinear dynamics.

Energy equality for the compressible Primitive Equations with vacuum

Sarka Necasova
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Maria Angeles Rodriguez-Bellido, Tong Tang

We study the energy conservation for the weak solutions to the compressible Primitive Equations (CPE) system with degenerate viscosity. We give sufficient conditions on the regularity of weak solutions for the energy equality to hold, even for solutions that may include vacuum. In this paper, we give two theorems, the first one gives regularity in the classical Sobolev and Besov spaces. The second one state result in the anisotropic space. We get new regularity results in the second theorem because of the special structure of CPE system, which are in contrast to compressible Navier-Stokes equations.

A weak solvability of the stationary MHD system with inhomogeneous boundary conditions

Jiri Neustupa
Institute of Mathematics, Czech Academy of Sciences, Czech Republic

We discuss the question of existence of a steady weak solution to the incompressible MHD equations in a bounded domain, not necessarily simply connected, with inhomogeneous Dirichlet's boundary condition for velocity and various kinds of inhomogeneous boundary conditions for magnetic field.

Existence of a steady flow through a rotating radial turbine with an arbitrarily large inflow and an artificial boundary condition on the outflow

Tomas Neustupa
Czech Technical University, Czech Republic

We prove the existence of a steady strong solution to the Navier-Stokes-type problem in a 2D multiply connected domain, modelling a flow of an incompressible viscous fluid through a rotating radial turbine. We consider the inhomogeneous Dirichlet boundary condition on the inflow and an artificial boundary condition of the "do nothing" type on the outflow. The conditions admit an arbitrarily large flux through the turbine. This is, however, compensated by the requirement that the distance between the outflow and the profiles is "sufficiently small". The solution is steady in the rotating frame, i.e. in the frame, attached to the rotating turbine.

Homogenization of Navier-Stokes-Fourier system in domains with tiny holes

Milan Pokorny
Charles University, Czech Republic
Yong Lu, Emil Skrisovsky, Florian Oschmann

We consider the compressible Navier–Stokes–Fourier system in a domain with large number of holes. Under the assumption that the holes are sufficiently small, together with certain standard assumptions on the adiabatic exponent and the behaviour of the heat conductivity, we show that if passing simultaneously with the number of holes to infinity and their size to zero, in the limit we obtain again a solution to the compressible Navier–Stokes–Fourier system in the domain without holes. The result holds both for the steady and evolutionary problem. The talk is based on a paper with Yong Lu (Nanjing University), a paper with Emil Skříšovský (Charles University, Prague) and recent results obtained together with F. Oschmann (Mathematical Institute of the Czech Academy of Sciences).

On the magnetohydrodynamic equations under different boundary conditions for the velocity and the magnetic field

Nour Seloula
Laboratoire de mathematiques Nicolas Oresme Université de Caen, France
Julien Poirier

The boundary value problem for the steady-state magnetohydrodynamic (MHD) equations under various boundary conditions for the velocity and the magnetic field is considered in a bounded domain. First, we state the case in which the tangential components of the velocity and the magnetic field are specified on the boundary together with a pressure boundary condition. We give some results of existence, uniqueness and regularity of solutions in Hilbertian case and in Lp theory. Next, we consider the (MHD) equations with Navier type boundary conditions for the velocity and the magnetic field.

Optimal boundary control problem for steady Navier-Stokes equations with regularized directional do-nothing boundary condition

Ana Silvestre
Universidade de Lisboa, Portugal
Pedro Nogueira, Jorge Tiago

We consider the steady Navier-Stokes equations with mixed boundary conditions, where, instead of the classical do-nothing (CDN) outflow boundary condition, a regularized directional do-nothing (RDDN) condition is imposed. For computational purposes, a saddle point approach is chosen for the weak formulation of the problem. An auxiliary reference flow is used, which also works as a lifting of the inhomogeneous Dirichlet boundary conditions.
After proving the well-posedness of the Navier-Stokes equations with RDDN condition, we consider the minimization of a quadratic cost functional of velocity tracking type by means of a control localized on a portion of the inflow boundary. We prove the existence of a solution for this optimal control problem and derive a system of first-order optimality conditions. All these results are obtained under suitable assumptions on the size of the data and the controls, which, however, are less restrictive compared with the case of a CDN outflow condition.

A geometric criterion for the Navier-Stokes equations in terms of velocity direction

Zdenek Skalak
Czech Technical University, Czech Republic

We present a geometric regularity criterion for the Navier-Stokes equations in the whole three dimensional space. We prove as our main result that the regularity of Leray weak solutions follows from the Lipschitz continuity assumption on the direction of the velocity. Our result is reminiscent of the paper by Constantin and Fefferman who proved the regularity in terms of the direction of the vorticity.

L.C.Berselli, Some geometric constraints and the problem of global regularity for the Navier-Stokes equations, Nonlinearity Vol.22, 2009, 2561–2581.
P.Constantin, C.Fefferman: Direction of vorticity and the problem of global regularity for the Navier-Stokes equations, Indiana Univ. Math. J. Vol. 42, 1993, 775–789.

Time-periodic Stokes- and Navier-Stokes problems in a layer

Maria Specovius-Neugebauer
University of Kassel, Germany
Konstanin Pileckas, Sebastian Rauchhaus

In this talk we present results about existence and spatial asymptotics for solutions to the time-periodic Stokes and Navier-Stokes problem with Dirichlet conditions in a layer. This includes also the case of nonhomogeneous boundary conditions with nonzero flux which is of particular interest if one wants to model real life problems.
Moreover, in unbounded domains the knowledge about the general spatial asymptotic behaviour is usually helpful to construct optimal so called artificial boundary conditions. The latter are needed if one wants to approximate the solutions by numerical calculations.

Anisotropically spatial-temporal behavior of the Navier-Stokes flow past a rigid body

Tomoki Takahashi
Tokyo Institute of Technology, Japan

We consider the spatial-temporal behavior of the Navier-Stokes flow past a three dimensional rigid body and deduce the temporal decay rate with the spatial weight caused by translation.
The key tool is the L^p-L^q estimate of the Oseen semigroup in exterior domains and we develop the weighted L^q theory of the Oseen semigroup.
New results on the Stokes semigroup in isotropic L^q spaces are also discussed.

On well-posedness of quantum fluid systems in the class of dissipative solutions

Tong Tang
Yangzhou University, Peoples Republic of China
Danica Basarič

The main objects of the present work are the quantum Navier–Stokes and quantum Euler systems; for the first one, in particular, we will consider constant viscosity coefficients. We deal with the concept of dissipative solutions, for which we will first prove the weak-strong uniqueness principle and afterwards, we will show the global existence for any finite energy initial data. Finally, we will prove that both systems admit a semiflow selection in the class of dissipative solutions.

On the Helmholtz decomposition in general domains

Werner Varnhorn
Kassel University, Germany
Reinhard Farwig, Christian Simader, Hermann Sohr

In the theory of the incompressible Navier-Stokes equatuons, the Helmholtz decomposition (HD) in $L^q(\Omega)$ plays a fundamental role.
We show for general domains $\Omega \subseteq \mathbb{R}^n$, $n \geq 2$, $1 < q < \infty$ that (HD) is necessary and sufficient for the validity of a certain gradient estimate (GE), and also for the validity of a certain estimate (DE) for divergence free functions. Moreover, we study the optimal constants in the estimates (GE) and (DE) and prove that these constants coincide.

From compressible to incompressible systems

Aneta Wroblewska-Kaminska
Institute of Mathematics, Polish Academy of Sciences, Poland

We will show asymptotic analysis for hydrodynamic system, as Navier-Stokes-Fourier system, as a useful tool in in the situation when certain parameters in the system - called characteristic numbers - vanish or become infinite. The choice of proper scaling, namely proper system of reference units, the parameters determining the behaviour of the system under consideration allow to eliminate unwanted or unimportant for particular phenomena modes of motion. We will concentrate on rigorous mathematical analysis of low Mach number limits with so called ill-prepared data and I will present some results which concerns passage from compressible to incompressible models of fluid flow emphasising difficulties characteristic for particular problems. In particular we will discuss Navier Stokes-Fourier system on varying domains, a multi-scale problem for viscous heat-conducting fluids in fast rotation and the incompressible limit of compressible finitely extensible nonlinear bead-spring chain models for dilute polymeric fluids.

Liouville type theorems for the stationary MHD equations

Minsuk Yang
Yonsei University, Korea
Youseung Cho, Jiri Neustupa

We establish the Liouville-type theorem of the stationary MHD equations under some conditions.

Strongly compact strong trajectory attractors for the nonautonomous 3D Navier-Stokes equations

Songsong Lu
Sun Yat-Sen University, Peoples Republic of China

We show that for any fixed accuracy and time length T, a finite number of T-time length pieces of the complete bounded solutions on the global attractor are capable of uniformly approximating all Leray-Hopf weak solutions within the accuracy in the natural strong metric after sufficiently large time when the 3D Navier-Stokes equations is with a fixed normal force and every complete bounded solution is strongly continuous. Moreover, we obtain the strong equicontinuity of all the complete bounded solutions on the global attractor. These results follow by proving the existence of a strongly compact strong trajectory attractor for such a system. The notion of a (weak) trajectory attractor was previously constructed for a family of auxiliary systems including the originally considered one. We developed a framework called evolutionary system, with which a (weak) trajectory attractor can be actually defined for the original system nearly ten years ago. Very recently, the theory of trajectory attractors is further developed in the natural strong metric for our purpose. The framework is general and can also be applied to other nonautonomous dissipative partial differential equations for which the uniqueness of solutions might not hold.

Special Session 28: Qualitative Theory of Nonlinear Elliptic and Parabolic Equations

Raul Manasevich, University of Chile, Chile
Satoshi Tanaka, Tohoku University, Japan

In this session, we will be dedicated to properties of solutions of nonlinear elliptic and parabolic equations and systems. We will study qualitative properties, such as existence, uniqueness, stability, bifurcation and symmetry of solutions and related methods. Results showing connection with real world applications are welcome also.

Elliptic Hamilton-Jacobi systems through Lane-Emden Hardy-Hénon equations

Marta Garcia-Huidobro
Pontificia Universidad Catolica de Chile, Chile
Marie Francoise Bidaut-Veron

We are concerned with the study of the solutions of any sign of the system

$$\begin{cases} -\Delta u_1 = |\nabla u_2|^p, \\ -\Delta u_2 = |\nabla u_1|^q, \end{cases}$$

in a domain of $\mathbb{R}^N$, $N \geqq 3$ and $p, q > 0$, $pq > 1$.. We show their relation with Lane-Emden Hardy-Hénon equations

$$-\Delta_{\mathbf{p}}^{\mathbf{N}} w = \varepsilon r^{\sigma} w^{\mathbf{q}}, \qquad \varepsilon = \pm 1,$$

where $u \mapsto \Delta_{\mathbf{p}}^{\mathbf{N}} u$ ($\mathbf{p} > 1$) is the $\mathbf{p}$-Laplacian in dimension $\mathbf{N}$, $\mathbf{q} > \mathbf{p} - 1$ and $\sigma \in \mathbb{R}$. We make a complete description of the radial solutions of the system and of the Hardy-Hénon equations and give nonradial a priori estimates and Liouville type results.

A power type approximation of Moser-Trudinger inequality

Masato Hashizume
Hiroshima University, Japan
Norisuke Ioku

The Moser–Trudinger inequality is considered as a limiting case of the Sobolev inequality in the framework of Orlicz spaces. However, the Moser–Trudinger inequality is not obtained via a direct limiting procedure for the Sobolev inequality. In this talk, we consider a Sobolev type inequality and we show that the Carleson–Chang limit on the Moser–Trudinger inequality is derived as a limit of the concentration level of the Sobolev type inequality. In addition to the property, we study the variational problem on the inequality.

Multiplicity of ground state solutions via magnitude changes

Pilar Herreros
P. Universidad Catolica de Chile, Chile
Carmen Cortázar, Marta García-Huidobro

We will study the radially symmetric solutions to the problem

$$\Delta u + f(u) = 0, \quad x \in \mathcal{R}^N, N > 2, \lim_{|x| \to \infty} u(x) = 0.$$

We will see that we can generate new solutions to this problem by introducing abrupt magnitude changes in the function f. Using this idea, we can construct functions f, defined by parts, such that the problem has any given number of solutions.

Nodal solutions for the Moore-Nehari differential equation

Ryuji Kajikiya
Osaka Electro-Communication University, Japan

We study the Moore-Nehari differential equation.
For a nonnegative integer n, we call a solution u an n-nodal solution if it has exactly n zeros in $(-1, 1)$.
We call a solution u symmetric if it is even or odd.
We shall show that for each nonnegative integer n, the equation has a unique n-nodal symmetric solution.
We call a solution u an (m, n) solution if it has exactly m zeros in $(-1, 0)$ and exactly n zeros in $(0, 1)$.
We shall prove that for each nonnegative integers m, n, the equation has an (m, n)-solution and an (m, m)-asymmetric solution.

On the monotonicity of the period for some equations with a p-Laplace operator

Raul Manasevich
University of Chile, Chile

In this talk we provide some new results for the monotonicity of the period of some nonlinear differential equations which are connected with a minimization problem on the circle.

The Euler-Lagrange equation for this problem turns out to be a nonlocal nonlinear differential equation containing the p-Laplace operator and we are looking for positive solutions on the circle. Under a suitable transformation this equation is transformed into one without local terms but with varying period

Effect of decay rates of initial data on the sign of solutions to Cauchy problems of some higher order parabolic equations

Nobuhito Miyake
Graduate School of Mathematical Sciences, the University of Tokyo, Japan

In this talk, we consider Cauchy problems of linear and semilinear polyharmonic heat equations. It is known that Cauchy problems of higher order parabolic equations have no positivity preserving property in general. On the other hand, it is expected that solutions to Cauchy problems of polyharmonic heat equations are eventually globally positive if initial data decay slowly enough. Our aim of this talk is to show the existence of the threshold for the decay rate of initial data which determines whether the corresponding solution to the Cauchy problem of the linear polyharmonic heat equation is eventually globally positive or not. As the application of this result, we construct eventually globally positive solutions to a Cauchy problem of a semilinear polyharmonic heat equation.

Exact Morse index of radial solutions for semilinear elliptic equations with critical exponent on annuli

Yasuhito Miyamoto
University of Tokyo, Japan

We are concerned with radial solutions of a Hénon equation with a critical exponent on annuli. For each n this problem has exactly two radial solutions with n nodal domains. We obtain an exact Morse index of these solutions if the radius of the inner hole is small. Various estimates about Morse index are also obtained for an arbitrary annulus. In a certain assumption we can obtain an exact Morse index of positive (and negative) solutions on all annuli. The proof is based on a combination of ODE techniques and variational methods.

Maximizing the total population and the bang-bang property in reaction-diffusion logistic models

Kentaro Nagahara
Tokyo Institute of Technology, Japan
Yuan Lou, Eiji Yanagida

In this talk, we consider the problem of maximizing the total population in a reaction-diffusion logistic model. This model has a logistic-type reaction term and a diffusion term, and the solution represents the population density distribution. This equation was proposed by J.G. Skellam in 1951 as a model for the population dynamics of organisms, and is also known as the Fisher-KPP equation. The logistic term includes a coefficient with spatial heterogeneity called the intrinsic growth rate. We show that in situations where species do not go extinct, i.e., where there is a nontrivial positive stationary solution, the total population is maximized when the intrinsic growth rate with spacial heterogeneity has a property called bang-bang type. In other words, according to this model, organisms increase their populations most when some locations are fertile and other locations are harsh. A discretized model in the relevant spatial direction will also be presented. This is a joint work with Prof. Yuan Lou and Eiji Yanagida.

Concentration phenomena on radial solutions to semilinear elliptic equations with the Trudinger-Moser growth

Daisuke Naimen
Muroran Institute of Technology, Japan

We study the asymptotic behaviour of radial solutions to semilinear elliptic equations with Trudinger-Moser critical nonlinearities. Via the blowup analysis, we classify the concentration compactness behaviour of them. Three different behaviour; concentration with zero weak limit, concentration with nontrivial weak limit, and strong convergence, are observed. We also deduce the limit equation and energy of them.

Singular solutions for semilinear elliptic equations with general supercritical growth

Yuki Naito
Hiroshima University, Japan
Yasuhito Miyamoto

We consider radial singular solutions of the semilinear elliptic equation $\Delta u + f(u) = 0$ in $\Omega \setminus \{0\}$, where Ω is a unit ball in $\mathbb{R}^N$ with $N \geq 3$ and $f \in C^2[0, \infty)$. We will show some qualitative properties of the singular solutions with general supercritical nonlinearities.

Our method can treat a wide class of nonlinearities $f(u)$ in a unified way, e.g., $u^p \log u$, $\exp(u^p)$ and $\exp(\cdots\exp(u)\cdots)$ as well as u^p and e^u.
We will verify an exact asymptotic expansion of the singular solution as well as its uniqueness in the space of radial functions.

Even ground state for nonlinear Schrödinger systems with repulsive interaction

Yohei Sato
Saitama University, Japan

It is known that the nonlinear Schrödinger systems with repulsive interaction has no ground states. Here ground state means a positive solution that attains a minimizing problem on the Nehari set with two constrains. On the other hand, when the minimizing problem is restricted to even functions, one calls a positive minimizer as an even ground state. We study the existence of even ground states of nonlinear Schrödinger systems with repulsive interaction. The keys of the proof are estimates of minimizing level and a classificaton of Palais-Smale sequences of even functions. In particular, the case $N = 1$ requires more detailed estimates than $N = 2, 3$.

Boundedness of the solutions of a kind of nonlinear parabolic systems

Lyoubomira Softova
University of Salerno, Italy
Emilia Anna Alfano, Luisa Fattorusso

We deal with nonlinear systems of parabolic type satisfying component-wise structural conditions. The nonlinear terms are Carathéodory maps having controlled growth with respect to the solution and the gradient and the data are in anisotropic Lebesgue spaces. Under these assumptions we obtain essential boundedness of the weak solutions.

On the structure of solutions for two mean field eqautions

Ryo Takahashi
Nara University of Education, Japan

In this talk, we consider two mean field equations describing hydrodynamic turbulence in equilibrium. Several differences of the structure of solutions between them are shown.

Uniqueness and multiplicity of positive solutions to the scalar-field equation on large annuli in the three-dimensional unit sphere

Satoshi Tanaka
Tohoku University, Japan

In this talk, we consider the Dirichlet problem: $\Delta_{\mathbb{S}^3} u - u + u^p = 0$ in Ω_ε; $u = 0$ on $\partial\Omega_\varepsilon$, where $\Delta_{\mathbb{S}^3}$ is the Laplace-Beltrami operator on the three-dimensional unit sphere $\mathbb{S}^3$, $p > 1$, $0 < \varepsilon < \pi/2$, and Ω_ε is an annular domain in $\mathbb{S}^3$ whose great circle distance (geodesic distance) from the North Pole is greater than ε and less than $\pi - \varepsilon$. We obtain the existence, uniqueness, and multiplicity results of the positive solutions depend only on the latitude. This is joint work with Naoki Shioji (Yokohama National University) and Kohtaro Watanabe (National Defense Academy).

On the antimaximum principle for the p-Laplacian and its sublinear perturbations

Mieko Tanaka
Tokyo University of Science, Japan
Vladimir Bobkov

I'll talk about qualitative properties of weak solutions of the Dirichlet problem for the equation $-\Delta_p u = \lambda m(x)|u|^{p-2}u + \eta + a(x)|u|^{q-2}u + f(x)$ in a bounded domain.

Complete classification of planar p-elasticae

Kensuke Yoshizawa
Institute of Mathematics for Industry, Kyushu University, Japan
Tatsuya Miura

Among planar curves, the p-bending energy is defined by the L^p-norm of the signed curvature, and critical points of the p-bending energy under the fixed length constraint are called p-elasticae. The aim of this talk is to give a complete classification of planar p-elasticae. A key point is to introduce a new type of generalization of the Jacobi elliptic functions, which also leads us to optimal regularity of planar p-elasticae.
This talk is based on a joint work with Prof. Tatsuya Miura (Tokyo Institute of Technology), and its arXiv identifier is 2203.08535.

Special Session 29: Reactions Diffusion Equations with Applications to Spatial Ecology and Infectious Disease

Rachidi Salako, University of Nevada Las Vegas, USA
King Yeung Lam, The Ohio State University, USA
Yuan Lou, Shanghai Jiao Tong University, Peoples Republic of China

This special session is concerned with recent advances on reaction diffusion equations and their applications to spatial ecology and infectious disease. The session aims to bring together experts and young researchers in this area to share their recent works, exchange ideas and foster future interdisciplinary collaborations.

On the evolution of slow dispersal in multi-species communities

Robert Stephen Cantrell
University of Miami, USA
King-Yeung (Adrian) Lam

For any N ≥ 2, we show that there are choices of diffusion rates d_i, $1 \leq i \leq N$ such that for N competing species which are ecologically identical and have distinct diffusion rates, the slowest disperser is able to competitively exclude the remainder of the species. In fact, the choices of such diffusion rates are open in the Hausdorff topology. Our result provides some evidence in the affirmative direction regarding the conjecture by Dockery et al. in 1998. The main tools include Morse decomposition of the semi-flow and the theory of normalized Floquet principal bundle for linear parabolic equations. A critical step in the proof is to establish the smooth dependence of the Floquet bundle on diffusion rate and other coefficients, which may be of independent interest.

Optimal dispersal in integrodifference models

Chris Cosner
University of Miami, USA
Robert Stephen Cantrell, Ying Zhou

This talk will describe optimal dispersal, and how it could evolve in some integrodifference models for migratory populations. We formulate integrodifference models that describe seasonal migrations where the environment is heterogeneous in space and varies between seasons, and where the integral kernels describing movement can depend on conditions at the arrival and departure points. We consider the kernels as movement strategies and use pairwise invasion analysis to find which kernels correspond to evolutionarily stable strategies (ESS). As in many other modeling contexts, those are the ones that can produce an ideal free distribution, which can be characterized in terms of line sum symmetry. The analysis requires our operators to be strongly positive. Thus, in cases where the environment is partially occupied in both seasons, we need to use order unit norms to define spaces whose positive cones have a nonempty interior. Perhaps this approach may be useful in other applications of integrodifference models where dispersal depends on environmental quality.
We also describe a way that a population can learn how to migrate optimally from the experience of past migrations.

Harvesting-mediated emigration can affect community structure in a competitive system

Jerome Goddard II
Auburn University Montgomery, USA
H. Abusammour, J. T. Cronin, J. Garrett, S. Humphries, & R. Shivaji

Trait-mediated behavioral responses (an indirect effect) to other species can affect population dynamics significantly. One example of such a response is modification of emigration probability, which has the potential to change species interactions and community structure. Habitat loss and fragmentation due to anthropogenic activities creates landscape-level spatial heterogeneity where remnant patches are often surrounded by a hostile matrix. Matrix composition or hostility is an important component of a landscape and can have profound effects on species movement and boundary behavior, persistence of a single species, and coexistence of interacting species. We model a system of two competitors dwelling in the same remnant patch surrounded by a hostile matrix. In this case, one competitor is being harvested (direct effect) and the other is not being harvested but is disrupted by the harvesting process, causing increased emigration (indirect effect). Thus, the second competitor exhibits a positive relationship between harvesting effort and emigration, i.e., harvesting-mediated emigration. In this talk, we will introduce our modeling framework and share some recent results demonstrating that community structure can be altered because of harvesting-mediated emigration and habitat fragmentation.

The principal Floquet bundle and a conjecture of Dockery et al.

King-Yeung Lam
The Ohio State University, USA
Robert Stephen Cantrell, Yuan Lou

In 1998, Dockery et al. showed that for two competing species which are identical except for their diffusion rate, the slower diffuser can exclude the faster counterpart regardless of initial data. It is then conjectured that the same result holds for N competing species, for any number N greater than or equal to 3. In this talk, we will discuss some recent progress towards a generalized reduction principle based on new estimates of the underlying principal Floquet bundle. Our discussion includes the case when the coefficients are autonomous, time-periodic, or more general non-autonomous coefficients under spatially heterogeneous conditions. This is joint work with Robert S. Cantrell and Yuan Lou.

Persistence of a two-stage structured population model with nonlocal dispersal

Maria Amarakristi Onyido
Northern Illinois University, USA
Rachidi B. Salako, Markjoe O. Uba, Cyril I. Udeani

This talk will discuss the effects of dispersal rates on the persistence and extinction of a two-stage structured population model with nonlocal dispersal. In particular, we will show that the model dynamics completely depend on the sign of the principal spectrum point of its linearization at the trivial solution. Furthermore, the asymptotic limits of the principal spectrum point and that of positive steady states of the model will also be addressed.

Some recent results in competitive systems

Rana Parshad
Iowa State University, USA

In this talk we discuss recent results when the classical Lotka-Volterra competition models are varied. We provide a number of ecological applications for these new models. We focus on finite time extinction effects as a basis for harvesting strategies. We also discuss non-consumptive or “fear” effects. Applications of these methods to the biological control of invasive pests is also discussed.

Human mobility and disease prevalence

Rachidi Salako
University of Nevada Las Vegas, USA
Yuan Lou, Rachidi B Salako, Pengfei Song

Globalization has made the world more connected and interdependent due to the increase in the rates of human migration. However, this also contributes significantly to the spread of deadly infectious diseases. Mathematical modeling of epidemics is vital to understand the spreading dynamics of an infectious disease and develop the best strategies to curb its spread. In this talk, we will examine the effect of human mobility on disease prevalence by studying the dependence of the total infected population at endemic equilibria with respect to population diffusion rates of a diffusive epidemic model. In particular, for small diffusion rates, our results indicate that the total infected population size is strictly decreasing with respect to the ratio of the diffusion rate of the infected population over that of the susceptible population.

Modelling phytoplankton-virus interactions: Phytoplankton blooms and lytic virus transmission

Junping Shi
College of William & Mary, USA
Jimin Zhang, Yawen Yan

A dynamic reaction-diffusion model of four variables is proposed to describe the spread of lytic viruses among phytoplankton in a poorly mixed aquatic environment. The basic ecological reproductive index for phytoplankton invasion and the basic reproduction number for virus transmission are derived to characterize the phytoplankton growth and virus transmission dynamics. The theoretical and numerical results from the model show that the spread of lytic viruses effectively controls phytoplankton blooms. This validates the observations and experimental results of Emiliana huxleyilytic virus interactions. The studies also indicate that the lytic virus transmission cannot occur in a low-light or oligotrophic aquatic environment.

Reaction-diffusion models for the spatial spread of cholera

Jin Wang
University of Tennessee at Chattanooga, USA

We study the spatial dynamics of cholera, a severe waterborne infection caused by the bacterium Vibrio cholerae, using reaction-diffusion models. Our focus is how the host movement and pathogen dispersal impact the spatial spread of the disease. We conduct an analysis on the traveling wave solutions of these reaction-diffusion systems. We also present a numerical approach to compute and analyze the basic reproduction numbers associated with such models.

A reaction-advection-diffusion model of cholera epidemics with seasonality and human behavior change

Xueying Wang
Washington State University, USA
Ruiwen Huang, Xiao-Qiang Zhao

Cholera is a water- and food-borne infectious disease caused by V. cholerae. To investigate multiple effects of human behavior change, seasonality and spatial heterogeneity on cholera spread, we propose a reaction-advection-diffusion model that incorporates human hosts and aquatic reservoir of V. cholerae. We first derive the basic reproduction number R0 for this system and then establish a threshold type result on its global dynamics in terms of R0. Further, we show that the bacterial loss at the downstream end of the river due to water flux can reduce the disease risk, and describe the asymptotic behavior of R0 for small and large diffusion in a special case (where the diffusion rates of infected human and the pathogen are constant). We also study the transmission dynamics at the early stage of cholera outbreak numerically and find that human behavior change may lower the infection level and delay the disease peak. Moreover, the relative rate of bacterial loss, together with convection rate, plays an important role in identifying the severely infected areas. Meanwhile spatial heterogeneity may dilute or amplify cholera infection, which in turn would increase the complexity of disease spread.

Analysis of a reaction-diffusion susceptible-infected-susceptible epidemic patch model incorporating movement inside and among patches

Yixiang Wu
Middle Tennessee State University, USA
Shanshan Chen

We propose and analyze a reaction-diffusion susceptible-infected-susceptible (SIS) epidemic patch model. The individuals are assumed to reside in different patches, where they are able to move inside and among the patches. The movement of individuals inside the patches is descried by diffusion terms, and the movement pattern among patches is modeled by an essentially nonnegative matrix. We define a basic reproduction number $\mathcal{R}_0$ for the model and show that it is a threshold value for disease extinction versus persistence. The monotone dependence of $\mathcal{R}_0$ on the movement rates of infected individuals is proved when the dispersal pattern is symmetric or non-symmetric. Numerical simulations are performed to illustrate the impact of the movement of individuals inside and among patches on the transmission of the disease.

Stationary solutions of a chemotaxis system with singular sensitivity and logistic source

Shuwen Xue
Northern Illinois University, USA
Wenxian Shen, Halil Ibrahim Kurt

Chemotaxis, the directed movement of cells or living organisms in response to the concentration gradient of chemical substances, plays a significant role in a large range of biological phenomena such as tumor growth, wound healing, and embryo development. One of the most important and interesting phenomena of chemotaxis is cellular aggregation, in which initially evenly distributed cells merge with each other and eventually aggregate into one or several groups. Mathematically, this phenomenon can be modeled by showing time-dependent solutions converge to bounded but spikey stationary solutions. In this talk, first, we will talk about the stability and instability of constant solution. Then, we will discuss the local bifurcation and stability of bifurcation solutions from the constant solution. Next, we will investigate global bifurcation and spiky bifurcation solutions. Finally, we will show some numerical simulations to visualize theoretical results and demonstrate some interesting cellular aggregation phenomena.

Special Session 30: Optimal Control of Finite and Infinite Dimensional Dynamic Systems and their Applications

Nasir U. Ahmed, University of Ottawa, Canada
Stanislaw Migorski, Jagellonian University, Poland
Saroj K. Biswas, Temple University, USA

The subject of optimal control is as extensive and as diverse as dynamic systems governed by ordinary differential equations, partial differential equations, abstract differential equations, stochastic differential equations, and their functional counterparts defined on finite and infinite dimensional Banach and topological spaces. Over the last six decades, there has been phenomenal development of control theory on more and more complex systems, and it seems it will continue with increasing interest. This field is mathematically very rich and employs almost every aspect of topology and functional analysis. Since the publication of Pontryagin Maximum Principle in the late fifties, the subject on optimal control has expanded in every possible direction of concern to humanity. Optimal control theory has been already applied extensively in engineering and physical sciences, social sciences, and global computer communication networks etc. In recent years, there has been increasing thrusts in other directions, such as biological sciences and medicine which are likely to make breakthroughs in the future. This session welcomes all papers concerned with theoretical development of optimal controls, their computational complexities as well as their applications in all areas of physical and social sciences.

Atmospheric greenhouse concentration and its optimal removal strategy

Nasir Ahmed
University of Ottawa, Canada
Saroj Biswas

In this paper we propose a space-time dynamic model for describing the temporal evolution of greenhouse gas concentration in the atmosphere. We use this dynamic system to develop an optimal control strategy for reduction of greenhouse gas concentration.

Differential inclusions with m-dissipative multioperators

Irene Benedetti
University of Perugia, Italy

The talk deals with multivalued differential equations in abstract spaces. Nonlocal initial conditions are assumed. The model includes an m-dissipative multioperator which generates a nonlinear semigroup that is equicontinuous, but not necessarily compact. The existence of integral solutions is discussed, with a topological index argument and it is based on the regularity of the nonlinear term with respect to the Hausdorff measure of noncompactness and on a transversality condition. The motivation for these studies is that nonlocal Cauchy problems may have better effects to describe real life phenomena than the classical initial value problem. For example, it is used to represent mathematical models for evolution of various phenomena, such as nonlocal neural networks, nonlocal pharmacokinetics, nonlocal pollution and nonlocal combustion. Moreover, the presence of a multivalued nonlinearity allows to consider optimal control problems as applications. The discussion is completed by an example of how to use these results in the study of the existence of solutions for partial differential inclusions of parabolic type in a bounded domain in $\mathbb{R}^n$, with a nonlinear term described by a subdifferential of a suitable map and with nonlocal integral initial conditions.

Deforming local minima of an optimal control problem

Andy Borum
Vassar College, USA

Given two local minima of an optimal control problem, can one be deformed into the other by continuously moving the boundary conditions, all the while remaining a local minimum? In other words, is the set of all local minima over all possible boundary conditions path-connected? In this talk, I will describe a sufficient condition for this set to be path-connected that relies on two types of symmetries—invariance under the action of a Lie group and invariance under a rescaling of time. I will then discuss some examples where this set is path-connected, some examples where it is not, and an application in robotics.

Decay for a Klein-Gordon-Schrödinger system with locally distributed damping

Michael Filippakis
University of Piraeus, Greece
M. Poulou

The aim of this paper is to study the following KGS system defined in Ω which is a bounded domain in $\mathbb{R}^2$

$$i\psi_t + \kappa\psi_{xx} + i\alpha b(x)\psi = \phi\psi\omega),$$
$$\phi_{tt} - \phi_{xx} + \phi + \lambda(x)\phi_t = -\mathrm{Re}\psi_x,$$

with smooth boundary Γ and ω is a neighborhood of $\partial\Omega$ satisfying the geometrical control condition. The aim of the paper is to prove the existence, uniqueness and uniform decay for the solutions. The publication of this paper has been partly supported by the University of Piraeus Research Center.

Necessary optimality conditions for a fractional integro-differential optimal control problem

Rafal Kamocki
University of Lodz, Poland

In this talk, we shall study an optimal control problem of a Lagrange type in which a control system is described by a nonlinear and singular integro-differential equation of Volterra type with a Caputo derivative. The necessary optimality conditions for the considered problem are derived. The posed problem is investigated using an extremum principle for smooth problems.

Inverse problems of identifying parameters in PDEs with random data

Akhtar Khan
Rochester Institute of Technology, USA
M. Sama, H. J. Starkloff, C. Tammer

This talk will focus on the inverse problem of identifying a variable parameter in stochastic PDEs. The inverse problems will be studied in a stochastic optimization framework. Theoretical, as well as numerical results, will be presented.

Optimal control problem governed by a fractional Goursat-Darboux problem

Marek Majewski
University of Lodz, Poland

We consider an optimal control problem governed by a fractional version of the Goursat-Darboux problem with integral cost functional. A sufficient condition for the existence of optimal solution based on the convexity of the so-called generalized velocity set is proved.

Set-valued Young integral, its properties and applications

Mariusz Michta
University of Zielona Gora, Poland

Since the pioneering work of Robert Aumann, the notion of set-valued integrals for multivalued functions has attracted the interest of many authors both from theoretical and practical points of view. In particular, the theory has been developed extensively, among others, with applications to optimal control theory, mathematical economics, theory of differential inclusions and set-valued differential equations. Later, the notion of the integral for set-valued functions has been extended to a stochastic case where set-valued Ito and Stratonovich integrals have been studied and applied to stochastic differential inclusions and set-valued stochastic differential equations. On the other hand, in a single-valued case, one can consider stochastic integration with respect to non-semimartingale integrators such as the Mandelbrot fractional Brownian motion which has Hölder continuous sample paths. Such integrals can be understood in the sense of Young. This kind of integrals have been developed and widely used in the theory of differential equations by many authors. Thus it seems reasonable to investigate also differential inclusions driven by a fractional Brownian motion and Young type integrals also.
The talk concerns both the properties of set-valued Young integrals and topological properties of solutions of Young differential inclusions. In particular, we present that the set of all solutions is compact in the space of continuous functions. Also its dependence on initial conditions as well as properties of reachable sets of solutions with respect to time will be established.

Well-posedness of parabolic variational-hemivariational inequalites with unilateral constraints

Stanislaw Migorski
Jagiellonian University, Poland

In this paper we study a novel class of parabolic evolution variational-hemivariational inequalites with a unilateral constraint. A theorem on the well-posedness for a weak solution is established. We illustrate the results by a quasistatic nonsmooth frictional viscoelastic contact problem with a unilateral constraint for which we derive results on the unique weak solvability and the stability of the solution.

Controllability in dynamics of diffusion processes without compactness

Valentina Taddei
University of Modena and Reggio Emilia, Italy

The talk deals with the controllability of semilinear evolution equations in Banach spaces. The final configuration is always achieved with a control of minimum norm. The results make use of topological techniques. which do not require the compactness of the evolution operator generated by the linear part nor any compactness condition on the multivalued nonlinearity The discussion is completed with some applications to dynamics of diffusion processes.

Special Session 32: Recent Developments in Mathematical Theories of Complex Fluids

Xianpeng Hu, City University of Hong Kong, Hong Kong
Yong Yu, Chinese University of Hong Kong, Hong Kong
Chenyun Luo, Chinese University of Hong Kong, Hong Kong

The aim of this special session is to bring together experts in the area of complex fluids, such as liquid crystals and viscoelasticity, to present their recent research results in theoretical analysis and applications in engineering. In this session, people are expected to exchange new ideas, to discuss challenging issues, to explore new directions and topics, and to foster new collaborations and connections.

The maximal classical development for shock forming solutions of the 3D compressible Euler equations

Leonardo Abbrescia
Vanderbilt University, USA
Jared Speck

It is well known that solutions to the inviscid Burgers' equation form shock singularities in finite time, even when launched from smooth data. A far less documented fact, at least in the popular works on 1D hyperbolic conservation laws, is that shock singularities are intimately tied to a lack-of-uniqueness for the classical Burgers' equation.
We prove that, locally, solutions to the Compressible Euler equations do not suffer from the same lack-of-uniqueness, even though they can be written as a coupled system of Burgers' in isentropic plane-symmetry. Roughly, the saving grace is that Euler flow involves two speeds of propagation, and one of them prevents the mechanism driving the lack-of-uniqueness. Analytically, this is done by explicitly constructing a portion of the boundary of classical hyperbolic development for shock forming data. This boundary is a connected co-dimension 1 submanifold of Cartesian space, and we will discuss the delicate geo-analytic degeneracies and difficulties involved. This is joint work with Jared Speck.

Partial regularity for the stochastic Ericksen–Leslie equations

Hengrong Du
Vanderbilt University, USA
Chuntian Wang

In this talk we study the singularities in the simplified stochastic Ericksen–Leslie (SEL) system with a Ginzburg–Landau type approximation in three dimensions. The SEL system is a mathematical model that describes the hydrodynamics of nematic liquid crystals under the influence of random forces. We obtain the existence of a suitable weak martingale solution. Furthermore, we show that for the statistically stationary solutions, almost surely at any time the set of singular points is empty. This is based on joint work with Chuntian Wang.

Incompressible limit of three dimensional compressible viscoelastic systems with vanishing shear viscosity

Xianpeng Hu
City University of Hong Kong, Peoples Republic of China

We will discuss the incompressible limit of compressible viscoelastic systems when the shear viscosity converges to zero. The incompressible limit is characterised by the large value of the volume viscosity. In the limit, the dispersive effect of pressure waves disappears and the global convergence to the limit system around an equilibrium is justified with the help of vector fields.

Suppression of chemotactic blow up by active scalar

Zhongtian Hu
Duke University, USA
Alexander Kiselev, Yao Yao

Chemotactic blow up in the context of the Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of fluid advection can arrest singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties and its amplitude is sufficiently strong - an effect that is conjectured to hold for more general classes of nonlinear PDE. In this talk, I will discuss some results on suppression of singularity formation in systems where Keller-Segel equation is coupled with fluid flow via buoyancy force. The talk is based on a joint work with Alexander Kiselev and Yao Yao.

Poiseuille flow of full Ericksen-Leslie system modeling nematic liquid crystal flows

Tao Huang
Wayne State University, USA
Geng Chen, Weishi Liu, Xiang Xu

We study the Cauchy problem of the Poiseuille flow of full Ericksen-Leslie model for nematic liquid crystals. The model is a coupled system of a parabolic equation for the velocity of fluid and a (quasilinear) wave equation for the director field of liquid crystal molecules. For a particular choice of several physical parameter values, we construct solutions with smooth initial data and finite energy that produce, singularities-blowups of gradients in finite time in both 1D and 2D. We are also able to establish the existence of global weak solutions that are Hölder continuous in 1D.

Energetic variational approaches in active materials and reactive fluids

Chun Liu
Illinois Institute of Technology, USA

We will discuss the energetic variational approaches in active materials and reactive fluids. Several applications will be discussed, as well as the challenges in analysis and simulations.

Capillary gravity water waves linearized at monotone shear flows: Eigenvalues and inviscid damping

Xiao Liu
University of Illinois Urbana-Champaign, USA
Chongchun Zeng

We consider the 2-dim capillary gravity water wave problem – the free boundary problem of the Euler equation with gravity and surface tension – of finite depth $x_2 \in (-h, 0)$ linearized at a uniformly monotonic shear flow $U(x_2)$. Our main results consist of two aspects, eigenvalue distribution and inviscid damping. We first prove that in contrast to finite channel flow and gravity wave, the linearized capillary gravity wave has two branches of eigenvalues for high wave numbers. Under certain conditions, we provide a complete picture of the eigenvalue distribution. Assuming there are no singular modes, we obtain the linear inviscid damping. We identify the leading asymptotic terms of velocity and obtain the stronger decay for the remainders.

A generalized Beale-Kato-Majda breakdown criterion for the 3D free-boundary problem in Euler equations with surface tension

Chenyun Luo
Chinese University of Hong Kong, Hong Kong
Kai Zhou

It is shown in Ferrari that if $[0, T^*)$ is the maximal time interval of existence of a smooth solution of the incompressible Euler equations in a bounded, simply-connected domain in $\mathbb{R}^3$, then $\int_0^{T^*} \|\omega(t, \cdot)\|_{L^\infty} \, dt = +\infty$, where ω is the vorticity of the flow. Ferrari's result generalizes the classical Beale-Kato-Majda's breakdown criterion in the case of a bounded fluid domain. In this manuscript, we show a breakdown criterion for a smooth solution of the Euler equations describing the motion of an incompressible fluid in a bounded domain in $\mathbb{R}^3$ with a free surface boundary. The fluid is under the influence of surface tension. In addition, we show that our breakdown criterion reduces to the one proved by Ferrari when the free surface boundary is fixed. Specifically, the additional control norms on the moving boundary will either become trivial or stop showing up if the kinematic boundary condition on the moving boundary reduces to the slip boundary condition.

On the interaction between a harmonic oscillator and a viscous fluid

Giusy Mazzone
Queen's University, Canada
Mahdi Mohebbi

In this talk, we will consider the time-periodic motion of a harmonic oscillator in a viscous incompressible fluid occupying an infinite pipe. The motion of the fluid is driven by a prescribed, time-periodic flow rate. We will show that the phenomenon of resonance does not occur in the class of weak solutions to the governing equations if the flow rate is "sufficiently small". In addition, at a large distance from the oscillator, the fluid velocity converges to the time-periodic generalization of the Poiseuille flow in an infinite pipe.

On a thermodynamically consistent model for magnetoviscoelastic fluids in 3D

Yuanzhen Shao
University of Alabama, USA
Hengrong Du, Gieri Simonett

In this talk, we consider a system of equations that model a non-isothermal magnetoviscoelastic fluid, which is thermodynamically consistent. The system is analyzed by means of the Lp-maximal regularity theory. First, we will discuss the local existence and uniqueness of a strong solution. Then it will be shown that a solution initially close to a constant equilibrium exists globally and converges to a (possibly different) constant equilibrium. Finally, we will show that that every solution that is eventually bounded in the topology of the natural state space exists globally and converges to the set of equilibria.

Variational problems on nematic liquid crystal droplets

Changyou Wang
Purdue University, USA
Qinfeng Li

In this talk, I will discuss the minimization problem of nematic liquid crystal droplets:

$$E(u,U) = 1/2 \int_U |Du|^2 \, dx + \int_{\partial\Omega} f(u \cdot \nu) dH^{n-1}$$

subject to $volume(U) = V_0 > 0$. The existence of minimization in a class of admissible domains, and some uniqueness (or rigidity) results in mean-convex, star-shaped domain will be presented. This is a joint work with Qinfeng Li (Hunan University, P.R.C).

PNP and Keller Segel equation and their related topics

Yong Yu
The Chinese University of Hong Kong, Hong Kong
Chia-Yu Hsieh

Poisson-Nernst-Planck equation and Keller-Segel equation are two different types of diffusive equation in physics and biology. In this talk, we will introduce their equilibrium solutions and the asymptotic behavior of the flows when time is large. The boundary layer phenomenon will also be discussed.

Special Session 33: Modeling and Data Analysis for Complex Systems and Dynamics

Jianzhong Su, University of Texas Arlington, USA
Padmanabhan Seshaiyer, George Mason University, USA
Lixia Duan, North China University of Technology, Peoples Republic of China
Pengcheng Xiao, Kennesaw State University, USA

Many dynamical systems, as examples from small scale neuronal systems and genomic systems in human, to large scale ecosystems of earth that impact climate change, are featured by nonlinear and complex patterns in spatial and temporal dimensions. These phenomena that are represented by massive amount of data, carry significant information and regulate down-stream dynamics. Understanding the mechanisms underlying such events by quantitative modeling represents a mathematical challenge of current interest. Yet all these systems share the similar dynamical system issues in ordinary/partial different equation such as bifurcation, stability, oscillations, stochastic noise as well as issues in determining hidden model parameters from experimental data sets and computational errors of the models. This special session offers a forum to exchange the state of the art theoretical advances related to this promising area as well as computational tools. It will foster and encourage communication and interaction between researchers in these directions. The common themes include mathematical models and data analysis, theoretical analysis, computational and statistical methods of dynamical systems and differential equations for the bio-system focused models, as well as applications in brain research. The topics may include but not restrict to:

(1) Dynamics and computation of neuronal systems

- Modeling and dynamical analysis of biological neurons and neuronal networks.
- Generation, encoding and transduction of neuronal signals and patterns.
- Modeling and analysis of cognitive information processing mechanisms
- Dynamic abnormality in neuronal systems due to diseases.

(2) Dynamics of immune systems

- Modeling biomedical processes, including tumor growth, cardio-vascular diseases, infection, and healing, mediated by immunologic mechanisms.
- Analysis of mathematical models for dynamics features such as instabilities, bifurcations that provide insight into the nature of the underlying bio-physical mechanisms.
- Modeling wound healing and inflammatory responses, including cell to cell interactions, foreign body reactions and quantitative as well as qualitative comparison with experimental data.

(3) Data analysis and modeling of whole brain activities

- Complexity theory applied to brain
- Perception, learning and memory functions in brain.
- Computational evolutionary biology.
- Models, analysis and algorithms in Bioinformatics.

(4) Data analysis and modeling in engineering, science, industry and agriculture, in particular these involving water, soil and other natural resources, that develop and utilizes data science methodology to interpret and reduce the complexity and dimensions of data sets that lead to predictive dynamics models

(5) Imaging analysis and software tools to help with modeling and analytics of complex systems and dynamics.

Multiscale continuum spine modeling: From structural plasticity to neural circuits

Steven Baer
Arizona State University, USA

Dendritic spines are small protrusions that stud the dendritic shaft of different neurons in the brain, including the retina. They are considered to be an important locus for plastic changes underlying memory and learning processes. In addition, neurological and psychiatric disorders are accompanied by pathological alterations in spine morphology and spine density. Spines exist in great numbers: e.g., over 200,000 spines per neuron on Purkinje cells. Continuum spine theory applies when a dendritic cable is populated by a large number of spines and the interspine distance is much less than the length scale of the cable. In this talk, continuum spine modeling is used to study the spatiotemporal dynamics of spine restructuring and spine population dynamics. Next, we show how the continuum approach can be generalized to formulate a multiscale continuum model of the neural subcircuits in the vertebrate outer retina. We conclude by introducing a new continuum model for the distribution of branches, rather than spines, over the primary neurite of the Drosophila MN5 motorneuron.

The complexity of neuromodulation

Janet Best
The Ohio State University, USA
Hilary Garcia-Marrero, Fred Nijhout, Michael Reed

Commonly prescribed selective serotonin reuptake inhibitors (SSRIs) inhibit the serotonin transporter to correct a presumed deficit in extracellular serotonin signaling during depression. These agents bring clinical relief to many who take them; however, a significant and growing number of individuals are resistant to SSRIs. There is increasing evidence that inflammation plays an important role in the clinical variability of SSRIs, though the interactions between SSRIs, inflammation and synaptic serotonin modulation are complex and difficult to discern. This talk will present modeling and data analysis for extracellular serotonin and its co-modulation with inflammation-associated histamine.

Neural coding and low-rank networks

Steven Collazos
University of Minnesota Morris, USA
Duane Nykamp

A theory in neuroscience proposes that groups of coactive neurons form a basis for neural processing. Following other researchers' work on threshold-linear networks, which are firing rate models where the activation function is a rectifier, we model the collection of all possible ensembles of neurons (i.e., the collection of permitted sets) as a collection of binary strings that indicate which neurons are considered responsive. Unlike the threshold-linear regime, however, we allow the activation function to be piecewise differentiable. We construct the collection of permitted sets by imposing a threshold on the responsiveness of the neuron to input at the steady state. Furthermore, when the synaptic weight matrix is almost rank one, we prove that the collection of permitted sets is a convex code. If time permits, we will present how our framework might be applicable other neuronal network models.

Regularized least absolute deviation-based sparse identification of dynamical systems

Lin Du
Northwestern Polytechnical University, Peoples Republic of China

This work develops a regularized least absolute deviation-based sparse identification of dynamics (RLAD-SID) method to address outlier problems in the classical metric-based loss function and the sparsity constraint framework. Our method uses absolute derivation loss as a substitute of Euclidean loss. Moreover, a corresponding computationally efficient optimization algorithm is derived on the basis of the alternating direction method of multipliers due to the non-smoothness of both the new proposed loss function and the regularization term. Numerical experiments are performed to evaluate the effectiveness of RLAD-SID using several exemplary nonlinear dynamical systems, such as the van der Pol equation, the Lorenz system, and the 1D discrete logistic map. Furthermore, detailed numerical comparisons are provided with other existing methods in metric-based sparse regression. Numerical results demonstrate that (1) RLAD-SID shows significant robustness toward a large outlier and (2) RLAD-SID can be seen as a particular metric-based sparse regression strategy that exhibits the effectiveness of the metric-based sparse regression framework for solving outlier problems in a dynamical system identification.

Dynamics of mixed bursting in the pre-Botzinger complex model

Lixia Duan
North China University of Technology, Peoples Republic of China

The dynamics of neuronal firing activity has important implications in understanding the pathological respiratory rhythm. Studies on electrophysiology have shown that the magnetic flow has an important influence on the firing activities of neurons. Based on the Butera neuron model, we investigate the effects of external stimulus on neuronal activities. By multi -time scale and bifurcation analysis, we study the dynamic mechanism of the generation and transition of firing patterns under the condition of external stimulus. The results show the bistable structure can be switched from one type to another by controlling the external stimulus, which means that different neuronal firing patterns will arise. Our current work gives an insight into the effects of the external stimulus on the activities of the neural network.

Traveling front solution stability in a lateral inhibition network in the neural field model

Yixin Guo
Drexel University, USA
Dominik Macaluso

We investigate the stability for traveling front solutions of the neural field model. This model has been studied intensively for propagating patterns with saturating Heaviside gain of neuron firing activity. Previous work has shown the existence of traveling fronts of the neural field model in a more complex setting using a non-saturating piecewise linear gain. We aim to study the stability of traveling fronts of the neural field model through the Evan's function. We attain the Evan's function of traveling fronts using an integration of analytical derivations and computational approach for the neural field model with previously uninvestigated piecewise linear gain. Using this approach, we are able to determine the stability for any given traveling fronts of the neural field model

Logarithmic asymptotics in fractional evolution equation: Analysis and computation

Changpin Li
Shanghai University, Peoples Republic of China

The so-called logarithmic asymptotics indicates that the solution to the differential equation has algebraic asymptotics in the sense of logarithmic function. It is often used to characterize the ultra slow process, such as creep in igneous rocks. In this talk, we mainly introduce the logarithmic asymptotics and regularity of the solution to the Caputo-Hadamard fractional evolution equation. Based on these theoretical analysis, we construct the reliable numerical algorithms to numerically solve it.

Epileptic neurodynamics associated with D-serine regulations

Suyu Liu
Hangzhou Dianzi University, Peoples Republic of China
Xiaohang Zhu

D-serine, as a co-agonist in the regulation of NMDA receptors by neuroglial cells, may be involved in fine-tuning the function of NMDA receptors and maintaining the synaptic plasticity mediated by NMDA receptors during the neuron-glia interaction. Whether and how this fine-tuning mechanism affects the signaling activity of neurons and glial cells in epileptic pathologies is uncertain. To account for this, we proposed a kind of dynamical network model associated with epilepsy coupled with neurons, astrocytes and microglia cells, and studied the effect of glial transmitter D-serine on neuronal discharge patterns in this network, as well as the transition dynamics among these activity patterns. The results showed that the D-serine regulation induced by positive feedback between astrocytes and microglia-astrocytes could make the neuronal discharge transition between a normal and an epileptic state. By using the bifurcation theory, the windows and the pathological values of the two parameters presented by the specific coupling parameters were obtained. These results indicate that the abnormal release of D-serine from astrocytes and the abnormal function of microglia may be the symptoms of epileptic seizure, and further understand the mechanism of abnormal synaptic function in epileptic seizure.

A validated brain metabolism model

Alice Lubbe
University of Texas Arlington, USA
Craig R Malloy, Jianzhong Su

A primary goal of brain metabolism research is to ascertain values of flux rate (or exchange rate) parameters involved in the tricarboxylic acid (TCA) cycle. A two-compartment mathematical model expresses metabolic interactions among and between glial and neuronal cells. Our two-compartment model consists of an ODE system that depicts the rates of accumulated concentrations of the isotopomers of key TCA cycle intermediates using the law of mass action. Previous models address TCA intermediates affected by glucose labeled with ^{13}C only the first and sixth carbons in studies with mice. Our model is among the first to characterize the propagation of glucose labeled in all six carbons with ^{13}C through key TCA cycle intermediates. We successfully validated our model against data from a recent experiment in which all six carbons of glucose were labeled with ^{13}C and infused in pigs, and the carbon multiplets of TCA cycle intermediates such as glutamate were subsequently measured from brain tissue samples using ^{13}C NMR spectroscopy at multiple time points. Our work to determine optimally accurate flux rate parameters lies on the horizon.

Coupling-induced mixed-mode oscillations by symmetry breaking via cusped singularities

Morten Pedersen
University of Padua, Italy
M Brons, KU Kristiansen, MP Sorensen

Mixed-mode oscillations consisting of alternating small- and large-amplitude oscillations are often caused by folded singularities and singular Hopf bifurcations. We show that coupling between identical nonlinear oscillators can cause mixed-mode oscillations because of symmetry breaking. This behavior is illustrated for FitzHugh-Nagumo oscillators with repulsive coupling, and we show that it is caused by a singular Hopf bifurcation related to singularities at a cusp – not a fold – of the critical manifold. Using blowup, we determine the number of small-amplitude oscillations analytically, showing – as for the folded nodes – that they are determined by the ratio of eigenvalues. The model undergoes a saddle-node bifurcation in the desingularized reduced problem, which also occurs on a cusp, and not a fold. We find excellent agreement between our analytical results and numerical computations.
Inspired by earlier work, we then identify a new type of bursting dynamics in a model of two coupled beta-cells. Small-amplitude oscillations in the action potential height precede transitions to square-wave bursting. This behavior is related to a pitchfork-of-limit-cycles bifurcation in the fast subsystem caused

by symmetry breaking. Moreover, we show that it is organized by a cusped saddle-node in the averaged system as seen for the coupled FitzHugh-Nagumo units.

Data-driven approaches for predicting transmission dynamics of infectious diseases

Padmanabhan Seshaiyer
George Mason University, USA

Modeling and Data analysis continues to provide useful insights to understand the early transmission dynamics of infectious diseases such as COVID-19. Along with the development of mathematical modeling, there have also been a variety of data-driven approaches that have been introduced to estimate the parameters in these models such as the transmission, infection, quarantine and recovery using real data. While there have been significant advances in estimating parameters, there is still a great need to develop efficient, reliable and fast data-driven approaches. Moreover, the need to develop realistic epidemic models incorporating behavioral responses of compliance and adherence, makes the associated modeling and prediction more complex. In this talk, we will introduce some models for understanding spread of COVID-19 along with physics-informed neural network data-driven approaches to study dynamics and to estimate parameters in these models. We will also discuss the importance of data science education to help students at all levels to engage in research in the areas of modeling and data analysis.

EEG Source Localization: new methods and applications

Jianzhong Su
University of Texas Arlington, USA
Hongguang Xi, Julio Enciso-Alva

Mathematics plays an increasing role in brain research and medicine. The well-known Hodgkin-Huxley model for neurons laid a foundation for computational neuroscience. However, understanding activities in the whole brain remains a focus of active research. Full brain Electroencephalography (EEG) and its source localization is a brain imaging modality based on multi-channel EEG signals. It measures the brain field potential fluctuations on the entire scalp for a period of time, and then we can mathematically calculate the electric current density inside the brain by solving an inverse problem for a partial differential equation. In this talk, we introduce mathematical methods for the EEG imaging problems, their validations through simulations and experimental data, and discuss its applications. One application is in identifying abnormality in brain activities during seizures of an infant patient with Glucose Transporter Deficiency Syndrome. Another application is to find the neuronal signatures in response to pain stimulation. Our research shows these brain data can be further used to study the brain properties that glean into the inner working of brain functions using mathematical and statistical tools.

Activity pattern analysis of the subthalamopallidal network under ChR2 photocurrent control

Honghui Zhang
Northwestern Polytechnical University, Peoples Republic of China
Ying Yu, Zichen Deng, Qingyun Wang

This paper incorporates the neural photostimulation to model firing activities of the subthalamopallidal network from basal ganglia, and the emphasis is to explore the possible effects of lightgated cation membrane channel on neural system. Motivated by previously proposed computational models of ChR2, we study the firing rhythms of subthalamic nucleus and GPe with different coupling modes, which are closely related with somatic movement. Firstly, the applicability of ChR2 models we employed has been tested and verified through the reaction of single GPe neuron. Then we analyzed the firing patterns of neural population under two prototype network architectures respectively. The sparsely coupled network can produce six general classes of sustained firing patterns with little regularity, while the tightly coupled network can provide five different general waveforms with high correlation, including cluster synchronization of in-phase and anti-phase. Most noteworthy, with the control of ChR2, we found the synchronization of continuous waves can be destroyed and even eliminated under appropriate conditions, which can be an enlightenment for the treatment and regulation of Parkinson's disease and other motor neural disease related to basal ganglia.

Special Session 34: Variational, Topological and Set-Valued Methods for Nonlinear Differential Problems

Giuseppina d'Aguì, University of Messina, Italy
Angela Sciammetta, University of Palermo, Italy
Patrick Winkert, University of Technology Berlin, Germany
Eleonora Amoroso, University of Messina, Italy

The focus of this session is the qualitative analysis of non-linear differential problems. In particular, the main purpose is to give an update on recent developments concerning ordinary and partial differential equations, variational-hemivariational inequalities, difference and algebraic systems obtained by exploiting different methods of the nonlinear analysis such as critical point theory, fixed point theorems, topological degree, Morse theory, set-valued analysis and so on.

Existence and localization of solutions for nonlocal differential problems

Irene Benedetti
University of Perugia, Italy

In this talk we will present results of existence and localization of solutions for nonlocal differential problems in abstract spaces. In particular, we will show an approach based on the so called transversality conditions, novel in this setting. This technique provides a unifying method for studying models describing reaction-diffusion processes in several frameworks. We will consider nonlocal initial conditions such as the Cauchy multipoint and the mean value conditions, and we can handle nonlinearity with superlinear growth, for instance cubic polynomials or maps depending on the integral of the solution, thus encompassing nonlocal diffusion behaviours.

Some results for perturbed (p, q)-quasilinear elliptic problems

Anna Maria Candela
Università degli Studi di Bari Aldo Moro, Italy

We consider a family of (p, q)-quasilinear elliptic problems which may lose their variational structure if there is a continuous perturbation term. Anyway, suitable assumptions and appropriate procedures allow us to prove the existence of one or more solutions if the perturbation is small enough.
These results are part of joint works with R. Bartolo and A. Salvatore.

Existence results for second order nonlinear ODEs

Pasquale Candito
Mediterranean University of Reggio Calabria, Italy

The existence of at least one solution for non-linear two-point boundary value problems is discussed. In particular, a solution is obtained as the limit of a sequence of linear interpolations of the solutions of a corresponding differences problem. Examples are presented to illustrate also the a priori estimates obtained on the infinite norm of a solution and its derivatives.

Symmetry breaking for a supercritical elliptic problem in an annulus

Francesca Colasuonno
Alma Mater Studiorum Università di Bologna, Italy
Alberto Boscaggin, Benedetta Noris, Tobias Weth

In this talk, I will present an existence result for the Dirichlet problem associated with the elliptic equation

$$-\Delta u + u = a(x)|u|^{p-2}u$$

set in an annulus of $\mathbb{R}^N$. Here $p > 2$ is allowed to be supercritical in the sense of Sobolev embeddings, and $a(x)$ is a positive weight with additional symmetry and monotonicity properties, which are shared by the solution that we construct. For this problem, we find a new type of positive, axially symmetric solution. Moreover, in the case where the weight $a(x)$ is constant, we detect a condition, depending only on the exponent p and on the inner radius of the annulus, that ensures that the solution is nonradial. In this setting, the major difficulty to overcome is the lack of compactness in a nonradial framework. The proofs rely on a combination of variational methods and dynamical system techniques.

Some refined criteria toward boundedness in an attraction-repulsion chemotaxis model with nonlinear productions

Alessandro Columbu
Università di Cagliari, Italy
Silvia Frassu, Giuseppe Viglialoro

We deal with a zero-flux attraction-repulsion chemotaxis model with nonlinear production rates for both chemorepellent and chemoattractant, and we will establish certain relations between such rates and some coefficients of the problem ensuring uniform boundedness of solution to the model. This is a joint work with Silvia Frassu and Giuseppe Viglialoro.

A new class of double phase variable exponent problems and a Nehari manifold approach

Angel Crespo-Blanco
Technische Universität Berlin, Germany
Petteri Harjulehto, Leszek Gasiǹski, Patrick Winkert

During the last decade the so-called double phase operator has drawn attention from researchers. Originally it was introduced by Zhikov in the context of homogenization and elasticity theory and as an example for the Lavrentiev phenomenon. It regained popularity after some novel regularity results for local minimizers of the corresponding functional.
In the first part of this talk we introduce a new class of quasilinear elliptic equations driven by the so-called double phase operator with variable exponents. We discuss useful properties of the corresponding Musielak-Orlicz Sobolev spaces and of this new double phase operator. In contrast to the previously known constant exponent case we are able to weaken the assumptions on the data.
After that we consider a problem with superlinear right-hand side and we show, under very general assumptions, a multiplicity result for such problems, whereby we show the existence of a positive solution, a negative one and a solution with changing sign. The sign-changing solution is obtained via the Nehari manifold approach and, in addition, we can also give information on its nodal domains. Furthermore, we derive a priori estimates on the solutions in the L^∞-norm under the very general setting used above.

A bifurcation phenomenon for the critical p-Laplace equation in the ball

Francesca Dalbono
University of Palermo, Italy
Matteo Franca, Andrea Sfecci

We focus on positive radial solutions to the Dirichlet problem associated with the p-Laplace eigenvalue scalar curvature equation. The scalar curvature function K is assumed to be smooth, positive and to satisfy the l-flatness condition in a neighborhood of zero. We show that the number of radial positive solutions undergoes a bifurcation phenomenon: if K is steep enough at 0, the problem admits one solution for every positive eigenvalue λ, while if K is too flat at 0, the problem admits no solutions for λ small and two solutions for λ large.
The existence of the second solution, ensured by an additional monotonicity assumption on K, is new, even in the classical Laplace case. The proofs are based on Fowler transformation, invariant manifold theory and phase plane analysis.

Symmetrization for linear and nonlinear fractional elliptic problems

Vincenzo Ferone
Università di Napoli Federico II, Italy

We describe a new method to obtain symmetrization results for fractional elliptic equations involving the s-laplacian. Some possible applications of the method to nonlinear equations and to equations with lower order terms will be discussed.

Critical double phase problems: Recent results and open questions

Alessio Fiscella
Università degli Studi di Milano-Bicocca, Italy

In this talk, we discuss about recent results for double phase problems involving different critical nonlinearities. Our problems present some difficulties due to the presence of singularities and the lack of compactness of the critical embeddings for the related Musielak-Orlicz Sobolev space. After introducing the different variational approaches which allows us to provide the existence and multiplicity of solutions, we conclude the talk presenting some open questions.

Nonlinear Dirichlet problem for the fractional p-Laplacian with jumping reactions

Silvia Frassu
University of Cagliari, Italy
Antonio Iannizzotto

In this talk we deal with a nonlinear elliptic equation driven by the degenerate fractional p-Laplacian, with Dirichlet type condition and a jumping reaction, i.e., (p-1)-linear both at infinity and at zero but with different slopes crossing the principal eigenvalue. Under two different sets of hypotheses, entailing different types of asymmetry, we prove the existence of at least two nontrivial solutions. Our method is based on degree theory for monotone operators and nonlinear fractional spectral theory.
This work is in collaboration with Antonio Iannizzotto.

Nontrivial solutions for indefinite Schrödinger type equations

Shibo Liu
Florida Institute of Technology, USA

In this talk I will present our results on stationary Schrödinger equations, Schrödinger-Poisson systems, quasilinear Schrödinger equations and other related problems such as the Schrödinger-Kirchhoff equations. In all these problems the Schrödinger operator is indefinite, thus the zero function is not a local minimizer of the variational functional and the mountain pass theorem is not applicable. It is important to observed that except for the semilinear Schrödinger equation, the classical linking theorem is also not applicable. To overcome this difficulty we employ the concept of local linking (introduced by Shujie Li and Jiaquan Liu in the 1980's), nontrivial solutions for these indefinite problems are obtained by variational methods.

Properties of solutions to a class of parabolic and hyperbolic problems

Monica Marras
University of Cagliari, Italy

We discuss blowup phenomena to solution of some classes of parabolic systems under Neumann boundary conditions and nonlinear hyperbolic coupled systems of fourth order under Dirichlet or Navier boundary conditions. The solutions may blow up in finite time t^* and under appropriate assumptions on data, a safe interval of existence of the solution is derived with a lower bound of the lifespan. The proofs are based on some inequalities and coupled estimates techniques.

Critical growth double phase problems

Kanishka Perera
Florida Institute of Technology, USA
Francesca Colasuonno

We obtain nontrivial solutions of critical growth double phase problems by variational methods.

Mean value formulas for classical solutions to subelliptic equations with non-smooth coefficients

Sergio Polidoro
Università di Modena e Reggio Emilia, Italy
Diego Pallara

We prove mean value formulas for classical solutions to second order linear differential equations in the form

$$\partial_t u = \sum_{i,j=1}^{m} X_i(a_{ij} X_j u) + X_0 u + f,$$

where $A = (a_{ij})_{i,j=1,\dots,m}$ is a bounded, symmetric and uniformly positive matrix with C^1 coefficients under the assumption that the operator $\sum_{j=1}^{m} X_j^2 + X_0 - \partial_t$ is hypoelliptic and the vector fields $X_1, \dots, X_m$ and $X_{m+1} := X_0 - \partial_t$ are invariant with respect to a suitable homogeneous Lie group. Our results apply e.g. to degenerate Kolmogorov operators and parabolic equations on Carnot groups $\partial_t u = \sum_{i,j=1}^{m} X_i(a_{ij} X_j u) + f$.

An elementary proof of the strong maximum principle is obtained as an application of the mean value formulas.

Bounded solutions for generalized quasilinear elliptic equations

Addolorata Salvatore
University of Bari, Italy

We give some existence results for weak bounded solutions of the generalized quasilinear Schrödinger equation

$$-\mathrm{div}(a(x,u,\nabla u)) + A_t(x,u,\nabla u) + V(x)|u|^{p-2}u = g(x,u) \qquad \text{in } \mathbb{R}^N$$

with $p > 1$, $N \geq 2$ and $V : \mathbb{R}^N \to \mathbb{R}$ suitable measurable positive function.
Here, $A : \mathbb{R}^N \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ is a $\mathcal{C}^1$-Carathéodory function which grows as $|\xi|^p$ and has partial derivatives

$$A_t(x,t,\xi) = \frac{\partial A}{\partial t}(x,t,\xi), \quad a(x,t,\xi) = \left(\frac{\partial A}{\partial \xi_1}(x,t,\xi), \dots, \frac{\partial A}{\partial \xi_N}(x,t,\xi)\right)$$

and $g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a given Carathéodory function with super$-p-$linear but subcritical growth or with sub$-p-$linear growth.
Since the principal term depends on u, other than on its gradient ∇u and on the spatial variable x, we study the interaction of two different norms in a suitable Banach space with the aim of obtaining a good variational approach.
These results are part of joint works with A. M. Candela, F. Mennuni, G. Palmieri and C. Sportelli.

Mild solutions of second-order semilinear impulsive differential inclusions in Banach spaces

Valentina Taddei
University of Modena and Reggio Emilia, Italy
Martina Pavlackova

the existence of a mild solution to the Cauchy problem for impulsive semilinear second-order differential inclusion in a Banach space is investigated in the case when the nonlinear term also depends on the first derivative. This purpose is achieved by combining the Kakutani fixed point theorem with the approximation solvability method and the weak topology. This combination enables obtaining the result under easily verifiable and not restrictive conditions on the impulsive terms, the cosine family generated by the linear operator and the right-hand side while avoiding any requirement for compactness. The talk concludes with an application to evolution problems.

Nonhomogeneous degenerate quasilinear problems with convection

Elisabetta Tornatore
University of Palermo, Italy
D. Motreanu

We consider a nonlinear differential Dirichlet problem whose equation is driven by a degenerate p-Laplacian with a weight depending on the solution and whose reaction is a convection term, thus depending on the solution and its gradient. The existence of a weak solution is proven by arguing through a truncated auxiliary problem.

Boundedness criteria for an attraction-repulsion chemotaxis model with consumed signals

Giuseppe Viglialoro
University of Cagliari, Italy
Silvia Frassu, Rafael Rodriguez Galvan

In this talk we focus on an attraction-repulsion chemotaxis model with consumed signals, formulated in n-dimensional bounded and smooth domains. We derive sufficient conditions on its data yielding global and bounded classical solutions to the related zero-flux Cauchy problem. This is a joint work with Silvia Frassu and Rafael Rodriguez Galvan.

Special Session 36: Stochastic Systems, SDEs/SPDEs, Games, Quantum-Computing and Storages

Wanyang Dai, Nanjing University, Peoples Republic of China

We will present the latest achievements of theory, methods, and computations for general stochastic systems, stochastic differential equations (SDEs), and stochastic partial differential equations (SPDEs). Their interactions with and/or applications in broad areas, e.g., optimal control and game theory, queuing theory and quantum storage, information theory and quantum-computing, smart contract and blockchain, FinTech, big data and artificial intelligence (AI), genomics and healthcare, smart material sciences and communication systems will be highlighted.

CNN & machine learning based simulation and analysis for a unified backward SPDE with application to a strongly nonlinear case

Wanyang Dai
Nanjing University, Peoples Republic of China

We develop a generic convolutional neural network (CNN) and machine learning based numerical scheme to simulate the 2-tuple adapted strong solution to a unified backward stochastic partial differential equation (B-SPDE) driven by Brownian motion with application to a strongly nonlinear case in finance. The dynamics of the scheme is modeled by a CNN through conditional expectation projection and machine learning. It consists of two convolution parts: W layers of backward networks and L layers of reinforcement iterations. Furthermore, it is a completely discrete and iterative algorithm in terms of both time and space with mean-square error estimation and almost sure convergence supported by both theoretical proof and numerical examples. In doing so, we need to prove the unique existence of the 2-tuple adapted strong solution to the system under both conventional and Malliavin derivatives with general local Lipschitz and linear growth conditions.

Asymptotics and ergodicity of nonlinear distorted Brownian motion and of nonlinear Fokker-Planck equations

Michael Röckner
University of Bielefeld, Germany
V. Barbu

In this talk we present recent results on the large time asymptotics of the one dimensional time marginal laws of the nonlinear distorted Brownian motion, obtained via analyzing the corresponding nonlinear Fokker-Planck equation. The first result is the existence and uniqueness of an invariant measure given by an explicit formula in the non-degenerate case. The second is in the degenerate case and states the existence of an invariant measure and the compactness in L^1 of the omega limit set including information about its location. The third is the mean ergodicity in the latter case.

Stochastic control/stopping problem with expectation constraints

Song Yao
University of Pittsburgh, USA

We study a stochastic control/stopping problem with a series of inequality-type and equality-type expectation constraints in a general non-Markovian framework. We demonstrate that the stochastic control/stopping problem with expectation constraints (CSEC) is independent of a specific probability setting and is equivalent to the constrained stochastic control/stopping problem in weak formulation (an optimization over joint laws of Brownian motion, state dynamics, diffusion controls and stopping rules on an enlarged canonical space). Using a martingale-problem formulation of controlled SDEs, we characterize the probability classes in weak formulation by countably many actions of canonical processes, and thus obtain the upper semi-analyticity of the CSEC value function. Then we employ a measurable selection argument to establish a dynamic programming principle (DPP) in weak formulation for the CSEC value function, in which the conditional expected costs act as additional states for constraint levels at the intermediate horizon.

Quantitative contraction rates for McKean-Vlasov stochastic differential equations with multiplicative noise

Chao Zhu
University of Wisconsin Milwaukee, USA

This work focuses on the quantitative contraction rates for McKean-Vlasov stochastic differential equations (SDEs) with multiplicative noises. Under suitable conditions on the coefficients of the SDE, we derive explicit quantitative contraction rates for the convergence in Wasserstein distances of McKean-Vlasov SDEs using the coupling method. The contraction results are then used to prove a propagation of chaos uniformly in time, which provides quantitative bounds on convergence rate of interacting particle systems, and establishes exponential ergodicity for McKean-Vlasov SDEs.

Special Session 37: Nonlinear Elliptic Problems in Geometry and Physics

Jinmyoung Seok, Seoul National University, Korea
Jaeyoung Byeon, KAIST, Korea
Norihisa Ikoma, Keio University, Japan

There are many interesting and challenging nonlinear elliptic problems coming from geometry and physics. We would like to invite young and leading specialists in the field of nonlinear elliptic problems, who are interested in the following research topics: solvability-nonsolvability, asymptotic behaviors, stability and regularity of solutions for nonlinear Schrödinger systems, Born-Infeld model, Chern-Simon-Schrödinger systems, etc.

Standing wave and travelling wave solutions for a fourth order Schrödinger equation

Jean-Baptiste Casteras
University of Lisbon, Portugal

In this talk, we will be interested in standing wave solutions to a fourth order nonlinear Schrödinger equation having second and fourth order dispersion terms. This kind of equation naturally appears in nonlinear optics. In a first time, we will establish the existence of ground-state and renormalized solutions. We will then be interested in their qualitative properties, in particular their stability.

On topological solutions of the self-dual Maxwell-Chern-Simons-Higgs vortex equations

Jongmin Han
Kyung Hee University, Korea
Kyungwoo Song

In this talk, we consider the self-dual equations arising from the Maxwell-Chern-Simons-Higgs model on $\mathbb{R}^{1,2}$ equipped with a background metric $(1, -b(x), -b(x))$. We assume that $b(x)$ is not a constant and satisfies $b(x) = O(|x|^{-\omega})$ at infinity with $\omega \in (0, 2)$. The main equations have two important parameters: the Maxwell coupling constant κ and the Chern-Simons coupling constant q. We show that there exists a constant $\beta_* > 0$ such that there exists a topological solution provided $\kappa q > \beta_*$. We also verify the Chern-Simons limit for those solutions.

On steady states for the Vlasov-Schrödinger-Poisson system

Sangdon Jin
Chung-Ang University, Korea
Younghun Hong

The Vlasov-Schrödinger-Poisson system is a kinetic-quantum hybrid model describing quasi-lower dimensional electron gases. In this talk, we discuss the derivation of kinetic quantum hybrid models using partial confinement. Also, for this system, we study the construction of a large class of 2D kinetic/1D quantum steady states in a bounded domain as generalized free energy minimizes, and we show their finite subband structure, monotonicity, uniqueness, and conditional dynamical stability. This talk is based on joint work with Younghun Hong (Chung-Ang University).

A supercritical elliptic problem in the half space with an inhomogeneous boundary condition

Sho Katayama
University of Tokyo, Japan
Kazuhiro Ishige

This talk concerns an elliptic boundary problem of a supercritical elliptic equation in the half space $\partial\mathbb{R}^N := \{(x_1, \ldots, x_N) \in \mathbb{R}^N : x_N > 0\}$ with an inhomogeneous Dirichlet boundary condition $u = \lambda\varphi$ on $\partial\mathbb{R}^N$. Here φ is a nonnegative non-zero function on $\mathbb{R}^N$ with suitable conditions and $\lambda > 0$ is a parameter. Under a Joseph–Lundgren subcritical condition, we give a complete classification of the existence/nonexistence of solutions. Briefly, our result states that there is a threshold constant $\lambda^* > 0$ such that the problem has a solution if and only if $0 < \lambda \leq \lambda^*$.

Existence and multiplicity of radially symmetric solutions for NLS equations

Tomoharu Kinoshita
Waseda University, Japan

In this talk, we consider the following NLS equations in $\mathbf{R}^N$:

$$-\Delta u + V(x)u = g(u),\ u \in H^1(\mathbf{R}^N),$$

where $N \geq 2$, $V \in C^1(\mathbf{R}^N, \mathbf{R})$ and $g \in C(\mathbf{R}, \mathbf{R})$. For a wide class of nonlinearities, which satisfy the Berestycki-Lions type condition, we show the existence and multiplicity of radially symmetric solutions. We use a new deformation argument under a new version of the Palais-Smale condition.

Nonlinear Schrödinger systems with traps potentials for mixed interaction

Sang-Hyuck Moon
UNIST, Korea
Jaeyoung Byeon, Tai-Chia Lin

In this talk, we will consider the three components nonlinear Schrödinger systems with mixed coupling forces (2 repulsive, 1 attractive) and nonconstant trap potentials in whole space $\mathbb{R}^N$. To get a compactness, we impose a potential wall at infinity, and we can construct a least energy vector solution. A main interest in this work is its asymptotic behavior of the solution for large interaction forces. One component repelling other two components survives and the other two components diminish and concentrate at a point diverging to infinity as the interaction forces are getting larger and larger. The location of the concetration point, which we could characterize in terms of the limit of a surviving component, a repulsive force and potentials of diminishing components under the assumption of the nondegeneracy for the limit problem of the surviving component.

Existence results of positive solutions to semi-linear elliptic problems on metric graphs

Masataka Shibata
Meijo University, Japan

We consider elliptic equations $-\epsilon^2 u'' + u = u^p$ on compact metric graphs, where $p > 1$ is a given constant and ϵ is a positive parameter. We focus on the multiplicity of positive solutions for sufficiently small ϵ and introduce some results about the existence of positive solutions.

Existence of non-topological solutions of the self-dual Einstein-Maxwell-Higgs equations on compact surfaces

Juhee Sohn
Korea National University of Transportation, Korea
Jongmin Han, Youngae Lee

In this talk, we show the existence of non-topological solutions to the self-dual Einstein-Maxwell-Higgs equations on a compact surface when the parameter $\varepsilon > 0$ is small enough and the total string number N is bigger than two. Based on degree theory and blowup analysis, we find non-topological solutions.
(This was supported by Korea National University of Transportation in 2023)

Normalized solutions for L2-critical NLS

Kazunaga Tanaka
Waseda University, Japan
Silvia Cingolani, Marco Gallo, Norihisa Ikoma

We study the existence of normalized solutions for nonlinear Schrödinger equations:

$$-\Delta u + \mu u = g(u) \quad \text{in } \mathbf{R}^N, \qquad \int_{\mathbf{R}^N} |u|^2\, dx = m,$$

where $N \geq 2$, $g \in C(\mathbf{R})$, $m > 0$ are given and $\mu > 0$, $u \in H^1(\mathbf{R}^N)$ are unknown. We consider the situation

$$g(s) \sim |s|^{4/N} s \quad \text{as } s \sim 0 \text{ and } s \sim \pm\infty$$

and we show the existence of positive normalized solutions for a suitable $m > 0$.
This is a joint work with Silvia Cingolani, Marco Gallo and Norihisa Ikoma.

A weak solution to a $(1, p)$-Laplace problem is continuously differentiable

Shuntaro Tsubouchi
University of Tokyo, Graduate School of Mathematical Sciences, Japan

This talk is concerned with gradient continuity for a weak solution to a very singular elliptic problem involving both one-Laplacian and p-Laplacian with $p \in (1, \infty)$. The main difficulty herein is that this problem becomes no longer uniformly elliptic near a facet, the degenerate region of a gradient. This fact prevents us from using standard methods from the De Giorgi–Nash–Moser theory to prove Hölder continuity of a gradient, especially near the facet. It is substantially because the one-Laplace operator is degenerate elliptic in a direction of a gradient, while this operator becomes singular elliptic in other directions. Such anisotropic diffusivity appears difficult to handle in existing regularity theory. Continuity of a derivative was first established by Yoshikazu Giga and the speaker, in a special case where a weak solution is both scalar-valued and convex. After that work was completed, the speaker, inspired by a recent work on higher regularity for a very degenerate elliptic problem, has found it possible to show gradient continuity in general cases. The aim of this talk is to explain briefly how to prove continuous differentiability of general weak solutions, even across facets.

Nodal solutions for coupled elliptic equations

Zhi-Qiang Wang
Utah State University, USA

We discuss nodal type solutions for coupled elliptic equations. We report work for constructing multiple nodal solutions having componentwisely-shared nodal numbers of coupled elliptic systems. This is done by further developing critical point theory with built-in flow invariance which has been a useful tool to give locations of critical points by minimum methods.

Special Session 39: Recent Results in Local and Nonlocal Elliptic and Parabolic Equations

Zu Gao, Wuhan University of Technology, Peoples Republic of China

This special session is to bring together experts and young researchers in the field of PDEs to share and exchange their ideas and recent results, to discuss open or challenging problems and also foster future collaborations and connections. The session mainly focuses on the existence and nonexistence, uniqueness, regularity, asymptotic behavior and qualitative properties of solutions to local and nonlocal elliptic and parabolic equations, besides, other related topics are welcome.

Nonlocal BVPs on extension domains

Simone Creo
Sapienza Università di Roma, Italy

In this talk we consider evolution BVPs for nonlinear fractional operators on irregular domains. It is known that these operators model the so-called anomalous diffusion.
In particular, we investigate parabolic Robin-Venttsel' problems for regional fractional p-Laplace operators on extension domains.
By using nonlinear semigroup theory, we prove that the problems at hand admit unique weak solutions. We then prove regularity properties of the associated nonlinear semigroups, i.e. ultracontractivity properties. This is achieved by means of generalized fractional Green formulas and fractional logarithmic Sobolev inequalities, adapted to the problems at hand.
We then conclude by mentioning some generalizations and possible future developments.
These results are obtained in collaboration with M. R. Lancia and P. Vernole.

The Bernstein technique for integro-differential equations

Serena Dipierro
University of Western Australia, Australia

We discuss how to extend the classical Bernstein technique to the setting of integro-differential operators. As a consequence, we provide first and one-sided second derivative estimates for solutions to fractional equations, for which we prove uniform estimates as their order approaches two.

Non autonomous BVPs in extension domains with dynamical boundary conditions

Maria Rosaria Lancia
Sapienza Università di Roma, Italy

I will consider a parabolic semi-linear non autonomous problem, both for local and nonlocal operators, with dynamical boundary conditions, in an extension domain with boundary a d-set.
Such problems are also known in the literature as Venttsel problems.
Existence and uniqueness of the mild solution of the associated semi-linear abstract Cauchy problem are proved as well as regularity results.
This is a part of a long term project with S.Creo and P.Vernole.

Diffusion over bronchial trees: Solvability and global regularity results

Alejandro Velez-Santiago
University of Puerto Rico - Mayaguez, USA
Kevin Silva-Perez

We consider a domains $\Omega_a \subseteq \mathbb{R}^2$ with ramified boundary Γ_a^∞, for a a parameter with $1/2 \leq a \leq a^* \simeq 0.593465$. This domain represents an idealization of bronchial trees in the lungs system. Since the exchanges between the lungs and the circulatory system take place only in the last generation of the bronchial trees, an accurate model for diffusion of oxygen may involve inhomogeneous Robin boundary conditions over Γ_a^∞. Therefore, we investigate the realization of the diffusion equation

$$\frac{\partial u}{\partial t} - \mathcal{A}u + \alpha u = f(x,t) \qquad \text{in } \Omega_a \times (0,\infty)$$

with mixed boundary conditions

$$\frac{\partial u}{\partial \nu_{\mathcal{A}}} + \beta u = g(x,t) \text{ on } \Gamma_a^\infty \times (0,\infty); \qquad u = 0 \text{ in } (\partial\Omega_a \setminus \Gamma_a^\infty) \times (0,\infty),$$

and $u(x,0) = u_0 \in C(\overline{\Omega}_a)$, where $\mathcal{A}$ stand as a linear (possibly non-symmetric) divergence-type differential operator, $\frac{\partial u}{\partial \nu_{\mathcal{A}}}$ represents a generalized notion of a normal derivative over irregular surfaces, $\alpha \in L^r(\Omega_a)$, $\beta \in L^s_\mu(\Gamma_a^\infty)^+$ with $\operatorname{ess}\inf_{x\in\Gamma_a^\infty} |\beta(x)| \geq \beta_0$ for some constant $\beta_0 > 0$ large enough, where $\min\{r,s\} > 1$. We show unique solvability of this diffusion equation, and moreover we establish that weak solution of this model equation are globally continuous in space and in time.

Special Session 40: Asymptotic Behaviour in Nonlinear Elliptic and Parabolic Problems

Yoshitsugu Kabeya, Osaka Prefecture University, Japan

Understanding the asymptotic properties of solutions to stationary and time dependent nonlinear PDEs often provides the key to finding the actual solutions and their properties, such as regularity, stability, front propagation. It is also fundamental in the numerical analysis of nonlinear PDEs. The aim of this special session is to bring together a group of leading international experts on asymptotic behaviour in linear and nonlinear PDEs for an exchange of ideas and update on most recent advances in these areas which will foster new collaborations. Relevant topics from the linear theory will include Green's functions and heat kernels estimates; maximum principles and regularity theory. The focus in nonlinear PDEs will be on elliptic and parabolic problems including equations on manifolds and problems with nonlocal interactions.

Singular solutions to a nonlinear elliptic equation on a unit sphere

Yoshitsugu Kabeya
Osaka Metropolitan University, Japan
Soohyun Bae, Jann-Long Chern, Shoji Yotsutani

We consider a nonlinear elliptic equation on the unit sphere and discuss the existence of positive singulalr solutions. We also investigate the properties of singular solutions.

Example of Turing's instability by equal diffusion

Hirokazu Ninomiya
Meiji University, Japan

In 1952, Turing proposed the mechanism of pattern formation in which a stable equilibrium of some kinetic system is destabilized by diffusion. In the case of two-component reaction–diffusion systems, however, the diffusion coefficients should be different for Turing's instability. I will give an example of a kinetic system with a asymptotically stable equilibrium, while the corresponding two-component reaction–diffusion system has a family of unstable stationary solutions that is arbitrarily close to the homogeneous stationary solution.

A quantitative stability estimate for a fourth order overdetermined problem

Michiaki Onodera
Tokyo Institute of Technology, Japan

I will talk about a joint work with Yuya Okamoto concerning a fourth order overdetermined boundary value problem in which the boundary value of the Laplacian of the solution is prescribed, in addition to the homogeneous Dirichlet boundary condition. It is known that, in the case where the prescribed boundary value is a constant, this overdetermined problem has a solution if and only if the domain under consideration is a ball. In this talk, we study the shape of a domain admitting a solution to the overdetermined problem when the prescribed boundary value is slightly perturbed from a constant. We derive an integral identity for the fourth order Dirichlet problem and a nonlinear weighted trace inequality, and the combination of them results in a quantitative stability estimate which measures the deviation of a domain from a ball in terms of the perturbation of the boundary value.

Special Session 41: Asymptotic Analysis and Bifurcations of Solutions for Nonlinear Models

Yoshitsugu Kabeya, Osaka Metropolitan University, Japan
Jann-Long Chern, Taiwan Normal University, Taiwan

The aim of this special session is to exchange recent results, ideas and techniques on nonlinear elliptic and parabolic PDEs, including reaction-diffusion systems and free boundary problems, from mathematical physics, chemical reactions, mathematical biology, medical science and some other fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulations and theoretical approaches with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunities to both established and junior researchers working in the related area to present their recent results.

Bifurctions and imperfect bifurcations of solutions to the scalar-field type eliptic equation under the Robin condition

Yoshitsugu Kabeya
Osaka Metropolitan University, Japan
Hiromazu Ninomiya

We discuss a nonlinear elliptic equation of the scalar-field type on a unit ball under the Robin condition. We investigate the radial solutions and also non-radial ones and the local bifurcations. This talk is based on the joint work with Professor Hirokazu Ninomiya of Meiji University, Japan.

On the ratio of population to resources in the diffusive logistic equation

Kousuke Kuto
Waseda University, Japan
Jumpei Inoue, Yanyue Meng

This talk is concerned with a class of stationary logistic equations for which Ni proposed an optimization problem to consider the supremum of the ratio of the total masses of species to resources by varying the diffusion rates and the configuration of resources, and moreover, he gave a conjecture that the supremum is 3 in a case when the habitat is a one-dimensional interval. Concerning this conjecture, Bai, He and Li (2016) found a sequence of diffusion rates and resource functions to make the corresponding ratios tend to the supremum 3 from below. In this talk, we first show the asymptotic profile of species corresponding to the maximizing sequence found by Bai et al. Next, we introduce a result that the supremum is infinity in a case when the habitat is a general multi-dimensional domain.

All global bifurcation diagrams of stationary solutions to a 1D phase field model

Tatsuki Mori
Graduate School of Engineering, Musashino University, Japan
Yasuhito Miyamoto, Sohei Tasaki, Tohru Tsujikawa, Shoji Yotsutani

We are interested in bifurcation diagrams of stationary solutions to a phase field model proposed by Fix and followed by Caginalp. In this talk, we show all the global bifurcation diagrams of stationary solutions to the model in the 1-dimension case. Bifurcation diagrams are surprisingly rich in variety depending on the latent heat and the initial total enthalpy. For instance, bifurcation diagrams include the secondary bifurcation point where symmetric breaking occurs, and curves which connect a limit of boundary layer solutions to the other limit of internal layer solutions.

On a numerical bifurcation analysis of a particle reaction-diffusion model for a motion of two self-propelled disks

Masaharu Nagayama
Hokkaido University, Japan
Yusuke Yasugahira

Theoretical analysis using mathematical models is often used to understand a mechanism of collective motion in a self-propelled system. Several kinds of characteristic motions have been observed in the experimental system using camphor disks due to the interaction of two camphor disks. In this talk, we understand the emergence mechanism of the motions caused by the interaction of two self-propelled materials by analyzing the global bifurcation structure using the numerical bifurcation method for a mathematical model.

Chimera behaviors in nonlocally coupled oscillator system

Kota Ohno
Chuo University, Japan
Toshiyuki Ogawa

Coupled oscillator systems have been studied in various fields of physical and biological phenomena. Among them, systems of nonlocally coupled oscillators can exhibit chimera states, which consist of spatially coherent and incoherent states. Chimera states have relations to neurological diseases such as epileptic seizures. Recently, several studies of chimera states based on numerical simulation have been reported by Omelchenko et al. However, as far as we know, the stability and bifurcation origin of chimera states have not been understood. Stuart-Landau type nonlinearity enables us to formulate the stability problem of traveling waves precisely through Floquet theory. We analyzed Floquet multiplier and confirmed the transition of stability of traveling wave by changing the parameters of nonlocal coupling. In this study, we will discuss the relationship between the stability of traveling waves and chimera states.

A perturbation theory of overdetermined boundary value problems

Michiaki Onodera
Tokyo Institute of Technology, Japan

Our main interest lies in the shape of a bounded domain for which a parametrized overdetermined boundary value problem admits a solution.
Unlike a typical nonlinear problem where the nondegeneracy of the linearized operator implies a local one-to-one correspondence between parameters and solutions, overdetermined problems generally fail to follow this scenario because of a loss of derivatives.
We develop a perturbation theory of overdetermined problems based on a characterization of an evolving domain by a geometric evolution equation.
We show that, if the linearized operator satisfies some monotonicity condition in addition to the nondegeneracy, then there exists a monotonically increasing family of domains admitting solvability of the corresponding overdetermined problem under a small continuous deformation of parameters.

Standing waves of coupled Schrödinger equations with quadratic interactions from Raman amplification in a plasma

Junping Shi
College of William & Mary, USA
Jun Wang

The standing wave solutions of a coupled nonlinear Schrödinger equations with quadratic nonlinearities from Raman amplification of laser beam in a plasma are considered. For both the original three-wave system and a reduced two-wave system, the existence/nonexistence, continuous dependence and asymptotic behavior of positive ground state solutions are established. In particular, multiple positive standing wave solutions are found via a combination of variational and bifurcation methods for the attractive interaction case, which has not been found for the conventional nonlinear Schrödinger systems with cubic nonlinearities.

Blowup of the critical norm for a supercritical semilinear heat equation

Jin Takahashi
Tokyo Institute of Technology, Japan

We consider the scaling critical Lebesgue norm of blowup solutions to a semilinear heat equation (the Fujita equation) in an arbitrary smooth domain. In the Sobolev supercritical range, we show that the critical norm must be unbounded near the blowup time. The range is optimal in view of the existence of blowup solutions with bounded critical norm for the Sobolev critical case. This talk is based on a joint work with Professor Hideyuki Miura (Tokyo Institute of Technology).

An approximation by a Keller-Segel system for nonlocal Fokker-Planck equation in bounded one-dimensional domain

Yoshitaro Tanaka
Future University Hakodate, Japan
Hideki Murakawa

To describe biological phenomena such as cell migration and cell adhesion many evolutional equations in which a nonlocal interaction of convolution type with a suitable integral kernel is imposed as an advection term are proposed. It is well known that such nonlocal equations can reproduce various behaviors depending on the shape of the integral kernel. For example, the sign of the integral kernel determines whether the cell density aggregates towards the gradient of its density or not. These nonlocal evolutional equations are often difficult to analyze, and the method of analysis is developing. In the light of these background we approximate the nonlocal Fokker-Planck equation by the combination of a Keller-Segel system which is a typical local dynamics. We will show that the solution of the nonlocal Fokker-Planck equation with any even continuous integral kernel can be approximated as a singular limit of the Keller-Segel system by controlling parameters. Furthermore, motivated by pattern formations, we will explain the result and comparison of the linear stability analysis around the equilibrium point of both equations.

Linearized eigenvalue problems, Lamé equation and modified elliptic integral of the third kind

Tohru Wakasa
Kyushu Institute of Technology, Japan
Yasuhiro Miyamoto, Shuya Aizawa, Haruki Takemura

The linearized eigenvalue problems for stationary solutions of the 1-dimensional reaction-diffusion equations are considered. In the previous studies by the author, expressions of all eigenfunctions in terms of the Jacobi elliptic functions are obtained for typical cases of the bistable nonlinearity. We will show the recent results on the expressions of linearized eigenvalue problems for the other cases of nonlinearity. In particular, we will introduce the modified elliptic intergral of the third kind, which appear in the characteristic equations of eigenvalues for these cases of nonlinearity. We will also focus on the relationship with the Lam'e equation.

Special Session 42: Regularity Results for Solutions of Nonlinear Systems and Applications

Maria Alessandra Ragusa, Catania University, Italy
Christopher S. Goodrich, University of New South Wales, Australia
Andrea Scapellato, Catania University, Italy

In continuation with the session of the previous edition, AIMS TAIPEI 2018, in the session the organizers focus the attention in the vaste problem of regularity for minimizers of quadratic and nonquadratic growth functionals where the integrand is dependent on x, u, Du. Is pointed out that about the dependence on the variable x is assumed only that $A(x, u, p)$ is in the class VMO, Vanishing Mean Oscillation class, as a function of x. Namely, is not assumed the continuity of $A(x, u, p)$ with respect to x. Are considered both partial and global regularity of the minimizer u.

Wavelets and calculus of variations in image processing

Emanuel Guariglia
Wenzhou-Kean University, Italy

In this talk, we show the most recent techniques in image processing. In particular, we use wavelets as tools to define constraints and variational functionals as measures of complexity of signals. More in general, these techniques are often hybrid methods that combine multiresolution analysis and calculus of variations. On one hand, a relevant advantage in these hybrid techniques is given by the sparsity of the wavelet representation, which directly leads us to study the compression image in terms of approximation error. On the other hand, calculus of variations allows us to improve the process of image denoising.

On the problem of regularity for minimizers of nonquadratic growth functionals

Maria Alessandra Ragusa
University of Catania, Italy

Is showed a problem studied in cooperation with Professor Atsushi Tachikawa. We treat the regularity problem for minimizers $u(x)$ of quadratic and nonquadratic growth functional having integrand $A(x, u, Du)$. We point out that concerning the dependence on the variable x is assumed only that $A(x, u, p)$ is in the class of Vanishing Mean Oscillation class, as a function of x. Namely, is not assumed the continuity of $A(x, u, p)$ with respect to x. Are treated partial regularity and global regularity of the minimizer u.

New results on anisotropic singular Dirichlet problems

Andrea Scapellato
University of Catania, Italy

Aim of this talk is to show some recent results on anisotropic singular Dirichlet problems. In particular, we consider a nonlinear elliptic Dirichlet problem driven by the anisotropic (p, q)-Laplacian and with a nonparametric reaction that is the sum of a singular and of a superlinear terms. In addition, we consider a Dirichlet problem driven by the $(p(z), q(z))$-Laplacian and a reaction that is the sum of a parametric singular term and a superlinear perturbation.

Special Session 43: Control and Long Time Dynamics of Evolutionary Partial Differential Equations

Louis Tebou, Florida International University, USA
Luz de Teresa, Universidad Nacional Autónoma de México, México

This session is centered around problems of control and long time dynamics of evolutionary distributed systems. Its purpose is to stimulate more discussion on those topics in order to further our understanding of some of the challenges posed by systems that describe natural phenomena. Our focus will be on the rigorous mathematical analysis of such systems, and involves well-posedness, controllability, stabilization, as well as existence, regularity, and dimensions of underlying attractors.

Controllability of telegrapher's equations on graphs

Sergei Avdonin
University of Alaska Fairbanks, USA
Gazi Mahmud Alam, Nina Avdonina

In this paper we study exact controllability for telegrapher's equations on metric graphs. First we consider trees, i.e. graphs without cycles. For such graphs, Serge Nicaise derived exact controllability from stabilizability without an estimate of the controllability time ("Stabilization and asymptotic behavior of a generalized telegraph equation", Z. Angew. Math. Phys. 66 (2015), no. 6, 3221–3247). We give a direct proof of controllability and provide a sharp time estimate in the cases when control is supported at all or all but one of the boundary vertices. If control is supported on a smaller number of the boundary vertices, we prove that the system is not exactly controllable in any finite time interval. Then we consider telegrapher's equations on general compact graphs with control supported at some boundary and internal vertices. We prove the exact controllability of the system with the optimal number of actuators and estimate the sharp controllability time interval.

Analysis of a multiscale interface coupling between fluid flow in deformable porous media and a lumped hydraulic circuit

Matthew Boussard
North Carolina State University, USA
Lorena Bociu, Giovanna Guidoboni, Sarah Strickwerda

We consider a multiscale interface coupling between a 3D partial differential equations system modeling fluid flow in deformable porous media and an ordinary differential equation modeling a lumped hydraulic circuit that accounts for the global features of the problem. The main application of interest is tissue perfusion. We provide recent results related to well-posedness analysis for the above mentioned system.

Boundary null controllability of beam with structural damping

Julian Edward
Florida International University, USA
Sergei Avdonin, Sergei Ivanov

This paper is concerned with boundary controllability of the linear beam equation with a spectrally defined structural damping. For any $T > 0$, null controllability is proven in time T.

Carleman inequalities for wave equations with oscillatory boundary conditions and application

Ciprian Gal
Florida International University, USA
Louis T. Tebou

We consider the wave equation with mixed boundary conditions in a bounded domain; on one portion of the boundary, we have dynamic Wentzell boundary conditions, and on the other portion, we have homogeneous Dirichlet boundary conditions. First, using an appropriate geometric partition of the boundary, we prove some Carleman estimates for this system. Then, we apply those estimates to prove a boundary controllability result for a nonconservative model of the system under consideration. Our results improve earlier Carleman estimates and boundary controllability results established in the Dirichlet boundary conditions setting.

Wellposedness of multilayered structure interaction PDE systems

Pelin Guven Geredeli
Iowa State University, USA

In this work, we will consider a multilayered structure interaction PDE system which has been used in the literature to describe the blood transport process within vascular walls. Our main objective is to establish semigroup wellposedness of the coupled PDE system by way of nonstandard elimination of associated pressure terms via appropriate nonlocal operators and subsequent Babuska-Brezzi formulations.

Control and identification problem for the distributed parameter system on the graph-star

Karlygash Nurtazina
L.N. Gumilyov Eurasian National University, Kazakhstan

The talk presents a result of recovering a differential operator from its spectral characteristics in the space of piecewise smooth functions on a star graph. The operator of the boundary value problem has a singularity generated by the structure of the graph. Differential expression is defined on the interior parts of all the edges of the graph. At an internal node of the graph the Kirchhoff-Neumann matching condition arises. The spectral approach is based on the spectral properties of an elliptic operator: the analyticity of the Green's function for boundary value problem on the spectral parameter, spectral completeness and the bases property of the set of eigenfunctions in the space of square summable functions. The identifiability of a system is closely related to its controllability.

Preliminary stability result for novel serially-connected magnetizable piezoelectric and elastic beam designs

Ahmet Ozkan Ozer
Western Kentucky University, USA
Mohammad Akil, Serge Nicaise, Virginie Regnier

In this talk, preliminary stability results for two models are shared: (i) a transmission problem of an elastic-piezoelectric-elastic design with only one local damping acting on the longitudinal displacement of the center line of the piezoelectric material and (i) a transmission problem of a piezoelectric-elastic design with only one local damping acting on the elastic part. The problem (i) is shown to be exponentially stable whereas the stability of the problem (ii) strongly depends on the arithmetic nature of a quotient involving all the physical parameters of the system. Therefore, exponentially or polynomial stabilities can be obtained. In the case of the polynomial stability, the decay rate is provided in terms of a measure of irrationality of the quotient. Several open problems and future directions will be discussed.

Controllability of a model system for strong interaction between internal solitary waves

Ademir Pazoto
Federal University of Rio de Janeiro, Brazil
J. A. Barcena-Petisco, R. Capistrano-Filho, F. Gallego and S. Guerrero

In this talk, we present local controllability results for a nonlinear coupled system of two Korteweg-de Vries equations posed on a bounded interval. The model was derived by Gear and Grimshaw to describe the interactions of two dimensional, long, internal gravity waves propagation in a stratified fluid.
We address both, the internal and the boundary control problems. Our approach consists mainly in proving the controllability of the linearized system, which is done by by using a classical duality approach. A fixed point argument and a local inversion theorem are then applied to get the results for the nonlinear system.

Special Session 45: Lie Symmetries, Conservation Laws and Other Approaches in Solving Nonlinear Differential Equations

Chaudry Masood Khalique, North-West University, South Africa
Wen-Xiu Ma, University of South Florida, USA
Maria Luz Gandarias, University of Cadiz, Spain

This session is devoted to research areas that are related to nonlinear differential equations and their applications in science and engineering. The main focus of this special session is on the Lie symmetry analysis, conservation laws and their applications to ordinary and partial differential equations. Other approaches in finding exact solutions to nonlinear differential equations will also be discussed. This includes, but not limited to, asymptotic analysis methodologies, the simplest equation method, the multiple exp-function method, inverse scattering transform techniques, the upper-lower solutions method, the Hirota method, and others.

Soliton solutions, travelling wave solutions and conserved quantities for a three-dimensional soliton equation in plasma physics

Oke Adeyemo
North-West University, South Africa

In this talk, we consider the study and analysis carried out on a three-dimensional soliton equation, which has applications in plasma physics and other nonlinear sciences such as fluid mechanics, atomic physics, biophysics, nonlinear optics, classical and quantum fields theories. In the real sense of it, solitons as well as solitary waves have been discovered in numerous situations and often dominate long-time behaviour. Thus, the Lie group method is applied to obtain the symmetries of the equation. These are consequently used to obtain various solutions of interests. Moreover, conservation laws of the soliton equation are derived.

Heat transfer study of electro kinetically modulated micropolar nanofluid flow for Cu and Ag nanoparticles in a propagating microchannel

Noreen Akbar
National University of Sciences and Technology, Pakistan

In this presentation i will discuss the theoretical investigation on the effect of natural convection on the flow of micropolar nanofluid through a symmetric channel. For a fixed solid volume fraction, properties of two different types of nanofluid will be discussed by considering two different nanoparticles (i.e. Cu and Ag Water nanoparticles). The investigation points revaluation of micro rotation associated with nanofluid particles will be presented for thermal profile of the fluid flow. The impact of micropolar fluid parameter on pressure rise per wavelength and pressure gradient will be also presented.

A front fixing Crank-Nicolson finite deference for the american put options model

Zakaria Idriss Ali
University of South Africa, South Africa
Minyahil Abebe Abera

In this paper, we introduce a novel approach to solving the American put options pricing model by hugely relying on a front-fixing Crank-Nicolson finite difference method. Since the American put option pricing model is a widely used financial model for valuing an option with the right to sell an underlying asset at a fated price which generally decided in advance. The method we proposed here, solves the problem of early exercise by introducing a front-fixing technique that permits for efficient and accurate valuation of an American put option. As in the comparison to other approaches in the existing literature, we can assert that this method is stable, accurate, and efficient. The results that we obtained here from the numerical experiments demonstrate not only the efficacy of the proposed method but also in accurately pricing American put options with a stable scheme.

New conserved integrals, invariants, symmetries and Casimirs of radial compressi ble fluid flow in $n > 1$ dimensions

Stephen Anco
Brock University, Canada
Amar Dar, Sara Seifi, Thomas Wolf

Conserved integrals and invariants (advected scalars) are studied for the equations of radial compressible fluid flow in $n > 1$ dimensions. Apart from entropy, which is a well-know invariant, three additional invariants are found from an explicit determination of invariants up to first-order. One holds for a general equation of state (EOS), and the two others hold only for entropic equations of state.

A recursion operator on invariants is presented, which produces two hierarchies of higher-order invariants. Each invariant yields a corresponding integral invariant, describing an advected conserved integral on transported radial domains. In addition, a direct determination of kinematic conserved densities uncovers two "hidden" non-advected conserved integrals: one describes enthalpy-flux, holding for barotropic EOS; the other describes entropy-weighted energy, holding for entropic EOS. A further explicit determination of a class of first-order conserved densities shows that the corresponding non-kinematic conserved integrals on transported radial domains are equivalent to integral invariants, modulo trivial densities.
One of the hierarchies of invariants is proved to consist of Hamiltonian Casimirs. The second hierarchy, which holds only for an entropic EOS, is explicitly shown to comprise non-Casimirs. Through the Hamiltonian structure of the radial fluid flow equations, these non-Casimir invariants yield a corresponding hierarchy of generalized symmetries. The first-order symmetries are shown to generate a non-abelian Lie algebra. Two new kinematic conserved integrals are likewise shown to yield additional first-order generalized symmetries holding for a barotropic EOS and an entropic EOS. These symmetries produce an explicit transformation group acting on solutions of the fluid equations.

Invariants for a system of two linear hyperbolic equations by complex methods

Adnan Aslam
National University of Sciences and Technology, Pakistan
F. M. Mahomed, A. Qadir, M. Safdar

Invariants of symmetry groups under transformations of dependent and independent variables lead to simplification of differential equations and their exact solutions if solutions of the transformed equations are known. Though Lie had developed his Symmetry Analysis for complex functions of complex variables, he did not explicitly use complex analyticity. We developed Complex Symmetry Analysis in which we make explicit use of the Cauchy-Riemann equations and find that one can solve systems of differential equations by it for equations not readily amenable to the usual real methods. We show that, via complex methods, one can deduce invariants that are not readily obtainable by real methods

Lie symmetries for coupled differential equations

Nicoleta Bila
Fayetteville State University, USA

The aim of this talk is to emphasize the existence of special relations between the Lie symmetries shared by specific partial differential equations and ordinary differential equations. Consequently, coupled manifolds may be considered due to their common infinitesimal generators of their shared Lie symmetry groups. As an example, the nonlinear differential equations represented by the Tzitzeica surface partial differential equation and the Tzizteica curve ordinary differential equation arising in differential geometry are considered as examples.

Conservation laws and explicit solutions of the Yajima-Oikawa-Newell system

Priscila da Silva
Universidade Federal do ABC, Brazil

In this talk we consider a novel integrable long wave-short wave model and study it from the point of view of symmetries and conservation laws. Moreover, from its quadrature we obtain explicit 3-parameter conservative, non-conservative and periodic traveling wave solutions.

Structural properties of an equation describing pseudospherical surfaces

Igor Freire
UFSCar, Brazil

In this talk we study Lie symmetries and conservation laws for an integrable equation describing pseudospherical surfaces. As an application, we use the Lie symmetries to construct explicit metrics for the pseudospherical surface determined by the solutions of the equation.

Finding symmetry-invariant solutions of partial differential equations by application of a multi-reduction conservation law method

Maria Luz Gandarias
University of Cadiz, Spain
Stephen Anco

A powerful application of symmetries is finding symmetry-invariant solutions of nonlinear differential equations. These solutions satisfy a reduced differential equation with one fewer independent variable. It is well known that a double reduction occurs whenever the starting nonlinear differential equation possesses a conservation law that is invariant with respect to the symmetry.
Recent work has developed a broad generalization of the double-reduction method by considering the space of invariant conservation laws with respect to a given symmetry. The generalization is able to reduce a nonlinear partial differential equation (PDE) in n variables to an ODE with $m - n + 2$ first integrals where m is the dimension of the space of invariant conservation laws.

In the present talk, we apply this general multi-reduction method to obtain travelling wave solutions of some physically interesting PDEs. An interesting side result is that we show how conservation laws that explicitly contain the independent variables can nevertheless be used under certain conditions to obtain a reduction.

Novel approach of nanofluids in medical science

Bilal Habib
National Institute of Health, Pakistan

In medical science the use of nanofluids are emerging trend. In this study we isolated Candida (Yeast) that causes many infections in humans. It is the main cause of biofilm formation on implants specially catheters etc that may contribute in the Fungal resistance to antifungals. In this study, samples were collected and new nanostratigies applied to overcome the drugs resistance.. The capacity of biofilm production by Candida species was very high. Almost all species produced biofilm. The strongest biofilms were produced by Candida tropicalis. . We performed antimicrobial sensitivity assay by using different antifungals drugs. Moreover, nanoscale strategies were also applied; Gold nanoparticles were synthesized with drug conjugates and these were tested against these fungal isolates. Gold nanoparticles and drug-Gold nanoparticles conjugates were anti-Candida agents in our study. Gold nanoparticles and drug-conjugated Gold nanoparticles inhibited biofilm formation significantly.

Instability and pinch-off of a slender fluid thread with variable surface tension

Muhammad Hameed
University of South Carolina, USA

The Instability and breakup of a fluid thread of low viscosity immersed in highly viscous exterior fluid at low Reynolds number is studied. With an aim to better understand the pinch-off dynamics, we use slender body theory, numerical simulations and experimental studies to investigate the effect of surfactant on the necking and eventual breakup. Evolution equations are derived using long wavelength approximations. These one dimensional partial differential equations are solved numerically for given initial interface and surfactant concentration. It is found that the presence of surfactant at the interface retards the pinch-off process. Surface diffusion and solubility of surfactant are found to have significant influence on the instability of the thread. It is found that greater diffusion of surfactant causes the thread to pinch faster. Results of the long wavelength model are also compared against the numerical simulations of the full problem. The solution of the full problem shows similar behavior to the simplified model. The experimental results support the prediction of theoretical model that the presence of surfactant slows down the pinch-off process.

Partial differential equations: The heat and Burgers' equation

Sameerah Jamal
University of the Witwatersrand, South Africa
U. Obaidullah, R. Maphanga

We present some recent advances of the applications of one-parameter Lie group transformations of famous partial differential equations. In particular, we discuss Burgers' equation and the classical heat equation, which are often the benchmarks in the study of differential equations.
In former case, we exploit the link between the equation and a recursion operator and in the latter we show how a fundamental solution may be manipulated to gain further insights.

A study of a generalized nonlinear advection-diffusion equation

Chaudry Masood Khalique
North-West University, South Africa

In this talk, we investigate a nonlinear partial differential equation of fluid mechanics, namely, the generalized nonlinear advection-diffusion equation, which describes the movement of a buoyancy-driven plume in an inclined porous medium. We shall consider three cases of n and in each case, we shall provide symmetry reductions and group-invariant solutions based on the optimal systems of one-dimensional subalgebras. Furthermore, we determine the conserved vectors by employing the multiplier approach.

Binary Darboux transformation for nonlocal nonlinear Schrödinger equations

Wen-Xiu Ma
University of South Florida, USA

A binary Darboux transformation is presented for nonlocal nonlinear Schrödinger equations. The key is to use eigenfunctions and adjoint eigenfunctions and a reduction leads to the classical Darboux transformation. An application starting from zero potentials generates soliton solutions.

Integrable Hamiltonian equations and matrix Lie algebras

Wen-Xiu Ma
University of South Florida, USA

We will talk about how to construct integrable Hamiltonian equations through matrix spectral problems. Hamiltonian structures are furnished by the trace identity and recursion operators are constructed from solutions to stationary zero curvature equations, which generate infinitely many symmetries and conservation laws. Illuminating examples are given.

Noether symmetries of a generalized coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry

Sivenathi Mbusi
North-West University, South Africa

In this talk, we carry out a complete Noether symmetry analysis of a generalized coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry. It is shown that several cases transpire for which the Noether symmetries exist. Moreover, we derive conservation laws connected with the admitted Noether symmetries. Furthermore, we fleetingly discuss the physical interpretation of the these conserved vectors.

A study of $(3+1)$-dimensional generalized KP-Boussinesq equation

Letlhogonolo Moleleki
North West University, South Africa

In this talk we investigate the (3 + 1)-dimensional generalized KP-Boussinesq equation, which was formulated recently as a generalization of (3 + 1)-dimensional KP equation. Using Lie group analysis we perform some symmetry reductions and obtain travelling wave solutions. We also present conservation laws of the underlying equation.

Finding Lie symmetry analysis of a system of equations describing the tumour invasion model

Basetsana Ntsime
University of South Africa, South Africa

In this study, we present the mathematical model of a tumor invasion model described by [1, 2]

$$\left.\begin{aligned}\frac{\partial\theta}{\partial t}=d_\theta\frac{\partial^2\theta}{\partial x^2}-\gamma\frac{\partial}{\partial x}(\theta\frac{\partial v}{\partial x})+\mu_1\theta(1-\theta-v),\\ \frac{\partial v}{\partial t}=-\eta m v+\mu_2 v(1-\theta-v),\\ \frac{\partial m}{\partial t}=d_m\frac{\partial^2 m}{\partial x^2}+\alpha\theta-\beta m,\end{aligned}\right\}$$

where d_θ, d_m, μ_1, μ_2, η, α and β are positive constants and θ, is the density tumour cells, v, is the extra cellular matrix density and m is the degradation enzymes θ, v and m depend on position and the time on a smooth bounded domain. The purpose of this paper is to apply the techniques of Lie symmetry to the model and present the group invariant solutions of the system of second order partial differential equations describing a tumor invasion model. The similarity solutions obtained, are presented in the general form.

References

[1] A. Kubo, A. and Y. Miyata, (2019) International Journal of Mathematical and Computational Methods, 4, 10–16.

[2] A. Kubo, A. and Y. Miyata, (2015) WSEAS Transactions on Biology and Biomedicine, 15, 101–111.

A study of generalized $(2+1)$-dimensional equal-width partial differential equation of engineering

Karabo Plaatjie
North-West University, South Africa
Chaudry Masood Khalique

In this talk we study the generalized (2 + 1)-dimensional equal-width equation which arises in various fields of science. Using Lie symmetry analysis along with power series expansion and Weierstrass methods, we construct closed-form solutions of this equation. Moreover, we derive the conserved vectors of the underlying equation by utilizing the multiplier method and Noether's theorem.

Analysis of a dynamical system describing the evolution of HIV/AIDS in a population

Jean Yves Semegni
North-West University, South Africa
Siba Vyambuera, Nizar Marcus, Kazeem Oare Okosun, Peter Joseph Witbooi,Gbenga Jacob Abiodun

We exhibit a Lyapunov function to prove the global stability of the disease-free equilibrium, then we provide sensitivity analysis of certain parameters of the system. Finally we provide an optimal control strategy to mitigate the spread of the disease among the population.

Capillarity and partially saturated porous material dynamics

Javed Siddique
Pennsylvania State University, York Campus, USA
Daniel M. Anderson

We explore the role of partial saturation and accompanying variations in permeability and capillary pressure in capillary rise dynamics into a rigid porous material. Experiments show a deviation from the classical Washburn model dynamics after early times and our aim in this work is to investigate this deviation. We use multiphase mixture theory for modeling to capture a single framework to understand the complex dynamics. We hope to compare the numerically computed results of our model to experimental data and other related literature.

Special Session 47: Singular Limits in Elliptic and Parabolic PDEs

Silvia Cingolani, Università degli Studi di Bari, Italy
Manuel del Pino, University of Bath, England
Serena Dipierro, University of Western Australia, Australia

Singular perturbation problems appear in a number of concrete phenomena in physics, biology, material sciences, etc, and produced in the recent years a variety of mathematical techniques and beautiful results. This session aims at gathering the world leading experts in this field, with specific focus on elliptic and parabolic PDEs.

Degenerate elliptic equations with nonlinear Hamiltonians: Existence results

Isabeau Birindelli
Sapienza Università di Roma, Italy
Galise, Rodriguez

In this talk we will present existence results for degenerate elliptic equations with a nonlinear gradient term of the form $F(x, D^2u) + H(Du) = f(x)$. in bounded uniformly convex domains Ω. I willpresent sufficient conditions for the existence and uniqueness of solutions in terms of the size of Ω, of the forcing term f and of H. The results apply to a wide class of equations, since very little is required from the principal part i.e. the degenerate elliptic operators. In particular the operator could be linear, or a weighted partial trace operators or e.g. the homogeneous Monge-Ampére operator.

Taylor dispersion in the non-cutoff Boltzmann equation on the whole space

Michele Dolce
EPFL, Switzerland
Jacob Bedrossian, Michele Coti Zelati

Consider the non-cutoff Boltzmann equation with soft potentials on the whole space in the large Knudsen number Kn $\gg 1$ regime, describing for instance molecules in the upper atmosphere. We study quantitative stability properties of a global Maxwellian background. Specifically, we prove that for initial data sufficiently small (independent of Kn), the solution displays several dynamics caused by the phase mixing/dispersive effects of the transport operator $v \cdot \nabla_x$ and its interplay with the singular collision operator. One consequence is the *Taylor dispersion*, showing that the perturbation decay on a time-scale $O(1)$. This is a faster relaxation time-scale compared to the $O(\mathrm{Kn})$ expected when only the collision operator is present. Additionally, we prove almost-uniform phase mixing estimates. For macroscopic quantities as the density ρ, these bounds imply almost-uniform-in-v decay of $(t\nabla_x)\rho$ in L^∞_x due to Landau damping and dispersive decay.

Multiple solutions to the nonlocal Liouville equation in $\mathbb{R}$

Antonio J. Fernández
Universidad Autónoma de Madrid, Spain
Luca Battaglia, Matteo Cozzi, Angela Pistoia

We construct multiple solutions to the Liouville type equation

$$(-\Delta)^{\frac{1}{2}} u = k(x)e^u, \quad \text{in } \mathbb{R}$$

More precisely, for k of the form $k(x) = 1 + \epsilon\kappa(x)$ with $\epsilon \in (0,1)$ small and $\kappa \in C^{1,\alpha}(\mathbb{R}) \cap L^\infty(\mathbb{R})$ for some $\alpha > 0$, we prove the existence of multiple solutions to the above equation bifurcating from the so-called Aubin-Talenti bubbles. These solutions provide examples of flat metrics in the half-plane with prescribed geodesic curvature $k(x)$ on its boundary. Moreover, they imply the existence of multiple ground state soliton solutions for the Calogero-Moser derivative NLS. The talk is based on joint works with L. Battaglia (Roma), M. Cozzi (Milano) and A. Pistoia (Roma).

Existence of small positive solutions to the nonlinear Schrödinger equation

Norihisa Ikoma
Keio University, Japan
Shinji Adachi, Tatsuya Watanabe

This talk is concerned with the existence of solutions to the nonlinear Schrödinger equation $-\Delta u + V(x)u = \lambda f(u)$ in $\mathbb{R}^N$. Here $\lambda > 0$ is a parameter and we only require the nonlinearity f to satisfy conditions around 0. Our results are the existence of positive solutions when λ is sufficiently large and the asymptotic behavior of positive solutions as $\lambda \to \infty$. This is joint work with Shinji Adachi (Shizuoka University) and Tatsuya Watanabe (Kyoto Sangyo University).

On nondegeneracy and stability in the one-phase singular perturbation problem

Nikola Kamburov
Pontificia Universidad Catolica de Chile, Chile

In this talk I will present some recent results on the singular perturbation problem that formally approximates the classical one-phase free boundary problem. We define a natural density condition on the transition layers of the solutions that guarantees the uniform nondegeneracy property is satisfied and preserved in the limit. We then apply our result to the problem of classifying global stable solutions of the underlying semilinear problem and we show that those have flat level sets in dimensions $n \leq 4$, provided the density condition is fulfilled.

Clustering phenomena in low dimensions for a boundary Yamabe problem

Giusi Vaira
University of Bari Aldo Moro, Italy
Sergio Cruz-Blazquez, Angela Pistoia

We consider the classical geometric problem of prescribing scalar and boundary mean curvature via conformal deformation of the metric on a n-dimensional compact Riemannian manifold. We deal with the case of negative scalar curvature and positive boundary mean curvature. It is known that if n=3 all the blowup points are isolated and simple. In this work we prove that this is not true anymore in low dimensions (that is n=4, 5, 6, 7). In particular, we construct a solution with a clustering blowup boundary point (i.e. non-isolated), which is non-umbilic and minimizes the norm of the trace-free second fundamental form of the boundary.

Asymptotic behavior of solutions to the Yamabe equation in low dimensions

Lei Zhang
University of Florida, USA
Zhengchao Han, Jingang Xiong

In this talk I will report recent progress on the Yamabe equation defined either on a punctured disk of a smooth manifold or outside a compact subset of $\mathbb{R}^n$ with an asymptotically flat metric. What we are interested in is the behavior of solutions near the singularity. It is well known that the study of the Yamabe equation is sensitive to the dimension of the manifold and is closely related to the Positive Mass Theorem. In my recent joint works with Jingang Xiong (Beijing Normal University) and Zhengchao Han (Rutgers) we proved dimension-sensitive results and our work showed connection to other problems.

Singularity formation for the Landau-Lifshitz-Gilbert equation in dimension two

Yifu Zhou
Johns Hopkins University, USA
Juncheng Wei, Qidi Zhang

Landau-Lifshitz-Gilbert equation (LLG), which models the evolution of spin fields in continuum ferromagnetism, can be viewed as a coupling between the harmonic map heat flow and the Schrödinger map flow. In this talk, we shall report some recent gluing construction of finite-time singularities for LLG in dimension two. To overcome the difficulties caused by the dispersion, technical ingredients such as distorted Fourier transform and sub-Gaussian estimates are employed. This is based on a joint work with J. Wei and Q. Zhang.

On the stability of radial solutions to an anisotropic Ginzburg-Landau equation

Andres Zuniga
University of O'Higgins, Chile
Xavier Lamy

We study the linear stability of entire radial solutions $u(re^{i\theta}) = f(r)e^{i\theta}$, with positive increasing profile $f(r)$, to the anisotropic Ginzburg-Landau equation $-\Delta u - \delta(\partial_x + i\partial_y)^2\bar{u} = (1 - |u|^2)u$, which arises in various liquid crystal models. In the isotropic case $\delta = 0$, Mironescu showed that such solution is nondegenerately stable. We prove stability of this radial solution in the range $\delta \in (\delta_1, 0]$ and instability outside this range. In strong contrast with the isotropic case, stability with respect to higher Fourier modes is not a direct consequence of stability with respect to lower Fourier modes. In particular, in the case where the anisotropy parameter is close to -1, lower modes are stable and yet higher modes are unstable.

Special Session 48: Mathematical Modeling and Optimization Techniques

Narinder Singh, Punjabi University, India
Mehar Chand, Baba Farid College, India
Satya Bir Singh, Punjabi University, India
Islamudin Rahim Khan, Punjabi University, India

The special session aims to bring together international experts working on mathematical models from various fields. These fields include control theory, material science, medicine, and aerospace, to name a few. Despite the different applications of mathematical models, the underlying concepts related to the modelling of complex processes and systems share many common features. The same holds for the application of scientific computing methods for the numerical implementation of the mathematical models. These methods nowadays constitute an integral part of the research task in applied fields. Topics such as parallel computing on high performance computer systems and the use of shared memory architectures are vital e.g. for the solution of partial differential equations arising in various fields. Rapidly developing technologies like machine learning methods became indispensable in data analysis. It is believed that bringing together experts from different fields will lead to new innovative ideas and create synergies between the scientific research topics.
List of Topics

(1) Mathematical and computer modeling of physical and biological processes
(2) Optimization, control and numerical methods for ODEs and PDEs
(3) Computer technologies and data analysis in engineering and bioinformatics
(4) High performance scientific computing for physical and biomedical applications

Examining B2C marketing strategies of IT companies in social media

Ebenezer Amakeh
Punjabi University, India

The rise of social media has transformed the way companies interact with customers, particularly in the business-to-consumer (B2C) context. Information technology (IT) companies are no exception to this trend, with many leveraging social media to promote their products and services to consumers. This research paper examines the B2C marketing strategies employed by IT companies in social media. The paper begins with a literature review of social media marketing and B2C marketing strategies, with a particular focus on IT companies. The review explores the various social media platforms that are commonly used by IT companies, the types of content that are typically shared, and the goals and objectives that underlie B2C marketing strategies. Overall, this research paper provides a comprehensive overview of the B2C marketing strategies employed by IT companies in social media. By analyzing the literature and presenting real-world examples, the paper provides insights into the most effective tactics for promoting IT products and services to consumers in the digital age. The findings of this research will be of interest to marketers, business leaders, and anyone interested in the intersection of technology and marketing.

Impacts of migration on family members left behind: The case of Damboya Woreda, Kembata Tembaro Zone South Nation Nationalities and People Region-Ethiopia

Teshale Bonje
Punjabi University, India

Migration studies are an interdisciplinary study with in a broader field of development studies due to its association with debates about development and underdevelopment. Irregular migration is one the most significant topic in migration studies and its complex nature has attracted many scholars around the world. In Ethiopia, large scale cross borders migration is increased, especially in the last decades and became more complex. Irregular migration has various impacts on the family left behind;however, there is dearth of information on the issue. Therefore, this investigation examines the impact of cross-borders irregular migration from kembata tembaro zone in southern Ethiopia. To achieve the study's objectives qualitative method will be employed as it help to gain deeper understanding of the topic in the real/ local context. The study will use data collection instruments like, in-depth interview and focus group discussion.
The primary information will be collected from purposively selected migrant's family members. Finally, thematic analysis will be used to analyse the data. The study will use the New Economics of Labour Migration Theory as a base to explore the impact of irregular migration on migrant's family members left at home country.

Solutions of truncated M-fractional differential equations with artificial neural network approach

Mehar Chand
Baba Farid College, India

In the present paper, an Artificial Neural Network (ANN) technique is developed to find the solution of Truncated M-Fractional differential equations (TFDE). Compared to integer order differential equations, (TFDE) has the advantage that it can better describe sometimes various real-world application problems of physical systems. Here we have employed multi-layer feed-forward neural architecture and error back propagation algorithm with unsupervised learning for minimizing the error function and modification of the parameters (weights and biases). Combining the initial conditions with the ANN output gives us a suitable approximate solution of (TFDE). To prove the applicability of the concept, some illustrative examples are provided to demonstrate the precision and effectiveness of this method. Comparison of the present results with other available results by traditional methods shows a close match which establishes the correctness and accuracy of this method.

Determinants of expenditure budget implementation in Ethipian Southern state public sector

Abera Elias
Andhra Univerisity, India
Petti Permanandam

The aim of this study is to investigare factors that affect expenditure budget implementation in Southern state public sector. The study will employ both quantitative and qualutative approaches and conducts through explanatory research design. The study's target population includes 1,341 directors and experts from 50 the state public sectors. Stratified, purpposive and simple random sampling techniques will be applied to choose the sample of 15 public sectors and 325 respondets for questionnaire. In addition, 15 key informants will be selected from Southern state's Finance Bureau and used to cheek the reliability of collected data.SPSs version 26 was employed to process and analyze the collected data using descriptive and inferential statistics. According to this study, concurrently, variables (Expenditure budget preparation and approval, Expenditure control, expenditure monitoring, human resource competency, legislative oversight and subsequent activities) all had a significant effect on the state public sector's annual expenditure budget implementation. An adjusted R square values of 58.2 percent indicates, the explanatory variables had statistically significant effect on the state public sector's anual expenditure budget implementation. Exclusively human resource competency with probablity value of 0.056, greater than 0.005 had an insignificant effect on state public sector's anual expenditure budget implementation. Finally, the researcher recommends that, the state government set quarterly expenditure budget targets for public sectors to avoid year-end spending accumulation

Empirical analysis of customers satisfaction regarding banking services

Obaidullah Hotak
Punjabi University, India

Today, USA banks are working under intense competition from the new generation banks and foreign banks. This is due to the advancement of information technology, which dramatically changes the banking industry. By this, customers are becoming very demanding. The extensive use of technology enables banks to adequately satisfy customers' requirements by adding a degree of variation in the services provided by the banks with the emergence of Payments Banks services. So, it becomes necessary to study the nature, features, and extent of Payment Banks' services and their impact on operational performance and service quality. Despite the increasing importance of Payment Banks' services, the research about Payment Banks in the USA context has been limited. Most of the research was conducted on national and international bases, but awareness and satisfaction of USA customers have not been studied much. In this reference, the present study attempted to analyze the awareness and satisfaction level of customers towards Payments Banks in the USA.

Role of CPM technique for managing a project and to determine the time schedule in project management with practical illustration

Islamudin Rahim Khan
Punjabi University, India

Many techniques exist for managing a project and to determine the time schedule for a project. Some of them as critical path, programming and review analysis method. the method and success of quantitative decision-making approaches in project management through the use of critical path method as regards project duration and critical activities of a project were discussed. The presentation concludes that the critical path method is more effective when the projects end time is certain. The presentation therefore recommends that to achieve maximum operative efficiency in utilizing resources in project management prior to commencing the execution of a project all required resources to cover every activity must be assembled and prioritized to eliminate interruptions that could bring delay and unnecessary cost implications. Again, policies focused on improving project execution should be implemented in developing nations to minimize the many cases of failed and delayed projects. Subsequently stakeholders in

projects must be given the right under the law to prosecute the government its agencies and the executors of projects for failure to execute a project on schedule.

An enhanced coot optimizer for multi-disciplinary optimization applications

Narinder Singh
Punjabi University, India

The coot optimizer is basically inspired by natural behaviour of the search engine (birds or coots). This optimizer originally was developed for tackling large-scale optimization applications. However, it faces lots of demerits like slow and premature convergence, due to which it gets trapped in local optima. To overcome these demerits the levy flight strategy has been integrated with coot optimizer, its called LF-COOT. The random walk phase of the levy flight is so familiar due to their high jumps. It helps in ignoring the local optima and enhancing the location of each search agent during the optimization process. Lévy flight also plays an important role in making the balance between exploration and exploitation phases. To evaluate the performance of the LFCOOT have been considered into account IEEE CEC' 2017 and CEC' 2020 standards benchmark suites. In addition, its accuracy has also been verified on internet vehicle routing and some real-world engineering applications. Experimental outcomes reveal that the LFCOOT is able to offer the finest quality of solutions than the competitors.

On certain integrals involving (p, k)-Mittag-Leffler function

Harbhajan Singh
Baba Farid College, India
Mehar Chand

In the present paper we establish certain integral formulae involving a new generalization of Mittag-Leffler function, which are expressed in terms of generalized Wright function and hypergeometric function. Further some interesting special cases of our main findings are also developed.

Advanced autonomous groups strategy based optimization algorithms for real world applications

Satya Bir Singh
Punjabi University, India
Narinder Singh

Multi-disciplinary engineering applications are widespread in numerous research disciplines. Lots of optimizers have been proposed in the literature to address these types of problems. However, the optimizer's performance considerably decreases with the growth in the complexity and other scale issues. Huge ranges of optimizers have been proposed to address the complex engineering application in the last few decades. Present study deals with enhancement strategies of Artificial Gorilla Troops Optimizer (GTO) based on divers' gorilla's groups. These techniques are named as AGTO, AGT-1 and AGTO-2. The objective is to improve the algorithm by avoiding the function from trapping in local minima and premature convergence in dealing with higher dimensional functions. To evaluate the superiority of the proposed strategies, 41 CEC standard test suites and six high dimensional multi-disciplinary engineering applications have been considered and its performance have been tested with the results obtained with various state- of -art- optimizers in terms of faster convergence rate and escaping in local minima etc. Simulated results establishes that splitting gorillas in groups and permitting them to have distinct separate and social intelligence can enhance the superiority of GTO significantly.

Special Session 50: Nonlinear Elliptic PDEs: Analysis and Computations

Florin Catrina, St. John's University, USA
Zhi-Qiang Wang, Utah State University, USA
Jianxin Zhou, Texas A & M University, USA

This group will focus on nonlinear equations and systems involving the weighted p-Laplacian operator, the fractional Laplacian and more general nonlocal operators, in both Euclidean spaces and on Riemannian manifolds. It will highlight new developments on the existence and qualitative properties of solutions, such as symmetry, monotonicity, uniqueness, and regularity. The group will also discuss broad applications of these equations to various branches of cciences, such as physics, chemistry, biology, probability, and finance.

A generalized radial Brezis-Nirenberg problem

Soledad Benguria Andrews
University of Wisconsin Madison, USA
Rafael Benguria

Given $n \in (2,4)$, we study the existence, nonexistence and uniqueness of positive solutions $u \in H_0^1(0,R)$ of

$$-u''(x) - (n-1)\frac{a'(x)}{a(x)}u'(x) = \lambda u(x) + u(x)^p,$$

with boundary condition $u'(0) = u(R) = 0$, under rather general conditions on $a(x)$. Here, as in the original problem, $p = (n+2)/(n-2)$ is the critical Sobolev exponent.
This is a joint work with Rafael Benguria, PUC, Santiago, Chile.

An energy conservation law and applications to PDE

Florin Catrina
St. John's University, USA

We discuss implications of an energy conservation identity on properties of solutions for some second order ODE's with Hamiltonian structure. Variational proofs of existence of solutions for semilinear elliptic PDE's are often based on compactness of embeddings between the appropriate functional spaces. In the case of critical or super-critical nonlinearities, the loss of compactness is manifested in the concentration of minimizing sequences at critical points or at singularities of the potential.
This identity is able to capture the loss of compactness for radial functions in some special cases of rotationally symmetric PDE's.

Heterogeneous radially symmetric semilinear elliptic equations with critical nonlinearities in $\mathbb{R}^2$

David Costa
UNLV, USA
Hossein Tehrani

This is a continuation of our multiplicity results for elliptic PDEs with heterogeneous critical nonlinearities á la Trudinger-Moser. Our results are ultimately connected to the increase in the threshold of compactness that can be achieved in the Trudinger-Moser inequality under radial symmetry and in the presence of a rapidly vanishing radial weight function in $H^1_{0,r}(B_R)$. They are generalizations to dimension $N = 2$ of the 1981 seminal result of W.-M.-Ni in dimension $N \geq 3$. In particular, we can prove the existence of a positive, a negative, and a sign-changing solution for the equation

$$-\Delta u = a(r)h(u)e^{\alpha u^2} \ \ in \ \ B(R_1, R_2) \ \ on \ \partial B(R_1, R_2)$$

without requiring the oddness assumption on the nonlinearity and, in fact, we can prove the existence of *Infinitely Many Sign-Changing Solutions* in the case of a ball. All of our results are obtained without any growth restriction on the lower-order terms of the nonlinearity and, to our knowledge, generalize most existing results in the literature of such problems.

Singularity formation for the Keller-Segel system in the plane

Manuel del Pino
University of Bath, England

The classical model for chemotaxis is the planar Keller-Segel system

$$u_t = \Delta u - \nabla \cdot (u\nabla v), \quad v(\cdot,t) = \frac{1}{2\pi}\log 1|\cdot| * u(\cdot,t).$$

in $\mathbb{R}^2 \times (0,\infty)$. A blowup of finite mass solutions is expected to take place by aggregation, which is a concentration of bubbling type, common to many geometric flows. We build with precise profiles solutions in the critical-mass case 8π, in which blowup in infinite time takes place. We establish the stability of the phenomenon detected under arbitrary mass-preserving small perturbations and discuss new constructions in the finite time blowup scenario. This is joint work with Juan Davila, Monica Musso, Federico Buseghin and Juncheng Wei.

Homotopy methods for solving nonlinear PDEs

Wenrui Hao
Penn State University, USA

Many nonlinear PDE systems have emerged from the fields of engineering and biology, and have garnered the attention of research scientists interested in studying their complex solution structures, such as pattern formation. In this talk, I will introduce several methods for computing the multiple solutions of such nonlinear PDEs. Specifically, I will discuss the use of the homotopy continuation technique for computing the multiple steady states of nonlinear differential equations and for exploring the relationship between the number of steady states and the system's parameters. Additionally, I will provide examples of benchmark problems that can be used to illustrate these concepts.

Existence and nonexistence of stable solutions to a fractional Hardy-Hénon equation

Norihisa Ikoma
Keio University, Japan
Shoichi Hasegawa, Tatsuki Kawakami

This talk is devoted to the existence and nonexitence of stable solutions to a fractional Hardy-Hénon equation $(-\Delta)^s u = |x|^l |u|^{p-1} u$ in $\mathbf{R}^N$. For this equation, we show the nonexistence of stable solutions when p is subcritical in the sense of Joseph-Lundgren and the existence of a family of stable solutions when p is critical or supercritical in the sense of Joseph-Lundgren. In addition, we reveal some properties of the family of stable solutions as well as the multiple existence of Joseph-Lundgren critical exponents for some range of s, N and l. This is based on joint work with Shoichi Hasegawa (Waseda University) and Tatsuki Kawakami (Ryukoku University).

A partial Newton-correction method for multiple fixed points of nonlinear differential operator by Legendre-Gauss-Lobatto pseudospectral method

Zhaoxiang Li
Shanghai Normal University, Peoples Republic of China
Jianxin Zhou

In this talk, we propose a partial Newton-correction method (PNCM) to find multiple fixed points of nonlinear differential operators. First a new augmented singular transform is developed to form a barrier surrounding previously found or known fixed points so that an algorithm search from outside cannot pass the barrier and penetrate into the inside to reach a previously found fixed point. Thus a fixed point found by an algorithm must be new. Its mathematical validations are established. A flow chart of PNCM is presented. Then a more accurate Legendre-Gauss-Lobatto pseudospectral scheme is constructed to converte a nonlinear fixed point problem into a linear partial differential equation and an algebraic equation. It greatly simplifies the computation. Finally numerical results are presented to show the effectiveness of these approaches. Our approach is quite general and simple. It has a great potential to be modified to solve other multiple solution problems.

A general perturbation theorem with applications to nonhomogeneous critical growth elliptic problems

Kanishka Perera
Florida Institute of Technology, USA

We prove a general perturbation theorem that can be used to obtain pairs of nontrivial solutions of a wide range of local and nonlocal nonhomogeneous elliptic problems. Applications to critical p-Laplacian problems, p-Laplacian problems with critical Hardy-Sobolev exponents, critical fractional p-Laplacian problems, and critical (p, q)-Laplacian problems are given. Our results are new even in the semilinear case $p = 2$.

Asymptotic behavior of least energy solutions to the Finsler Lane-Emden problem with large exponents

Futoshi Takahashi
Osaka Metropolitan University, Japan

We are concerned with the least energy solutions to the Lane-Emden problem driven by an anisotropic operator, so-called the Finsler N-Laplacian, on a bounded domain in $\mathbb{R}^N$. We prove several asymptotic formulae as the nonlinear exponent gets large. This talk is based on a joint work with Sadaf Habibi (Osaka City University).

On a heterogeneous diffusive logistic equation with a harvesting term under strong growth rate

Hossein Tehrani
UNLV, USA

We study existence of positive solutions of the following heterogeneous diffusive logistic equation with a harvesting term,

$$\begin{cases} -\Delta u = \lambda a(x)u + b(x)u^2 - ch(x) & \text{in}\,\Omega \\ u(x) = 0 & \text{on}\ \ \partial\Omega \end{cases}$$

where Ω is either a bounded smooth domain or all of $\mathbb{R}^N$, in which case the boundary condition reads $\lim_{x\to\infty} u(x) = 0$. Also λ and c are positive constant, h(x), b(x) are nonnegative and there exists a bounded smooth region Ω_0 such that $\overline{\Omega_0} = \{x : b(x) = 0\}$. Under the strong growth rate assumption, that is, when $\lambda \geq \lambda_1(\Omega_0)$, the first eigenvalue of the weighted eigenvalue problem $-\Delta v = \mu a(x)v \,\text{in}\,\Omega_0$ with Dirichlet boundary condition, we will show that if $h = 0\,\text{in}\,\Omega \setminus \Omega_0$, then our equation has a *unique positive solution for all c large*, provided that λ is in a right neighborhood of λ_1. In addition we present some results on the positive solution set of this equation in the weak growth rate case complementing existing results in the literature.

Coupled nonlinear elliptic equations with mixed couplings

Zhi-Qiang Wang
Utah State University, USA

We survey works on a class of coupled nonlinear elliptic equations with mixed couplings, in particular on solution structure of existence and qualitative property of positive solutions. Depending upon the system being attractive or repulsive, solutions may tend to be component-wisely synchronized or segregated. We then report our results on the effect of mixed coupling for which coexistence of synchronization and segregation may occur, in particular, we examine the asymptotic behavior of least energy solutions for large and small mixed coupling of multi-scales.

Gradient estimates for the insulated conductivity problem

Zhuolun Yang
Brown University, USA
Hongjie Dong, Hanye Zhu

We consider two insulators embedded in a homogeneous medium where the current-electric field relation is the power law. The electric field (represented by the gradient of solutions) may blow up as ε, the distance between two insulators, approaches 0. In a recent joint work with Hongjie Dong and Hanye Zhu, we establish some gradient estimates in terms of ε.

A global branch approach to normalized solutions for Schrödinger equations

Jianjun Zhang
Chongqing Jiaotong University, Peoples Republic of China

We present a novel approach to study the existence, non-existence and multiplicity of prescribed mass positive solutions to a Schrödinger equation of the form

$$-\Delta u + \lambda u = g(u), \quad u \in H^1(\mathbb{R}^N),\, N \geq 1.$$

This approach permits to handle in a unified way nonlinearities $g(s)$ which are either mass subcritical, mass critical or mass supercritical. Among its main ingredients is the study of the asymptotic behaviors of the positive solutions as $\lambda \to 0^+$ or $\lambda \to +\infty$ and the existence of an unbounded continuum of solutions in $(0, +\infty) \times H^1(\mathbb{R}^N)$. This talk is based on joint work with Prof. Louis Jeanjean and Prof. Xuexiu Zhong.

On finding multiple solutions to nonvariational nonlinear partial differential equations

Jianxin Zhou
Texas A & M University, USA
Bingbing Ji, Zhiqiang Wang

In this talk, a method to find multiple solutions to nonvariational nonlinear partial differential equations will be developed. Its mathematical justifications will be established. Some numerical examples will be presented where numerical solutions are the first time to be computed and visualized. Since the framework of the method is quite general, it is open to discuss various modifications.

Special Session 51: Phase Field Models and Real World Applications

Andrea Giorgini, Politecnico di Milano, Italy
Maurizio Grasselli, Politecnico di Milano, Italy
Alain Miranville, University of Poitiers, France

- In memoriam Gunduz Caginalp who taught us the importance of phase field models -

Many important phenomena are characterized by the presence of interfaces that separate, for example, different chemical species. Significant examples are the behavior of multi-phase fluids, intracellular phase separation, tumor growth dynamics, reconstruction of deteriorated images (inpainting) or self-assembly processes in diblock copolymers. A very effective approach to deal with these real world interface problems consists in formulating suitable phase field or diffuse interface problems whose paradigmatic equations are of Allen-Cahn or Cahn-Hilliard type, possibly coupled with other equations like, e.g., the Navier-Stokes system. This area has been very active in the last few decades both from the theoretical and the numerical viewpoint. The present session intends to focus on the recent progresses made in both issues and give the opportunity to both experts in the field and young research scholars to share their ultimate techniques, novel viewpoints and future goals.

Discontinuous Galerkin methods for growth Cahn-Hilliard models

Andreas Aristotelous
University of Akron, USA

Growth Cahn-Hillard type models are presented. For their numerical solution energy dissipative Discontinuous Galerkin Finite Element (DG-FE) schemes are developed and numerically analyzed. Simulations that verify some of the numerical analysis results are shown.

Phase-field approaches in elastic inverse problems

Andrea Aspri
University of Milan, Italy
Elena Beretta, Cecilia Cavaterra, Elisabetta Rocca, Marco Verani

In this talk I will present some recent results on elastic inverse problems related to the shape reconstruction of cavities and inclusions in a bounded linear isotropic medium by means of boundary measurements. We adopt the point of view of the optimal control, that is we rephrase the inverse problems as a minimization procedure where the goal is to minimize, in the class of Lipschitz domains, a misfit boundary functional or an energy-type functional with the addition of a regularization term which penalizes the perimeter of the cavity/inclusion to be reconstructed. The optimization problem is addressed by a phase-field approach, approximating the perimeter functional with a Modica-Mortola relaxation.
This is a joint work with E. Beretta, C. Cavaterra, E. Rocca and M. Verani.

Optimal control of a tumor growth model with brain lactate kinetics

Laurence Cherfils
La Rochelle University, France
S. Gatti, A. Miranville, H. Raad

I will give in this talk a mathematical model for high grade glioma, taking into account brain lactate kinetics and chemotherapy treatments. I will discuss the well-posedness of solutions, as well as the existence of an optimal treatment for reducing the tumor and lactate concentrations, up to desired targets. I will end with numerical simulations based on different therapeutic situations that can be found in the literature.

C0 interior penalty methods for phase field crystal equations

Amanda Diegel
Mississippi State University, USA
Natasha S. Sharma

A relatively new class of mathematical models known as phase field crystal models has emerged as a way to simulate physical processes where automic- and microscales are tightly coupled. In this talk, we present numerical schemes for two such models which rely on a C0 interior penalty finite element method spatial discretization. We show that the numerical methods are unconditionally energy stable and unconditionally convergent and support our conclusions with a few numerical experiments.

Interfacial free energy ansiotropy in Lennard-Jones systems: a phase field approach

Emre Esenturk
Oxford University, England

A novel version of phase field method is introduced for calculation of interfacial free energy (IFE) of the solid-gas interfaces of liquid-melt interface taking into account the discrete lattice structure. As an example, Lennard-Jones (LJ) systems on actual fcc lattice is considered. The approach provides an easy way to understand the process of transfer of microscopic anisotropy to the macroscopic scale. As an example, IFE for LJ systems is calculated at the triple point for different orientations. It is found that on an fcc lattice IFE is larger in the [100] direction than [110] direction.

Doubly nonlocal Cahn-Hilliard equations

Ciprian Gal
Florida International University, USA
J. Shomberg

The Cahn-Hilliard equation was proposed in the late 1950s and has become nowadays central in understanding phase transition phenomena in many complex materials. The equation aims to describe the process of phase separation, by which the two components of a binary material spontaneously separate and form domains that are pure in each material component. After we revisit much of the history behind the classical form of the Cahn-Hilliard equation we move onto the modern approach which ultimately gives a generalized form of the Cahn-Hilliard equation that can be applied in more general situations (for instance, when the phase separation takes place in a heterogeneous environment). The latter equation reduces to the classical form under certain conditions or assumptions. Interesting mathematics is to be discovered in this new setting and surprisingly a better understanding of the classical form may be also accomplished within this setting.

Existence of weak solutions to a diffuse interface model involving magnetic fluids

Martin Kalousek
Institute of Mathematics, Czech Academy of Sciences, Czech Republic
Sourav Mitra, Anja Schlömerkemper

We address the question of global in time existence of weak solutions for a diffuse interface model in a bounded domain involving incompressible magnetic fluids with unmatched densities. The model couples the incompressible Navier-Stokes equations, gradient flow of the magnetization vector and the Cahn-Hilliard dynamics describing the partial mixing of two fluids. The density of the mixture depends on an order parameter.

Phase field structural optimization in additive manufacturing

Kei Fong Lam
Hong Kong Baptist University, Hong Kong
Harald Garcke, Robert Nurnberg, Andrea Signori

3D printing is an umbrella term for a set of additive manufacturing technologies that fabricate highly intricate and complex designs not feasible with traditional die-casting or injection molding methods. But despite their popularization in recent years, several limitations prevent further integration into existing production lines. One recurring issue relates to overhangs, which are regions of the constructed object that when placed in a certain orientation extend outwards without any underlying support. Some of these overhangs can deform under their own weight and, if not supported from below, present a risk in damaging the printed object.
Beside printing additional support structures which increase material and processing costs, one acceptable remedy is to modify the design to be self-supporting as much as possible without compromising its intended functionality. In this talk we propose a phase field structural topology optimization framework, which realize an overhang angle constraint with the help of anisotropic perimeter functionals. Numerical examples are provided to demonstrate how we discourage designs that develop overhangs not respecting the angle constraint. It turns out that for our approach we have to work with non-differentiable functionals, and thus we turn to subdifferential calculus to derive the first order optimality conditions.

On a convergent SAV scheme for the stochastic Allen-Cahn equation

Stefan Metzger
Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

In this talk, we propose a new approach for the numerical approximation of (weak) solutions to nonlinear stochastic partial differential equations. Using the stochastic Allen-Cahn equation as a prototype for nonlinear stochastic partial differential equations with multiplicative noise, we present an augmented version of the scalar auxiliary variable technique that provides an unconditionally energy stable, fully discrete finite element scheme that is linear with respect to the unknown quantities. By recovering a discrete version of the energy estimate and establishing Nikolskii estimates with respect to time we are able to prove convergence of appropriate subsequences of discrete solutions towards martingale solutions by applying Skorokhod-type arguments and the martingale representation theorem.

On a two-phase two-fluxes degenerate Cahn-Hilliard model

Flore Nabet
École Polytechnique, France
Clément Cancés, Daniel Matthes

We study a nonlocal version of the Cahn-Hilliard dynamics for phase separation in a two-component incompressible and immiscible mixture with linear mobilities. Differently to the celebrated local model with nonlinear mobility, it is only assumed that the divergences of the two fluxe–but not necessarily the fluxes themselve–annihilate each other.
We propose a time implicit Finite-Volume scheme for this problem. The scheme is shown to preserve the key properties of the continuous model, namely mass conservation, positivity of the concentrations, the decay of the energy and the control of the entropy dissipation rate. We prove existence of a solution to the nonlinear scheme and convergence of the approximate solution towards a weak solution of the continuous problems. The existence of a weak solution has been established by showing the convergence of a minimizing movement scheme á la Jordan et al.
Numerical results illustrate the behavior of the numerical model and we also compare the nonlocal model to the classical Cahn-Hilliard model.
This is a joint work with Clément Cancès and Daniel Matthes.

Gradient stability of high order BDF methods and some applications

Morgan Pierre
Université de Poitiers, France
Anass Bouchriti, Noureddine Alaa

It is well known that the Allen-Cahn equation is a gradient flow.
In 1993, Elliott and Stuart proved that the k-step backward differentiation formula (BDFk) applied to the Allen-Cahn equation preserves its gradient structure for k=1, 2 and 3, if the time step is small enough. In 1996, Stuart and Humphries generalized this result for the BDFk method applied to the gradient flow of a semiconvex function and they left open the question for k=4, 5 and 6. In this talk, we show that the BDF4 and BDF5 schemes are gradient stable and we give a negative answer to the question for the BDF6 scheme. We also give some applications of these results to the Allen-Cahn and Cahn-Hilliard equations.

On some stochastic phase-field models of Cahn-Hilliard-Cook type with logarithmic potential

Luca Scarpa
Politecnico di Milano, Italy

We give an overview of some recent results on stochastic phase-field models with logarithmic potential, which cover in particular the well-celebrated Cahn-Hilliard-Cook equation. The proposed techniques allow to treat both the conservative and the non-conservative cases, as well as degenerate and non-degenerate mobilities.
Well-posedness, regularity, and long-time behaviour of solutions are discussed, with a mention of uniqueness-by-noise too. In the last part of the talk, related stochastic phase-field models will be presented, such as the conservative Allen-Cahn equation and coupled systems of Allen-Cahn-Navier-Stokes type.
The works presented in the talk are based on joint collaborations with A. Di Primio, Prof. M. Grasselli, and Dr. M. Zanella (Politecnico di Milano, Italy).

Phase segregation drives RNA-Protein dynamics

Andrea Signori
Politecnico di Milano, Italy
Maurizio Grasselli, Luca Scarpa

Phase separation has recently become a paradigm in Cell Biology. A phase-field model describing the formation of protein-RNA complexes subject to phase segregation is examined. The dynamics involve a single protein, two RNA species, and two complexes. Protein and RNA species are governed by coupled reaction-diffusion equations which also depend on the two complexes. The latter ones are driven by two Cahn-Hilliard equations with singular potentials and reaction terms depending on the solution variables. Some selected modeling and analytic features of the system are discussed. Among the several technical difficulties, the most remarkable one arises from the fact that the two complexes are initially nonexistent, so their initial conditions are zero, i.e., they start from a pure phase: this is a major obstacle in handling the Cahn-Hilliard equation with source term due to the singular nature of the considered potentials. The existence of weak solutions is established in both two and three dimensions.

Well-posedness of a phase field model in fluid-structure interaction

Krutika Tawri
University of California Berkeley, USA

Historically, phase-field methods have been used to model two-phase flows of macroscopically immiscible fluids. However, there have been a few recent efforts in using phase-field methods to model fluid-structure interactions. In this talk, we will discuss a well-posedness result and applications of a new model describing the interaction between an incompressible, viscous fluid and a (poro)viscoelastic structure where the interface between the two different phases is given by a thin smooth transition layer.

Special Session 52: Harmonic Analysis and Partial Differential Equations

William Bray, Missouri State University, USA
Dorina Mitrea, Baylor University, USA

Historically, harmonic analysis and partial differential equations form key cornerstones of mathematics, with close ties. Focal points for the session are topics in either area and, in particular,the interplay between them. Keywords include, but are not limited to: boundary value problems, norm estimates, Fourier analysis, Radon transforms, singular integrals, notions from geometric measure theory such as harmonic measure and rectifiability.

The role of geometry in the theory of function spaces

Ryan Alvarado
Amherst College, USA

Function spaces measuring size and smoothness, such as Sobolev spaces, Besov spaces, and Triebel–Lizorkin spaces, naturally materialize in the formulation of boundary value problems and it has been particularly important, in this regard, to fully understand the fundamental properties of these function spaces in very general geometric settings. In this talk we will survey some recently obtained results pertaining to the extension and embedding properties for certain brands of these function spaces and we will highlight how the geometric makeup of the underlying ambient space directly influences the very nature of these function spaces (in a quantitative manner).

Elliptic problems in Lipschitz and in $\mathcal{C}^{1,1}$ domains

Cherif Amrouche
Université de Pau et des Pays de l'Adour, France
Mohand Moussaoui

We are interested here in questions related to the **maximal regularity** of solutions of **elliptic** problems with **Dirichlet** boundary condition (see ([1]). For the last 40 years, many works have been concerned with questions when Ω is a **Lipschitz domain**. Some of them contain incorrect results that are corrected in the present work.
We give here new proofs and some complements for the case of the **Laplacian** (see [3]), the **Bilaplacian** ([2] and [6]) and the operator $\operatorname{div}(A\nabla)$ (see ([5]), when **A** is a matrix or a function. And we extend this study to obtain other regularity results for domains having an adequate regularity. We give also new results for the **Dirichlet-to-Neumann** operator for Laplacian and Bilaplacian.
Using the duality method, we can then revisit the work of Lions-Magenes [4], concerning the so-called **very weak solutions**, when the data are less regular.

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Space-like strong unique continuation for some fractional parabolic equations

Donatella Danielli
Arizona State University, USA
Vedansh Arya, Agnid Banerjee, Nicola Garofalo

In this talk we will present the space-like strong unique continuation property for a class of nonlocal equations, where the leading operator is the fractional heat. The proof of our main result is achieved via a conditional elliptic type doubling property for solutions to the appropriate extension problem, followed by a blowup analysis.

Two-weight bounds for paraproducts and sparse operators

Irina Holmes Fay
Texas A & M University, USA
Valentia Fragkiadaki

We discuss a sparse operator approach to Bloom-type bounds for paraproducts, including a new type of Bloom-weighted sparse operator.

Far field broadband approximate cloaking for the Helmholtz equation with a Drude-Lorentz refractive index

Narek Hovsepyan
Rutgers University, USA
F. Cakoni, M. Vogelius

We consider an approximate, transformation optics-based cloaking scheme for the Helhmoltz equation, that incorporates a Drude-Lorentz model to account for the dispersive properties of the cloak. We show that on one hand, perfect (far field) cloaking is impossible at any frequency for any incident field, but on the other hand, one can achieve approximate cloaking for any finite band of frequencies, as the resonant frequency of the Drude-Lorentz term approaches infinity.

The higher order regularity problem with data in generalized Banach function spaces

Marcus Laurel
Baylor University, USA
Marius Mitrea

The goal of this talk to present recent developments in boundary value problems for weakly elliptic, second-order systems with constant coefficients. Specifically, for such systems we consider the higher-order regularity problem in the upper-half space when the boundary datum is arbitrarily prescribed from a Generalized Banach Function Space. A Generalized Banach Function Space is a more inclusive version of the usual notion of a Banach Function Space, which admit various function spaces, e.g. the class of Muckenhoupt weighted Morrey spaces and their preduals, a.k.a, Block spaces, that traditional Banach function spaces fail to include. We are able to successfully demonstrate well-posedness in this regime for an arbitrary amount of smoothness at the boundary by working with a general notion of a Poisson kernel, as well as a Calderón-Zygmund theory, tailored for Generalized Banach Function Spaces. This is joint work with Marius Mitrea.

On the eigenvalue distribution of time-frequency limiting operators on higher dimensions

Azita Mayeli
City University of New York, USA

Most wireless communications (e.g., WiFi, cellular networks, or modern mobile phone protocols like 5G) rely on a fixed time band and frequency-limited signals, despite the uncertainty principle showing they are technically incompatible.
However, frequency-limited signals can be created that are almost limited to a given time band. In this talk, we will investigate how to determine the number of orthogonal functions that are almost limited to specific time and frequency regions.
This question can be investigated using the asymptotic and clustering behaviour of eigenvalues of time-frequency limiting operators. These operators are compact, self-adjoint and positive semi-definite, and understanding their eigenvalue distribution is crucial for improving wireless communication protocols through multiplexing. While the one-dimensional setting is well explored by a series of Bell Labs papers by H. Landau, H. Pollak, D. Slepian, H. Widom and I. Daubechies between 1960-1980, the clustering behaviour of eigenvalues in higher dimensions is a hard and open problem in general.
The higher-dimensional case is a crucial aspect in many applications, particularly in scientific imaging problems such as cryoelectron microscopy (cryo-EM) and MRI, and certain optimal orthogonal systems that are approximately space-limited and bandlimited functions in two or more variables play a vital role in achieving accurate and efficient representation of complex multi-dimensional data.
Although the question is well studied in dimension d=1 for the signals of one-variable, the situation becomes much more complex in higher dimensions, and numerous questions still need to be addressed in this regard, as we aim to address some of them in this talk. This is a joint work with Arie Israel.

On the Radon-Carleman problem in irregular domains

Irina Mitrea
Temple University, USA

The talk focuses on recent advances in the Radon-Carleman Problem in irregular domains. Specifically we compute and/or estimate the essential norm and/or Fredholm radius of singular integral operators of double layer type, associated with second order elliptic PDE, on function spaces naturally intervening in the formulation of boundary value problems for said PDE. This program is carried out in a very general geometric setting and is based on joint work with Dorina Mitrea and Marius Mitrea, Baylor University.

Integral operators and boundary value problems for weakly elliptic systems

Marius Mitrea
Baylor University, USA

I will report on progress in the direction of understanding when a boundary value problem for a weakly elliptic system is well posed, or at least Fredholm solvable, on a variety of function spaces, and in rather general geometric settings. At the center of this discussion is the notion of distinguished coefficient tensor, which has a decisive influence on the Fredholmness and invertibility properties of the boundary layer potential operators associated with the given system.

Estimates for Brascamp-Lieb forms in L^p-spaces with power weights

Katharine Ott
Bates College, USA
Russell Brown

We give an update on ongoing work to study a family of Brascamp-Lieb forms acting on families of weighted spaces where the weight is a power of the distance to the origin. We aim to find the largest set of indices for which we have weighted estimates for these multilinear forms. In this talk I will discuss a set of necessary and sufficient conditions for a special class of forms and then discuss the state of the problem in a more general setting.

Neumann boundary regularity for free boundary problems

Sarah Raynor
Wake Forest University, USA
T. Beck, D. Jerison, G. Moon

In this talk we will analyze the regularity of solutions to the free boundary problem of Alt-Caffarelli-Friedman near a fixed boundary with Neumann fixed boundary conditions,. We will compare several recent works in two dimensions (with Gary Moon) and in three dimensions (with Thomas Beck and David Jerison).

The Dirichlet problem on rough domains with data in Herz spaces

Pedro Takemura Feitosa da Silva
Baylor University, USA
Marius Mitrea

In this talk we present recent results in the direction of solving boundary value problems for general second-order systems on rough domains with boundary data taken in non-standard function spaces. More precisely, we study the Dirichlet problem with boundary data in Herz spaces. We develop a comprehensive Calderón-Zygmund theory for a relevant class of singular integral operators acting on (and from) this brand of spaces via a powerful extrapolation result, and succeed in employing the method of boundary layer potentials to establish a well-posedness result for the aforementioned boundary problem. This is joint work with Marius Mitrea.

The spectrum of boundary integral operators on Sobolev spaces

Matt Wright
Missouri State University, USA

This talk will focus on using well-known connections between certain boundary integral operators and their associated PDE's to draw conclusions about the spectral properties of those operators.

Special Session 53: Qualitative and Quantitative Techniques for Differential Equations arising in Applied and Natural Sciences

Rehana Naz, Lahore School of Economics, Pakistan
Stephane Lafortune, The College of Charleston, USA
Imran Naeem, Lahore University of Management Sciences, Pakistan

Differential equations are of great importance as a tool for modelling, describing and understanding natural and applied phenomena coming from biology, chemistry, economy and physics. This session aims to bring together scientists of different fields of such as mathematics and applied and natural sciences having differential equations as subject or tool of work. Potential topics to this session include, but are not limited to:

- Economic growth theory
- Optimal control
- Differential equations modelling natural and economic models
- Financial models e.g. Hamilton-Jacobi equation, Hamilton-Jacobi-Bellman equations, option models, Black-Schole models
- Equivalence transformations
- Stability analysis
- Numerical techniques for special problems in mathematical models
- Symmetries, differential equations, and applications
- Mathematical modeling in biological problems
- Fluid mechanics
- Evolution equations coming from hydrodynamics
- Difference equations and dynamic equations on time scales

Exact solitary wave solutions for a coupled gKdV-NLS system

Stephen Anco
Brock University, Canada
Thomas Wolf

We study a coupled gKdV-NLS system $u_t + \alpha u^p u_x + \beta u_{xxx} = \gamma(|\psi|^2)_x$ and $i\psi_t + \kappa\psi_{xx} = \sigma u\psi$ with a general nonlinearity power $p > 0$, which has been introduced in the literature to model energy transport in an anharmonic crystal material [1,2]. There is a strong interest in obtaining exact solutions describing frequency-modulated solitary waves $u = U(x - ct)$, $\psi = e^{i\omega t}\Psi(x-ct)$, where c is the wave speed, and ω is the modulation frequency. For the KdV case $p = 1$, some solutions have been found in [1], while for the mKdV case $p = 2$, no exact solutions were found [2]. Nothing has been done for higher nonlinearities $p \geq 3$.
In the present work, we derive exact solutions for $p = 1, 2, 3, 4$, starting from the travelling wave ODE system satisfied by U and Ψ. The method is new: (i) obtain first integrals by use of multi-reduction symmetry theory [3]; (ii) apply a hodograph transformation which leads to triangular (decoupled) system; (iii) introduce an ansatz for polynomial solutions of the base ODE; (iv) characterize conditions under which solutions yield solitary waves; (v) solve an algebraic system for the coefficients in the ansatz under those conditions.
The resulting solitary waves exhibit a wide range of features: bright and dark peaks; single peaked and multi-peaked; zero and non-zero backgrounds.

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On a side condition for Wronskian-involving differential equations

Nicoleta Bila
Fayetteville State University, USA

In this talk a few connections among specific concepts occurring in differential geometry and the theory of differential equations are emphasized. The aim of this work is to identify an intriguing class of undetermined nonlinear ordinary differential equations whose solutions satisfy a specific side condition consisting in a homogeneous third-order linear ordinary differential equation. A method for solving this class of Wronskian-involving differential equations based on the proposed side condition is presented. The Tzitzeica curve equation arising in the theory of space curves is considered as an example, and new closed and integral-form solutions for this equation are obtained.

Extreme nonlinear excitations in lattice and continuum models

Efstathios Charalampidis
California Polytechnic State University, USA

In this talk, we will provide an overview of results on extreme events called rogue waves in nonlinear Schrödinger (NLS) equations both in discrete and continuum settings. Motivated by the physics of ultracold atoms, i.e., atomic Bose-Einstein condensates (BECs), we will attempt to address the question about what type of experimental initial conditions should be utilized for producing waveforms which are strongly reminiscent of the Peregrine soliton. The underlying initial-boundary-value problems with Gaussian wavepacket initial data will be considered. Then, large amplitude excitations strongly reminiscent of the Peregrine, Kuznetsov-Ma breather or regular solitons will be identified when the width of the Gaussian initial pulse is varied. Then, we will systematically perform a bifurcation analysis of Kuznetsov-Ma breathers in the Salerno model which itself interpolates the completely integrable Ablowitz-Ladik (AL) model and discrete NLS equation. Novel results in the form of nanopteronic solutions will be presented both at the AL limit but also at the DNLS one where the stability of the identified solutions will be discussed. Finally, associated open questions and directions for future study will also be outlined. The findings presented in this talk might be of particular importance towards realizing experimentally extreme events in BECs but also in optics.

A novel explicit solution for a Novikov equation

Priscila da Silva
Universidade Federal do ABC, Brazil

In this talk we consider a symmetry-integrable equation proposed by V. Novikov (J. Phys. A, 2009) and exhibit an explicit pseudo-peakon solution. From the dynamical system that generates n-solutions, we also present an explicit 2-solution.

Entrainment effects of a sphere settling in viscous stratified fluid

Claudia Falcon
Wake Forest University, USA
Dylan Bruney, Richard McLaughlin, Roberto Camassa

Using the Stokes equations with variable density, we derive a first-principle model for a sphere settling in stratified viscous fluid. Taking advantage of the linearity of the governing equations, we split the fluid flow into the Stokes flow with static density distribution and the stratification induced flow. The solution reduces to a highly coupled system involving a convolution over the fluid domain of the fundamental solution and the forcing term. We discuss the challenges of the Greens function solution to the system, the difficulty that arises from integrating the three dimensional integral, and the appearance of removable singularities. In the cases where the stratification induced flow is not dominant, we propose an asymptotic approach that simplifies the computation. Diffusion effects of the entrainment will also be discussed and explained.

Asymptotic profiles for solutions of a generalised shallow water model

Igor Freire
UFSCar, Brazil

In this talk we study persistence and asymptotic properties of solutions emanating from Cauchy problems with an initial datum with certain decay at infinity. The model under consideration includes some relevant models, such as the Camassa-Holm equation.

Convergence to a self similar solution for a one phase Stefan problem arising in corrosion theory

Danielle Hilhorst
CNRS and University Paris-Sud, France
M. Bouguezzi, Y. Miyamoto, J.F. Scheid

Steel corrosion plays a central role in different technological fields. We will consider a simple case of a corrosion phenomenon which describes a pure iron dissolution in sodium chloride. We will prove that under rather general hypotheses on the initial data, the solution of this iron dissolution model converges to a self similar profile for large times. We will do so for an equivalent formulation which takes the form of a one dimensional one phase Stefan problem. In order to prove the convergence result, we apply a comparison principle together with suitable upper and lower solutions.

Gradient flow of the SBR entropy

Miaohua Jiang
Wake Forest University, USA

We prove the local existence of a gradient flow of the SRB entropy functional on a Hilbert manifold of expanding maps on a circle with a Sobolev norm in its tangent space. In a simple case, we obtain an explicit formula for the flow's ordinary differential equation representation. This gradient flow has a close connection to an interesting nonlinear partial differential equation: a gradient-dependent diffusion equation, $u_t u_x = u_{xx}$.

Higher order normality in the maximum principle as no-infimum gap condition

Monica Motta
University of Padua, Italy
Michele Palladino, Franco Rampazzo

In optimal control theory one sometimes extends the minimization domain of a given problem, with the aim of achieving the existence of an optimal control. However, this issue is naturally confronted with the possibility of a gap between the original infimum value and the extended one. Avoiding this phenomenon is not a trivial issue, especially when the trajectories are subject to endpoint constraints. Since the seminal works by J. Warga in 1970s, some authors have recognized 'normality'of an extended minimizer as a condition guaranteeing the absence of an infimum gap. (Let us recall that an extremal is called abnormal provided the corresponding cost multiplier in the Maximum Principle can be chosen equal to zero, and normal otherwise.) In particular, in 2020 Palladino and Rampazzo proposed a generalization of Warga's criterion to a vast class of endpoint-constrained minimum problems' extensions through the combined use of the notion of abundance (due to Warga and Kaskosz) and of a suitable set separation theorem.
Yet, normality is far from being necessary for this goal, a fact that makes the search for weaker assumptions a reasonable aim. In relation with a control-affine system with unbounded controls, we provide a sufficient no-gap condition based on a notion of higher order normality, which is less demanding than the standard normality and involves iterated Lie brackets of the vector fields defining the dynamics.

A new discretization of the singularly perturbed Burgers-Huxley equation

Justin Munyakazi
University of the Western Cape, South Africa
Eshetu B. Derzie, Tekle G. Dinka

The simultaneous presence of a singular perturbation parameter and the nonlinearity raise the challenge of finding a reliable and efficient numerical solution for the singularly perturbed Burgers-Huxley equation. We propose a nonstandard finite difference scheme which is developed in the following manner. The time variable is discretized using the backward Euler method. This gives rise to a system of nonlinear ordinary differential equations which are then dealt with using the concept of nonlocal approximation. Through a rigorous error analysis, the proposed scheme has been shown to be parameter-uniform convergent. Simulations conducted on two numerical examples confirm the theoretical result. A comparison with other methods in terms of accuracy and computational cost reveals the superiority of the proposed scheme.

Analysis of an incompressible cell-fluid Navier-Stokes model

Gabriela Planas
Universidade Estadual de Campinas, Brazil
Juliana Honda Lopes

In this talk, we consider a general cell-fluid Navier-Stokes model with the inclusion of chemotaxis. This general model relays on a mixture theory multiphase formulation. It consists of two mass balance equations and two general momentum balance equations, respectively, for the cell and fluid phase, combined with a convection-diffusion-reaction equation for oxygen. We investigate the existence of weak solutions in a two or three-dimensional bounded domain when the fluids are assumed to be incompressible with constant volume fraction.

Scalar backward stochastic differential equations

Bin Xie
Shinshu University, Japan
K. Adachi

In this talk, we will discuss recent results on one-dimensional backward stochastic differential equations. We mainly show the existence and uniqueness of L^p solutions for the case of backward stochastic differential equation with random terminal time and potential which is monotonic and uniformly continuous. Different approaches are introduced for the construction of solution.

Special Session 54: Applied Mathematics for Modern Challenges

Jennifer Mueller, Colorado State University, USA
Samuli Siltanen, University of Helsinki, Finland

This special session highlights the research of members of the editorial board of the new AIMS journal Applied Mathematics for Modern Challenges. Application areas are from the physical and life sciences, including medicine, climate modeling, and engineering. Mathematical areas include mathematical modeling, scientific computation, dynamical systems, inverse problems, imaging science, data science, optimization, and control theory.

Scientific data compression

Rick Archibald
Oak Ridge National Laboratory, USA

The US Department of Energy (DOE) makes substantial investments in the production and collection of massive amounts of scientific data through supporting the user facilities and scientific software. The high-performance computing (HPC) resources supported by the Office of Advanced Scientific Computing Research (ASCR) provide an ideal platform for applying scientific machine learning (SciML) on these massive data to accelerate scientific discoveries. However, an efficient, scalable, federated algorithm is necessary to apply SciML to distributed data produced at scientific user facilities. There is a push at Oak Ridge National Laboratory (ORNL) to develop the next generation of smart laboratories (https://www.ornl.gov/intersect), locally developing connections between experimental and computational facilities at ORNL. This talk will focus on recent efforts by IBM/INTERSECT/ORNL/REDHAT/SLAC to connect experimental facilities at different laboratories using federated learning.

Analysis and control in poroelastic systems with applications to biomedicine

Lorena Bociu
NC State University, USA

We answer questions related to tissue biomechanics via well-posedness, sensitivity analysis, and optimal control problems for fluid flows through deformable porous media. These results are relevant for many applications in biology, medicine and bio-engineering, including tissue perfusion, fluid flow inside cartilages and bones, and design of bioartificial organs. We focus on the local description of the problem, which involves implicit, degenerate, nonlinear poroelastic systems, as well as scenarios where the global features of the problem are accounted for through a multi-scale coupling with a lumped hydraulic circuit.

A multiscale preconditioner for Darcy flow

Eric Chung
Chinese University of Hong Kong, Hong Kong
Changqing Ye, Shubin Fu

In this talk, a two-level overlapping domain decomposition preconditioner is developed for solving linear algebraic systems obtained from simulating Darcy flow in high contrast media. Our preconditioner starts at a mixed finite element method for discretizing the partial differential equation by Darcy law with the no flux boundary condition and is then followed by a velocity elimination technique to yield a linear algebraic system with only unknowns of pressure. Then, our main objective is to design a robust and efficient domain decomposition preconditioner for this system, which is accomplished by engineering a multiscale coarse space that is capable of characterizing high contrast features of the permeability field. A generalized eigenvalue problem is solved in each non-overlapping coarse element in a communication-free manner to form the global solver, which are accompanied by local solvers originated from additive Schwarz methods but with a non-Galerkin discretization to derive the two-level preconditioner. We provide a rigorous analysis indicates that the condition number of the preconditioned system could be bounded above with several assumptions. The research of Eric Chung is partially supported by the Hong Kong RGC General Research Fund (Project numbers 14304719 and 14302620).

Particle stochastic reaction-drift-diffusion methods for studying cellular processes

Samuel Isaacson
Boston University, USA
Max Heldman

We will discuss the modeling and simulation stochastic reaction-drift-diffusion models, formulating a particle-based model appropriate for studying spatial transport and chemical reactions at the single-cell scale. The model accounts for molecular diffusion, drift due to one and two-body potentials, and chemical reactions between molecules, and is formulated to ensure the preservation of key equilibrium properties such as detailed balance of reversible reactions. We develop continuous-time Markov chain-based numerical methods for efficiently simulating the dynamics of the particle model, demonstrating new methods that can resolve interaction potentials on general unstructured meshes, provide second-order convergence in the mesh spacing, and preserve detailed balance of both reaction and drift-diffusion fluxes.

Inverse problems associated with determining gradients and direction of signaling molecules

Alan Lindsay
University of Notre Dame, USA
Andrew Bernoff, Adrian Hernandez Navarro

Communication at a cellular level is conducted by releasing signaling molecules which diffuse in a crowded environment until binding with receptors located on the membranes of other cells. At a glance this mechanism seems noisy and inefficient, yet cells reliably decode these signals to deduce directions in which to move or grow. In this talk I will describe a variety of inverse problems that arise from recovering directional information from such noisy arrivals. Mathematically, this involves a combination of asymptotic analysis, computational methods for PDEs, and statistical approaches.

Fast nonlinear imaging of pediatric patients with electrical impedance tomography

Jennifer Mueller
Colorado State University, USA

Electrical impedance tomography (EIT) is an emerging medical imaging technique in which electric fields are used to form real-time images of organ function and structure. To form these images, it is necessary to solve a severely ill-posed nonlinear inverse problem with computational efficiency. The D-bar method is a direct (non-iterative) method with a proven nonlinear regularization strategy and real-time implementation, making it particularly suitable for bedside imaging. This talk will present recent developments to improve speed and resolution and highlight its use with clinical data from the ACT 5 system.

Special Session 55: Sparse Signal Learning and its Applications in Data Science

Xuemei Chen, University of North Carolina Wilmington, USA
Longxiu Huang, Michigan State University, USA
Jing Qin, University of Kentucky, USA

Many signal processing problems utilize sparsity or its low dimensional structure such as compressed sensing, matrix completion, and low rank tensor recovery, all of which have prominent applications such as image processing, social network, and machine learning in general. This special session aims to present recent developments in this area, whether theory or applications, utilizing tools in sampling theory, random matrix/tensor theory, optimization, numerical linear algebra, approximation theory, graph theory, etc.

Data adaptive multiscale bases inducing joint compressibility

Julia Dobrosotskaya
Case Western Reserve University, USA
Weihong Guo

Finding a good representation for a given data set is often providing the key to solving a variety of signal processing problems. We propose to explore the design of a data adaptive representation with low redundancy that also incorporates multi- scale structure via minimizing a jointly weighted l^1 functional that induces scale separation and fast coefficient decay. Given a dataset of elements that share structural similarities, we aim at obtaining a tight frame with low redundancy and predefined scaling properties, yet without the explicitly required self-similar structure.
We replace the task of finding a representation inducing joint sparsity within the given dataset by the task of finding a system that provides an l^1 -optimal weighted sparsity, which is further shown to be suitable for many classical image analysis and recovery tasks.
The efficiency of the acquired representations are illustrated with digital removal of the cracks on Monet's paintings, identifying the leading geometric features of a dataset qualitatively better than SVD and more.

Instability of the infinite dimensional operator recovery problem

Christopher Dock
Tufts University, USA
Radu V. Balan

This talk expands on a well-known result of Daubechies, Cahill, and Casazza which says that the infinite dimensional phase-retrieval problem is never stable with respect to certain natural choices of metric. Following their proof, we bootstrap all the way up to the operator recovery problem for compact operators on a Hilbert space. The next part of the talk is a novel and far simpler proof which generalizes the result to bounded operators on a Hilbert space, and dispenses with some of the criteria required in the original proof. The result in this form has several interesting corollaries in terms of the non-existence of certain types of Banach Frames.

Fast algorithms via matrix subsampling

Keaton Hamm
University of Texas Arlington, USA

We will overview some matrix factorizations that allow one to observe only small randomly chosen submatrices of a data matrix, and how these factorizations can be applied in the design of fast algorithms for certain tasks such as Robust PCA or matrix completion. We show how to obtain state-of-the-art runtime for these tasks and apply the algorithms to some image and video processing tasks. We will discuss some natural generalizations of this approach to tensor data.

Fast hyperspectral band selection based on matrix CUR decomposition

Katherine Henneberger
University of Kentucky, USA
Longxiu Huang, Jing Qin

Band selection is an important technique for eliminating spectral redundancy of hyperspectral imagery while preserving critical information. Recently, correlations among neighboring bands or pixels have been exploited in the form of graph regularizations to reduce the data dimensionality efficiently. However, the manipulation of graph regularizations is typically a computational bottleneck. In this presentation, we propose a fast and robust method for hyperspectral band selection based on spatial/spectral graph Laplacians and matrix CUR decomposition. The efficiency of the proposed method is shown on two real datasets by comparing with several other state-of-the-art band selection methods.

Neural network approximation of continuous functions in high dimensions with applications to inverse problems

Santhosh Karnik
Michigan State University, USA
Rongrong Wang, Mark Iwen

The success of neural networks in a variety of inverse problems has fueled their adoption in disciplines ranging from medical imaging to seismic analysis. However, the high dimensionality of such inverse problems has simultaneously left current theory, which predicts that networks should scale exponentially in the dimension of the problem, unable to explain why the seemingly small networks work as well as they do in practice. To reduce this gap between theory and practice, we provide a general method for bounding the complexity required for a neural network to approximate a Hölder (or uniformly) continuous function on a high-dimensional set with a low-complexity structure. Many sets of interest in high dimensions have low-distortion linear embeddings into lower dimensional spaces. We can exploit this fact to show the size of a neural network needed to approximate a Hölder (or uniformly) continuous function on a low-complexity set in a high dimensional space grows exponentially with the dimension of its low-distortion embedding, not the dimension of the space it lies in. The result is a general theoretical framework which can be used to better explain the observed empirical success of smaller networks in a wider variety of inverse problems than current theory allows.

Digital beamforming robust to time-varying carrier frequency offset

Shuang Li
University of California Los Angeles, USA
Payam Nayeri, Michael B. Wakin

Adaptive interference cancellation is rapidly becoming a necessity for our modern wireless communication systems, due to the proliferation of wireless devices that interfere with each other. To cancel interference, digital beamforming algorithms adaptively adjust the weight vector of the antenna array, and in turn its radiation pattern, to minimize interference while maximizing the desired signal power. While these algorithms are effective in ideal scenarios, they are sensitive to signal corruptions. In this work, we consider the case when the transmitter and receiver in a communication system cannot be synchronized, resulting in a carrier frequency offset that corrupts the signal. We present novel beamforming algorithms that are robust to signal corruptions arising from this time-variant carrier frequency offset. In particular, we bring in the Discrete Prolate Spheroidal Sequences (DPSS's) and propose two atomic-norm-minimization (ANM)-based methods in both 1D and 2D frameworks to design a weight vector that can be used to cancel interference when there exist unknown time-varying frequency drift in the pilot and interferer signals. Both algorithms do not assume a pilot signal is known. Noting that solving ANM optimization problems via semi-definite programs can be a computational burden, we also present a novel fast algorithm to approximately solve our 1D ANM optimization problem. Finally, we confirm the benefits of our proposed algorithms and show the advantages over existing approaches with a series of experiments.

Optimal recovery from inaccurate observations

Chunyang Liao
Texas A & M University, USA
Simon Foucart

In Optimal Recovery, the task of learning a function from observational data is tackled deterministically by adopting a worst-case perspective tied to an explicit model assumption made on the functions to be learned. Working in the framework of Hilbert spaces, we considers a model assumption based on approximability and the observational inaccuracies modeled via additive errors bounded in either ℓ_2 or ℓ_1. This talk shows how to construct the recovery procedure, which can be chosen as linear maps under our problem setting.

Special Session 56: Variational Methods for Nonlinear PDEs

Kanishka Perera, Florida Institute of Technology, USA
Pasquale Candito, University of Reggio Calabria, Italy
Roberto Livrea, University of Palermo, Italy

This special session will cover variational methods for nonlinear PDEs including existence and multiplicity results, qualitative properties of solutions, critical point theory, etc.

Nonlinear elliptic problems involving the double phase operator with variable exponents

Eleonora Amoroso
University of Messina, Italy
Giuseppina d'Aguì, Patrick Winkert

In this talk we deal with existence results for nonlinear elliptic problems involving the double phase operator with variable exponents. The approach is based on variational methods.

Sharp Quantitative stability of Poincare-Sobolev inequality in the hyperbolic space

Mousomi Bhakta
Indian Institute of Science Education and Research, India
Debdip Ganguly, Debabrata Karmakar, Saikat Mazumdar

Consider the Poincaré-Sobolev inequality on the hyperbolic space: for every $n \geq 3$ and $1 < p \leq \frac{n+2}{n-2}$, there exists a best constant $S_{n,p,\lambda}(\mathbb{B}^n) > 0$ such that

$$S_{n,p,\lambda}(\mathbb{B}^n) \left(\int_{\mathbb{B}^n} |u|^{p+1} \, \mathrm{d}v_{\mathbb{B}^n} \right)^{\frac{2}{p+1}} \leq \int_{\mathbb{B}^n} \left(|\nabla_{\mathbb{B}^n} u|^2 - \lambda u^2 \right) \mathrm{d}v_{\mathbb{B}^n},$$

holds for all $u \in C_c^\infty(\mathbb{B}^n)$, and $\lambda \leq \frac{(n-1)^2}{4}$, where $\frac{(n-1)^2}{4}$ is the bottom of the L^2-spectrum of $-\Delta_{\mathbb{B}^n}$. It is known from the results of Mancini and Sandeep (Ann. Sc. Norm. Super. Pisa, 2007) that under appropriate assumptions on n, p and λ there exists an optimizer, unique up to the hyperbolic isometries, attaining the best constant $S_{n,p,\lambda}(\mathbb{B}^n)$. In this talk we will discuss the quantitative gradient stability of the above inequality and the associated Euler-Lagrange equation locally around a bubble.

We will show sharp quantitative stability of the above Poincaré-Sobolev inequality: if $u \in H^1(\mathbb{B}^n)$ almost optimizes the above inequality then u is close to the manifold of optimizers in a quantitative way. Secondly, we will discuss the quantitative stability of its critical points: if $u \in H^1(\mathbb{B}^n)$ almost solves the Euler-Lagrange equation corresponding to the above Poincaré-Sobolev inequality and the energy of u is close to the energy of an extremizer, then the following quantitative bound holds

$$\mathrm{dist}\,(u, \mathcal{Z}) \leq C(n,p,\lambda) \| \Delta_{\mathbb{B}^n} u + \lambda u + u^p \|_{H^{-1}(\mathbb{B}^n)},$$

where $\mathcal{Z}$ denotes the manifold of non-negative finite energy solutions of $-\Delta_{\mathbb{B}^n} w - \lambda w = |w|^{p-1} w$. Our result generalizes the sharp quantitative stability of Sobolev inequality in Euclidean space of Bianchi-Egnell ((J. Funct. Anal. 1991) and Ciraolo-Figalli-Maggi (Int. Math. Res. Not. IMRN, 2021) to the Poincaré-Sobolev inequality on the hyperbolic space.

New "critical" thresholds for some quasilinear elliptic problems

Anna Maria Candela
Università degli Studi di Bari Aldo Moro, Italy

We consider a family of quasilinear elliptic problems where the classical p-Laplacian is replaced by an operator which admits some coefficients depending on the solution itself. Even if such coefficients make the variational approach more difficult, a suitable supercritical growth for the nonlinear term is allowed.
These results are part of joint works with G. Palmieri, K. Perera and C. Sportelli.

A multiplicity result for a p-Laplacian supercritical Neumann problem

Francesca Colasuonno
Alma Mater Studiorum Università di Bologna, Italy
Benedetta Noris, Gianmaria Verzini

In this talk, we will discuss the existence and variational characterization of two distinct non-constant, radial, radially non-decreasing solutions to the supercritical equation

$$-\Delta_p u + u^{p-1} = u^{q-1}$$

under Neumann boundary conditions, in the unit ball of $\mathbb{R}^N$. Here $p \in (1,2)$ and q is large. Using a variational approach in an invariant cone, we can distinguish the two solutions on their energy: one has minimal energy inside a Nehari-type set and the other is obtained via a mountain pass argument inside the same set. In the talk, we will also highlight the differences with the cases $p = 2$ and $p > 2$. We will show that for $p \in (1, 2)$, the constant solution 1 is a local minimizer on the Nehari set: this is a peculiarity of this case and is responsible for the appearance of the second (higher-energy) solution. Finally, we will detect the limit profiles of the two solutions as $q \to \infty$.

On some properties of unbounded solutions to a class of chemotaxis models

Alessandro Columbu
Università di Cagliari, Italy
Silvia Frassu, Giuseppe Viglialoro

In this talk we consider an attraction-repulsion chemotaxis model with logistic, and with nonlinear rates for diffusion, sensitivities and productions of both chemoattractant and chemorepellent. We detect Lebesgue spaces where solutions blowup and, additionally, we estimate the blowup time. Finally, we establish some blowup criteria in a simplified case of the model. This is a joint work with Silvia Frassu and Giuseppe Viglialoro.

Long-range phase transition equations

Serena Dipierro
University of Western Australia, Australia

We discuss an improvement of flatness result for nonlocal phase transitions and, for a class of nonlocal equations that includes the fractional Allen-Cahn equation, we obtain a result in the same spirit of a celebrated theorem of Savin for the classical Allen-Cahn equation.

Nonlinear Dirichlet problem with the anisotropic Laplacian operator

Giuseppina d'Aguì
University of Messina, Italy
G. Bonanno, A. Sciammetta

In this talk, we focus on the existence of two nontrivial solutions to the Dirichlet problem associated with the anisotropic Laplacian operator. The proofs are based on critical point theory and the functional framework involves anisotropic Sobolev spaces.

Uniqueness for a class of nonlinear elliptic equations with lower order terms

Vincenzo Ferone
Università di Napoli Federico II, Italy
Angelo Alvino, Anna Mercaldo

We discuss existence and uniqueness results for weak solution to a class of homogeneous Dirichlet boundary value problems for equations containing a p-laplacian, $1 < p < 2$, and a lower order term which grows as a power of the gradient. The source term is suitably summable and satisfies a smallness condition.

On a class of indirect and direct chemotaxis-consumption models in high dimensions

Silvia Frassu
University of Cagliari, Italy
Giuseppe Viglialoro

In this talk we consider a zero-flux chemotaxis model with indirect signal absorption and we establish conditions on the data in order to have global boundedness of related solutions.
This work is in collaboration with Giuseppe Viglialoro.

Stationary Schrödinger type equations with nonlinearities sublinear at zero

Shibo Liu
Florida Institute of Technology, USA

We discuss the existence of infinitely many solutions for a class of quasilinear Schrödinger equations with concave and convex nonlinearities, where the convex nonlinearity may be supercritical. A semilinear Schrödinger equation with indefinite potential and nonlinearity sublinear at zero is also considered.

Constant sign and nodal solutions for singular quasilinear elliptic problems

Abdelkrim Moussaoui
Bejaia University, Algeria

We establish the existence of multiple solutions for singular quasilinear elliptic problems with a precise sign information: one nodal (sign-changing) and two opposite constant sign solutions. The approach combines sub-supersolution technique and Leray-Schauder topological degree.

Existence of solutions to superlinear double phase problems

Angela Sciammetta
University of Palermo, Italy

The present talk is devoted to the existence and multiplicity of solutions for some classes of double phase problem with a parametric superlinear right-hand side that has subcritical growth. Under very general assumptions on the data, we prove the existence of at least two nontrivial bounded weak solutions to such problem by using variational methods and critical point theory.

On existence results for semilinear heterogeneous problems in $\mathbb{R}^2$ involving critical nonlinearities of Trudinger-Moser Type

Hossein Tehrani
UNLV, USA
David Costa, Siegfried Carl

In this talk we explore the effect of the presence of a nonnegative heterogeneous weight function $a(x)$ on the existence of solutions to semilinear elliptic equations with critical nonlinearity a la Trudinger-Moser:

$$-\Delta u = a(x)h(u)e^{\alpha u^2} \,\text{in}\, \Omega \quad u = 0 \,\text{on}\, \partial\Omega$$

in dimension 2, i.e. when Ω is a smooth bounded domain in $\mathbb{R}^2$. We consider this under the assumption that either $a(x)$ grows sufficiently fast at an interior point of Ω or is radially symmetric and rapidly vanishing weight function. We are then able to prove existence of positive or sign changing solutions *without the usual growth restriction on the lower order term of critical nonlinearities (that is, $h(u)$).* By highlighting the effect of the heterogeneity in relaxing growth restrictions on the lower order terms, our results complement and generalize a number of existing results in the literature of such problems.

A nonlocal capillarity theory

Enrico Valdinoci
University of Western Australia, Australia
Alessandra de Luca, Serena Dipierro, Francesco Maggi

We describe some recent results motivated by a new nonlocal theory of capillarity, as related to the formation of droplets due to long-range interaction potentials. We will discuss the notion of contact angle in this setting, considering a nonlocal version of the classical Young's Law, together with some regularity properties and asymptotics. We will also present the case of anisotropic media and general interaction kernels.

A two-dimensional Ventcel problem modeling the equilibrium of a prestressed membrane

Giuseppe Viglialoro
University of Cagliari, Italy
Antonio Greco

This talk is concerned with a mathematical problem modeling the equilibrium of a thin membrane structure (tensile structure), with rigid and cable boundaries. The mathematical formulation is expressed by means of a second order elliptic Ventcel mixed boundary-value problem, in which take part the shape of the membrane and its stress tensor. We discuss some partial results, both by the theoretical and numerical point of view. This is a joint work with Antonio Greco.

Special Session 57: Mathematical Models for Traffic Monitoring and Control

Sean McQuade, Rutgers-Camden, USA
Maria Teresa Chiri, Queens University, Canada
Maria Laura Delle Monache, University of California Berkeley, USA

This session covers research used in practical highway traffic experiments for both monitoring traffic, and for mobile traffic control. Monitoring traffic reveals properties, such as the source of congestion which can be dynamic “stop and go traffic waves” or a stationary bottleneck resulting from road topology. The former can be smoothed in order to increase the overall MPG while not necessarily decreasing the throughput of traffic, and sometimes improving throughput as well. Models of the background traffic at large (macro) and small (micro) scale serve as the basis for control design.

Leveraging connected and automated vehicle data for queue-informed and incident-aware ramp metering strategies to improve highway operations

Jingqin Gao
New York University, USA
Kaan Ozbay, Yu Tang, Chuan Xu, Fan Zuo, Di Sha

Connected and automated vehicles (CAV) allow for the generation and sharing of enriched data. When these data are collected and utilized, it presents opportunities to enhance operational strategies aimed at better managing and improving traffic flow and safety. This study aims to develop and evaluate advanced queue-informed and incident-aware ramp metering algorithms. The queue-informed algorithm uses more accurate on-ramp queue estimation from CAV data to smooth metering rates, while the incident-aware algorithm integrates feedforward control into feedback ramp metering for distant bottlenecks. These control strategies are evaluated at both local and system-wide levels using a simulation-based approach to assess their impact on highway mobility, safety, efficiency, and reliability.

On the mathematical properties of some multi-scale traffic models

Xiaoqian Gong
Arizona State University, USA

In this talk, we will present the mathematical properties of some microscopic, mesoscopic and macroscopic descriptions of traffic flow models. From the microscopic perspective, we will discuss the limitations and improvements of the Intelligent Driver Model (IDM), as well as the well-posedness of the Bando-Follow-the-Leader (Bando-FtL) Model and a time-delayed version of the Bando-FtL. As one of the applications of the microscopic car-following models, we will talk about optimal cruise control for traffic smoothing. From the mesoscopic perspective, we will derive rigorously the mean-field limit of a finite-dimensional hybrid system describing multi-lane and multi-class traffic flow in presence of human-driven and autonomous vehicles. From the mesoscopic perspective, we will briefly talk about the well-posedness of a nonlocal LWR model with memory. Numerical simulations and field experiment results will also be presented.

Multiscale characteristics of traffic waves and sparse control

Nour Khoudari
Temple University, USA

Car-following models can reproduce instabilities and traffic waves that are observed in real traffic flow. Low density autonomous vehicles (AVs) have the potential to dampen and prevent these undesirable non-equilibrium phenomena. By connecting traffic models from micro to macro scales we: explore some of the multiscale characteristics of traffic instabilities, outline the key macroscopic flow consequences of microscopic traffic waves, show the possibilities and conditions of stabilizing traffic flow via AV-based smoothing, and point out the effects on the overall system-level fuel consumption balance by using simple vehicle-specific energy models.

Using a bi-level optimization algorithm to calibrate traffic simulations for the CIRCLES experiment

Sean Mcquade
Rutgers-Camden, USA

The CIRCLES consortium performed the largest live traffic control experiment to smooth traffic waves during rush hour on Interstate 24 West heading into Nashville, TN. It was performed during the week of November 14, 2022. To plan for the 100 vehicle test, a Bi-level optimization for an agent-based dynamic traffic assignment model (DTA) was used to calibrate a simulator. The supply parameters, such as traffic network topology, capacity, and speed limits, define the environment. The demand inputs represent the travelers and their behavior, such as origin-destination matrices, routing, and lane changing. The environment was calibrated to the background traffic where we ran the live experiment, using flow-speed data. Then, the control vehicles were

added to the simulator to estimate characteristics of their drive in the calibrated background traffic, such as how they would affect traffic on arterial roads, and how they may cluster at off ramps.

Conservation law in the presence of moving bottleneck and discontinuity in the flux

Hossein Nick Zinat Matin
University of California Berkeley, USA
Maria Laura Delle Monache

In this talk, we will discuss a (PDE-ODE) Cauchy problem in the presence of a moving bottleneck and discontinuity in flux. In particular, we consider the following Cauchy problem:

$$(P): \begin{cases} \partial_t \rho + \partial_x [f(\gamma, \rho)] = 0; \\ \rho(0, x) = \rho_\circ(x); \\ f(\gamma, \rho) - \dot{y}\rho \leq F(y); \\ \dot{y} = w(y, \rho), \quad y(0) = y_\circ; \end{cases}$$

In a macroscopic presentation of traffic flow, the existence of moving bottlenecks, e.g. slow-moving vehicles, can significantly affect traffic behavior by influencing the road capacity and consequently dynamics of the flow and is mathematically modeled by imposing a capacity constraint.
The discontinuities in the flux function arise as a result of the spatial dependence of flux functions, for instance, the variable speed limits in different regions of the road. The presence of such discontinuities introduces new types of waves which imply the need for defining a new Riemann solution. We prove the existence and uniqueness of the solution to the Cauchy problem (P) through a rigorous analysis of the interaction of these waves.

Learning inverse solver for scalar nonlinear hyperbolic PDEs: Application to the LWR traffic flow model

Bilal Thonnam Thodi
New York University Abu Dhabi, United Arab Emirates
Sai Venkata Ramana Ambadipudi, Saif Eddin Jabari

First-order macroscopic traffic flow models, which are instances of nonlinear hyperbolic partial differential equations, are conventionally solved using numerical schemes which are grid-dependent and require complete knowledge of initial and boundary data. We study learning a generic inverse solver for approximating weak solutions to arbitrary input conditions, e.g., spatial boundary or random collocation points, using an operator learning framework. Under this framework, the inverse solver is a parametric operator that learns a family of weak solutions offline from data (historical simulation). Computing solution to new inputs is then a single forward evaluation of the operator. This avoids resolving the problem for every new input instance, lowering the computational cost. We also present algorithms to generate sparse training datasets and efficiently learn the essential features of the hyperbolic solutions, namely, shocks and rarefaction waves. We illustrate the proposed method for solving Lighthill-Witham-Richards (LWR) traffic flow model and discuss the generalization error growth. These fast inverse solvers can be potentially used for real-time traffic monitoring and control.

Learning to simulate with real-world traffic data

Hua Wei
New Jersey Institute of Technology, USA

This talk explores the use of real-world traffic data for simulating traffic flow in urban areas. The objective is to improve the accuracy and efficiency of traffic simulations by training machine learning models to learn from real-world traffic data. The proposed approaches include learning data-driven models and calibrating existing physics-driven models. Specifically, this talk will introduce some of our latest work on learning to simulate with real-world data, i.e., when the traffic data is sparse and hard to obtain, and the follow-up simulation model and control model facing this real-world data. The findings have implications for traffic control management in real-world settings.

Special Session 59: Interplays between Statistical Learning and Optimization

Qiang Wu, Middle Tennessee State University, USA
Xuemei Chen, University of North Carolina Wilmington, USA
Yiming Ying, State University of New York Albany, USA

The recent exchange between statistical learning and computational optimization has never been so fruitful. Merging techniques from learning theory, approximation theory, and numerical optimization, researchers nowadays try to understand the convergence dynamics of the optimization procedure and unveil the generalization mystery of learning algorithms as well as the intimate interaction between them. The objective of this session is to identify the recent progress and trends in statistical learning and optimization and to serve as a multi-disciplinary forum to promote interaction among researchers from statistical learning and applied mathematics to exchange new ideas and techniques.

On learning with bounded loss functions

Yunlong Feng
State University of New York Albany, USA
Qiang Wu

In machine learning, bounded loss functions have been more and more frequently used owing to their robustness to outliers and heavy-tailed noise. However, the understanding of bounded loss functions, especially from a theoretical viewpoint, is still limited due to their nonconvexity. In this talk, I will report some of our recent efforts made in this regard. First, I will show that in the context of empirical risk minimization, bounded loss functions can be interpreted from a minimum distance estimation viewpoint. Second, the prediction ability of estimators resulting from bounded loss functions will also be assessed and discussed.

Pairwise learning for imbalanced data classification

Shu Liu
Middle Tennessee State University, USA
Qiang Wu

Imbalanced data classification problems appear quite commonly in real-world applications and impose great challenges to traditional classification approaches which work well only on balanced data but usually perform poorly on the minority class when the data is imbalanced. Resampling preprocessing by oversampling the minority class or downsampling the majority class helps improve the performance but may suffer from overfitting or loss of information. In this paper we propose a novel method called pairwise robust support vector machine (PRSVM) to overcome the difficulty of imbalanced data classification. It adapts the non-convex robust support vector classification loss to the pairwise learning setting. In the training process, samples from the minority class and the majority class always appear as pairs. This automatically balances the impact of two classes. Simulations and real-world applications show that PRSVM is highly effective.

Learning rates, corrupted linear systems, and randomized Kaczmarz

Nicholas Marshall
Oregon State University, USA
Oscar Mickelin

In this talk, we consider how the learning rate affects the performance of a relaxed randomized Kaczmarz algorithm for solving $Ax \approx b + \varepsilon$, where $Ax = b$ is a consistent linear system and ε has independent mean zero random entries. We derive a learning rate schedule that optimizes a bound on the expected error that is sharp in certain cases; in contrast to the exponential convergence of the standard randomized Kaczmarz algorithm, our optimized bound involves the reciprocal of the Lambert-W function of an exponential.

Exploring dynamical parameters of interacting galaxies using deep learning and optimization

Matthew Ogden
Middle Tennessee State University, USA
John Wallin, Graham West, Anthony Holincheck

Gravitational interactions between galaxies play a pivotal role in galaxy formation and evolution creating tidal distortions, new star formation, galactic mergers, and active galactic nuclei. Due to observational limitations, simulations play an integral part of galaxy research. Gravitational n-body and restricted three-body simulations, which capture underlying gravitational dynamics, can be used to create the complex morphologies observed in galaxies. However, when attempting to model observed interacting galaxies, there are over a dozen unknown dynamical parameters due to the same limitations. We are developing a method to explore the unknown dynamical parameters by using the observed morphologies. One significant challenge is accurately matching the morphology of real and simulated systems. To address this issue, we use Computer Vision and Deep Learning to create a fitness function between simulation and observational data using citizen science data. We then optimize this fitness function generating models matching the observed target better than previous models. By analyzing the best-fit models and their dynamical parameters, we can postulate values and boundaries for previously unknown values. Ultimately, this research will become a valuable tool to explore dynamic gravitational systems otherwise limited by observational data.

Data-driven deep learning neural networks for predicting the number of individuals infected by COVID-19 omicron variant

Ebenezer Oluwasakin
Middle Tennessee State University, USA
Abdul Q. M. Khaliq, Khaled M. Furati

Infectious disease epidemics are a challenge for medical and public health practitioners. They requires prompt treatment, but it is challenging to recognize and define epidemics in real time. By knowing the short-term prediction of an infectious disease epidemic, the disease's impact can be evaluated by preventive efforts. Real-time mathematical epidemic models such as logistic differential equations and deep learning methods are key preventative tools. Data-driven deep learning enables effective algorithms for identifying parameters in mathematical models. This paper introduces a logistic-informed neural networks algorithm inspired by applying a physics-informed neutral network to a logistic differential equation to learn the constant parameter and the time-dependent function of the Omicron variant. The learned parameter and time-dependent function, as well as the analytical solution of the logistic differential equation, are used to make a short-time prediction on the daily, the time that a plateau will be reached, and the cumulative number of individuals reported to be infected with the Omicron variant. In a data-driven simulation, the accuracy of this model is demonstrated using error metrics on Omicron variant data for Portugal, Italy, and China.

Dynamical sampling: Dynamical duals and quantization

Alex Powell
Vanderbilt University, USA
Jonathan Ashbrock

Dynamical sampling addresses the problem of recovering a signal from space-time samples of an evolution process. Many problems in dynamical sampling can be conveniently formulated in terms of frame theory. A frame is said to be dynamical if the frame vectors are generated by the iterates of a linear operator. Aceska and Kim proved that if a frame is dynamical then its canonical dual frame is also dynamical. However, not all frames are dynamical. Given a frame which is not necessarily dynamical, we study the dynamical structure of its dual frames. We prove that every redundant finite frame has infinitely many dual frames that are dynamical. We then show an application of dynamical dual frames to error diffusion algorithms for quantizing finite frame expansions. This is joint work with Jon Ashbrock.

An introduction to distributed machine learning

Qiang Wu
Middle Tennessee State University, USA

Distributed machine learning is an effective way to process large scale data. In this talk, I will present an introduction to the divide and conquer approach to implement distributed machine learning. We proved the optimality of kernel based machine learning methods for regression analysis and ranking. We proposed a bias correction trick to improve the performance of distributed kernel regression while preserving the theoretical optimality. We also proposed effective strategies for distributed classification by conducting a comparative analysis.

Learning theory for contrastive representation learning

Yiming Ying
State University of New York Albany, USA
Yunwen Lei, Tianbao Yang, Ding-Xuan Zhou

The performance of machine learning (ML) models often depends on the representation of data, which motivates a resurgence of contrastive representation learning (CRL) to learn a representation function. Recently, CRL has shown remarkable empirical performance and it can even surpass the performance of supervised learning models in various domains such as computer vision and natural language processing. In this talk, I present our recent progress in establishing the learning theory foundation for CRL. In particular, we address the following two theoretical questions: 1) how would the generalization behavior of downstream ML models benefit from the representation function built from positive and negative pairs? 2) Especially, how would the number of negative examples affect its learning performance?
Specifically, we can show that generalization bounds for contrastive learning do not depend on the number k of negative examples, up to logarithmic terms. Our analysis uses structural results on empirical covering numbers and Rademacher complexities to exploit the Lipschitz continuity of loss functions. For self-bounding Lipschitz loss functions, we further improve our results by developing optimistic bounds which imply fast rates in a low noise condition. We apply our results to learning with both linear representation and nonlinear representation by deep neural networks, for both of which we derive explicit Rademacher complexity bounds.

Learning ability of interpolating deep convolutional neural networks

Tian-Yi Zhou
Georgia Institute of Technology, USA
Xiaoming Huo

It is frequently observed that overparameterized neural networks generalize well. Regarding such phenomena, existing theoretical work mainly devotes to linear settings or fully-connected neural networks. This paper studies the learning ability of an important family of deep neural networks, deep convolutional neural networks (DCNNs), under both underparameterized and overparameterized settings. We establish the first learning rates of underparameterized DCNNs without parameter or function variable structure restrictions presented in the literature. We also show that by adding well-defined layers to a non-interpolating DCNN, we can obtain some interpolating DCNNs that maintain the good learning rates of the non-interpolating DCNN. This result is achieved by a novel network deepening scheme designed for DCNNs. Our work provides theoretical verification of how overfitted DCNNs generalize well.

Special Session 61: Qualitative Properties and Numerical Approximations of PDE Systems which Govern Fluid Flows and Flow-Structure Interactions

George Avalos, University of Nebraska-Lincoln, USA
Pelin Guven Geredeli, Iowa State University, USA

This special session will feature speakers who have wide-ranging research expertise in the continuous and numerical analysis of those partial differential equation (PDE) models which describe fluid or interactive fluid-structural dynamics. The analysis of such PDE models constitutes a broad area of research with applications in variety of real world problems. The main goal of this special session will be to convey recent results concerning the wellposedness, longtime behavior and numerical approximation for certain PDEs. In this regard, while some of this special session's speakers will present their results on the qualitative properties of solutions, other speakers will focus on quantitative analysis for such PDE systems, including the formulation of efficient and robust numerical solvers for Navier Stokes, Navier Stokes-elasticity interactive PDE systems as well as the development of associated mixed variational formulations and their companion numerical schemes for these dynamical systems.

Polynomial decay properties of multi-layered elastic-thermal interactions

George Avalos
University of Nebraska-Lincoln, USA

In this work we consider a multilayered wave-heat system where a 3-D wave equation is coupled with a 3-D heat equation via a 2-D interface whose dynamics is described by a 2-D wave equation. In particular, we undertake the problem of obtain explicit uniform rates of decay for smooth solutions of said FSI system; i.e., for solutions which correspond to initial data in the domain of the associated strongly continuous semigroup generator. To the best of our knowledge, this is the first such polynomial stability result obtained for multilayered FSI. By way of obtaining the rational decay result, we operated in the frequency domain and so dealt with a static FSI system: this static PDE system is essentially the image of the resolvent of the semigroup generator, as it acts on given finite energy data.

Analysis of a multiscale model based on the coupling of ODEs and PDEs for tissue perfusion

Lorena Bociu
NC State University, USA

Complex problems in biomedicine often need to be modeled by multiscale couplings between partial differential equations (PDEs) and ordinary differential equations (ODEs), in order to combine the accurate three-dimensional (3D) description of a local region of interest with the global features represented by reduced lumped models. We focus on a PDE/ODE system stemming from the multiscale interface coupling between a local description of tissue perfusion via a 3D deformable porous medium and a 0D lumped hydraulic circuit accounting for the blood circulation to and from the tissue, and study its well-posedness.

Controllability properties of coupled Stokes and Navier Stokes systems

Luz de Teresa
Universidad Nacional Autónoma de México, México
Takeo Takahashi, Yingying Wu-Zhang

In this talk, we discuss null controllability properties for two or more Stokes or Navier Stokes coupled systems. We present a Kalman rank condition that generalizes a similar condition to ODE and coupled heat equations.

Analysis of a rotationally constrained convection model

Yanqiu Guo
Florida International University, USA
Chongsheng Cao, Edriss Titi

This talk is about the analysis of an asymptotically reduced system for rotationally constrained convection. The presence of a dominant balance in equations for fluid flow can be exploited to derive a simpler set of governing equations that permits analytical explorations. For rotation dominated flows, the geostrophic balance occurs: the pressure gradient force is balanced by the Coriolis effect. The Taylor-Proudman constraint suggests that the dominant Coriolis force leads to flows that are organized into vertical columns whose horizontal scale is small compared to the layer height. Applying the asymptotic theory for small Rossby number and tall columnar structures, Julien and Knobloch derived a closed set of reduced equations from the three-dimensional Boussinesq equations. This reduced system is interesting yet challenging for analytical study. On the one hand, the nonlinear convection term has a reduced complexity since it contains only the horizontal gradient. On the other hand, the physical domain remains three dimensional, while the regularizing viscosity acts in the horizontal direction only, creating a major difficulty for establishing the global existence theory. I will present some of our results motivated by the global regularity problem. We show that the model is globally well-posed if regularized by a very weak dissipation. I will also discuss the case of infinite Prandtl number convection, and the situation when both of the Prandtl and Rayleigh numbers approach infinity. This is a joint project with Cao and Titi.

Approximation schemes for the null controllability of structurally damped plate dynamics

Pelin Guven Geredeli
Iowa State University, USA

This work considers a structurally damped elastic equation under hinged boundary condition. Fully-discrete numerical approximation schemes are generated for the null controllability of these parabolic-like PDEs. We mainly use finite element method (FEM) and finite difference method (FDM) approximations to show that the null controllers being approximated via FEM and FDM exhibit exactly the same asymptotics of the associated minimal energy function. These null controllers are also amenable to our numerical implementation in which we discuss the aspects of FEM and FDM numerical approximations and compare both methodologies.

An inverse problem for the Mindlin-Timoshenko system

Shitao Liu
Clemson University, USA
Jason Kurz, Pei Pei

In this talk, we consider an inverse problem for the Mindlin–Timoshenko plate system, which is a strongly coupled two dimensional system consisting of a wave equation and a system of isotropic elasticity that arises in modeling plate vibrations especially at high frequencies and thicker plates. More precisely, we prove the global uniqueness of recovering the plate density from a single boundary measurement of the system under appropriate geometrical assumptions.

Shocks interaction for the Burgers-Hilbert equation

Tien Khai Nguyen
North Carolina State University, USA

In 2009 J. Biello and J. Hunter derived a balance law modeling nonlinear waves with constant frequency, obtained from Burgers' equation by adding the Hilbert transform as a source term. For a general initial data $u \in \mathbb{R}$, the global existence of entropy weak solutions was proved by Bressan and Nguyen, together with a partial uniqueness result. Moreover, piecewise continuous solutions with a single shock and the shock formation have been recently studied. This talk will describe a further type of local generic singularities for solutions, namely, points where two shocks interact.

A robust model reduction for the boundary feedback stabilization of piezoelectric beams

Ahmet Ozkan Ozer
Western Kentucky University, USA
Ahmet Kaan Aydin, Rafi Emran, Jacob Walterman

Piezoelectric materials exhibit electric responses to mechanical stress, and mechanical responses to electric stress. The electrostatic and magnetizable PDE models, describing the longitudinal oscillations on the beam, with boundary feedback sensors/actuators are known to have exponentially stable solutions.
Firstly, a thorough analysis for the maximal decay rate via the optimal choice of feedback sensor amplifiers is discussed. Next, standard Finite Differences and novel order-reduction-based Finite Differences model reductions for these PDEs are proposed. In certain cases, numerical filtering for the spurious high-frequency modes may be unavoidable. These modes simply cause the loss of uniform gap among the eigenvalues as the discretization parameter tends to zero. The exponential decay of the solutions, mimicking the PDE counterparts, can be retained uniformly. A thorough analysis for the maximal decay rate via the optimal choice of feedback sensor amplifiers and the discretization parameter is discussed.
Finally, several interactive numerical tests by Wolfram Demonstrations Projects (WDP) are shared to support our results. These are simply interactive visualizations of the controlled dynamics preserving control-theoretic properties of the PDEs such as observability, controllability, stabilizability. As you move a Demonstration's controls, you see a change in its output that helps you understand the controlled dynamics with optimal feedback controllers.

Existence and uniqueness of solutions to a model describing gas dynamics

Weinan Wang
University of Arizona, USA
Christopher Henderson

The Boltzmann and Landau equations are two fundamental models in kinetic theory. They are nonlocal and nonlinear equations for which (large data) global well-posedness is an extremely difficult problem that is nearly completely open. In this talk, I will discuss two more tractable and related questions: (1) Local well-posedness for the Boltzmann equation and (2) Schauder estimates and their application to uniqueness of solutions to the Landau equation. At the end of the talk, I will discuss some open problems and future work. This is based on joint work with Christopher Henderson.

Sharp stability for the interaction energy

Xukai Yan
Oklahoma State University, USA

For a nonnegative density f and radially decreasing interaction potential W, the interaction energy is given by $E[f] = \int f(x)f(y)W(x-y)dxdy$. The celebrated Riesz rearrangement inequality says that $E[f] \le E[f^*]$, where f^* is the radially decreasing rearrangement of f. In this talk, I will discuss the quantitative estimate of this inequality. I will first make an introduction about the problem and describe some previous results about the stability estimate for characteristic functions. I will then present a recent work with Yao Yao, where we establish the stability estimate for general densities.

Special Session 62: Group Invariant Machine Learning

Jameson Cahill, University of North Carolina Wilmington, USA
Dustin Mixon, Ohio State University, USA

Many data sets display symmetry which can be modeled as a group acting on the data. Accounting for this symmetry can lead to improved performance for a variety of tasks. Such problems require insight from various areas of mathematics.

Coorbit representations of homogeneous metric spaces

Radu Balan
University of Maryland, USA
Efstratios Tsoukanis

We consider unitary representations of finite groups acting on $\mathbb{R}^n$. Such a representation induces a natural metric over the space of orbits, that we call homogeneous metric space.
This paper presents a universal construction of bi-Lipschitz Euclidean embeddings of such homogeneous metric spaces which are intrinsically group invariant.

Invariant machine learning on point clouds

Ben Blum-Smith
Johns Hopkins University, USA
Ningyuan (Teresa) Huang, Alexandra Pevzner, Soledad Villar

Physical systems represented by point clouds tend to be symmetric with respect simultaneously to Euclidean isometries of space and also relabelings of the points. While there are beautiful results from classical invariant theory that characterize functions separately invariant with respect to either of these types of symmetry, there is currently no analogous result for simultaneous invariance under both types of symmetry at once (unless the number of points is small). We review what is known, and discuss workarounds in machine learning contexts for parametrizing the invariant functions in the absence of complete results from the underlying invariant theory. Joint work with Ningyuan (Teresa) Huang, Alexandra Pevzner, and Soledad Villar.

Separating orbits using invariants of low degree

Dan Edidin
University of Missouri, USA

Given a representation V of a compact group G, the ℓ-th moment is a tensor parametrizing the invariant polynomials of degree ℓ. A classical theorem invariant theory implies that the Invariant ring is finitely generated which implies that for ℓ sufficiently large closed orbits can be separated by the moments of order up to ℓ. Unfortunately, the computational cost of computing the ℓ-th moment grows exponentially in ℓ. A problem that was originally motivated cryo-EM is to study representations for which moments of low degree separate (generic) orbits. In this talk we discuss the information determined by the second moment and describe a class of representations of the compact groups of classical type for which the third moment can separate generic orbits. While our original motivation for studying this problem was cryo-EM, we believe that understanding the structure of polynomial invariants has an important role in equivariant machine learning as well.

Numerically stable group invariants

Joseph Iverson
Iowa State University, USA
Jameson Cahill, Dustin G. Mixon, Daniel Packer

Machine learning algorithms are designed for Euclidean space, and the first step of many applications is to represent data in a finite-dimensional Hilbert space V. Frequently, many elements of V represent the same data point, and furthermore, the equivalence classes are orbits of a finite group $G \leq \mathrm{O}(V)$. For instance, a point cloud may be represented as columns of a matrix, but the columns could be permuted in any order. To properly model such data, we seek a Euclidean embedding of the quotient metric space V/G. This amounts to finding a G-invariant function $f\colon V \to \mathbb{R}^n$ that separates orbits. With applications in mind, we would like to choose f so that the resulting map $\tilde{f}\colon V/G \to \mathbb{R}^n$ is bilipschitz.
In this talk, we share some bad news and some good news about such numerically stable group invariants. The bad news is that there usually does **not** exist an invariant polynomial f for which $\tilde{f}$ is bilipschitz, even when restricted to the quotient image of the unit sphere. In fact, if $\tilde{f}$ is lower Lipschitz, then f fails to be differentiable at any point fixed by any non-identity element of G. The good news is that we offer a large and flexible class of bilipschitz invariants, which we call *max filter banks*. Max filters are easy to implement, and many special cases are familiar from the literature. Furthermore, when G is finite, a sufficiently large bank of generically chosen max filters provides a bilipschitz Euclidean embedding of V/G. Based on joint work with Jameson Cahill, Dustin G. Mixon, and Daniel Packer.

Fast principal component analysis for cryo-EM images

Oscar Mickelin
Princeton University, USA
Nicholas F. Marshall, Yunpeng Shi, Amit Singer

This work devises a fast method for estimating the covariance matrix of tomographic projection images affected by high levels of noise in cryo-electron microscopy. The mathematical model for these images consists of observing noisy samples of projections of randomly rotated variables, convolved with oscillatory functions. The method relies on a novel algorithm to expand discretized images in the Fourier-Bessel basis (the harmonics on the disk), which enables a compressed representation and fast estimation of the covariance matrix.

Max filtering with reflection groups

Daniel Packer
The Ohio State University, USA
Dustin Mixon

Given a finite-dimensional real inner product space V and a finite subgroup G of linear isometries, max filtering affords a bilipschitz Euclidean embedding of the orbit space V/G. We identify the max filtering maps of minimum distortion in the setting where G is a reflection group. Our analysis involves an interplay between Coxeter's classification and semidefinite programming

Geometric scattering on measure spaces

Michael Perlmutter
UCLA, USA
Joyce Chew, Matthew Hirn, Smita Krishnaswamy, Deanna Needell, Holly Steach, Siddharth Viswanath, Hau-Tieng Wu

The scattering transform is a multilayered, wavelet-based transform initially introduced as a model of convolutional neural networks (CNNs) that has played a foundational role in our understanding of these networks' stability and invariance properties. Subsequently, there has been widespread interest in extending the success of CNNs to data sets with non-Euclidean structure, such as graphs and manifolds, leading to the emerging field of geometric deep learning. In order to improve our understanding of the architectures used in this new field, several papers have proposed generalizations of the scattering transform for non-Euclidean data structures such as undirected graphs and compact Riemannian manifolds without boundary. In this paper, we introduce a general, unified model for geometric scattering on measure spaces. Our proposed framework includes previous work on geometric scattering as special cases but also applies to more general settings such as directed graphs, signed graphs, and manifolds with boundary. We propose a new criterion that identifies to which groups a useful representation should be invariant and show that this criterion is sufficient to guarantee that the scattering transform has desirable stability and invariance properties. Additionally, we consider finite measure spaces that are obtained from randomly sampling an unknown manifold. We propose two methods for constructing a data-driven graph on which the associated graph scattering transform approximates the scattering transform on the underlying manifold. Moreover, we use a diffusion-maps based approach to prove quantitative estimates on the rate of convergence of one of these approximations as the number of sample points tends to infinity. Lastly, we showcase the utility of our method on spherical images, directed graphs, and on high-dimensional single-cell data.

Injectivity, stability, and positive definiteness of max filtering

Yousef Qaddura
The Ohio State University, USA
Dustin G. Mixon

Given a real inner product space V and a group G of linear isometries, max filtering offers a rich class of G-invariant maps. In this paper, we identify nearly sharp conditions under which these maps injectively embed the orbit space V/G into Euclidean space, and when G is finite, we estimate the map's distortion of the quotient metric. We also characterize when max filtering is a positive definite kernel.

Universality of Andrews networks

Nathaniel Strawn
Georgetown University, USA

Andrews plots provide lossless visualizations of high-dimensional data sets by linearly mapping data points to 1D functions. Employing the subtle nonlinear transformation which sends smooth functions to path integrals over a function's graph, we obtain a neural network architecture admitting visualizations at every single step. In this talk, we demonstrate a universal approximation property of such networks, and discuss applications.

Special Session 63: Analysis and Optimization of Biological and Medical Systems

Brittni Hall, Auburn University, USA
Xiaoying Han, Auburn University, USA
Hans Werner Van Wyk, Auburn University, USA

The development of novel analytical techniques and optimization methodologies for applications in the applied sciences has been attracting much attention during the past decades. This special session focuses on mathematical models for biological and medical applications, in particular, those described by deterministic/stochastic/random ordinary/partial differential equations. A collection of recent advances in theoretical approaches and optimization methodologies to study these types of systems will be presented at this special session.

When can we cluster data? insights from convex optimization and semidefinite programming

Brendan Ames
University of Alabama, USA

Recent years have seen an incredible increase in the use of artificial intelligence and machine learning (AI/ML) methods in essentially all areas of science, technology, engineering, humanities, and education. However, despite this widespread adoption of AI/ML models as analytical tools, relatively very little is known about the theoretical properties of these models. In particular, there is minimal theory justifying their use, despite a large and growing body of empirical evidence of their efficacy. In this talk, I will attempt to partially bridge this gap between theory and performance for a particular machine learning problem: clustering. Clustering is a classical machine learning task where one seeks to partition a given data into subgroups of similar items called clusters. I will argue that the clustering task can be modeled as a family of combinatorial optimization problems, and illustrate how to use methods from semidefinite programming to design efficient and accurate numerical methods for clustering. In particular, I will propose several models for well-behaved or clusterable data, where we can expect these clustering heuristics to correctly identify the hidden cluster structure.

Additional food causes predators to explode - unless the predators compete

Kwadwo Antwi-Fordjour
Samford University, USA
Rana Parshad, Sureni Wickramasooriya, Aniket Banerjee

The literature posits that an introduced predator population is able to drive its target pest population to extinction, if supplemented with high quality additional food of sufficient quantity. We show this approach actually leads to infinite time blowup of the predator population, so is unpragmatic as a pest management strategy. We propose an alternate model in which the additional food induces predator competition. Analysis of this model indicates that depending on the competition parameter c, one can have global stability of the pest free state, bi-stability dynamics, or up to three interior equilibria. As c and the additional food quantity ξ are varied standard co-dimension one and co-dimension two bifurcations are observed. We also use structural symmetries to construct several non-standard bifurcations such as saddle-node-transcritical bifurcation (SNTC) in co-dimension two and a cusp-transcritical bifurcation (CPTC), also in co-dimension two. We further use symmetry to construct a novel pitchfork-transcritical bifurcation (PTC) in co-dimension two, thus explicitly characterizing a new organizing center of the model. Our findings indicate that increasing additional food in predator-pest models can hinder bio-control, contrarily to some of the literature. However, additional food that also induces predator competition, leads to rich dynamics and enhances bio-control.

Comparative analysis of different vector-host epidemic models with direct transmission

Dawit Denu
Georgia Southern University, USA
Sedar Ngoma, Bolaji Rachidi

In this talk, we will propose and analyze four vector-host epidemic models by considering different assumptions on how the infection is transmitted from one group to another. We first examine some common features and similarities of the four models, such as the existence and stabilities of the equilibrium solutions in relation to the basic reproduction number and the existence and uniqueness of a positive global solution. Moreover, we investigate the main differences between these models and study some specific properties of their solutions. Numerical simulations will be included to illustrate some of the theoretical results.

Optimal Control of a Beneficial Bacterial Population in the Gut Mircobiome

Brittni Hall
Auburn University, USA
Hans-Werner Van Wyk, Maggie Han

A mathematical model describing the growth of gut microbiome with one beneficial bacterial microorganism and a limiting nutrient source is developed based on the chemostat model. Motivated by the interesting discovery that time-dependent input of nutrient may result in asymptotic behavior different from constant input, we are interested in investigating how the growth of a particular type of bacteria can be boosted or inhibited to maintain a healthy gut. In this talk, we analyze the microbial population dynamics using an optimal control approach. In particular, we consider a modified model for the dynamics of an intestinal microbiome with one beneficial bacterial microorganism that we are trying to promote by the infusion of some probiotic along with one limiting resource of nutrient population over a given finite time interval, $[0, T]$.

Efficiency of locomotion of n-link snake robots

Daniel Irvine
Georgia Institute of Technology, USA

Consider the problem of optimizing the forward locomotion of a snake robot. Being limbless, this robot achieves motion by changing the shape of its body. We can therefore separate the configuration space of the robot into variables that represent the shape and variables that represent the displacement of the robot. This is the starting point for geometric mechanics. In this talk I will describe how a principal bundle describes the geometry present in the snake robot mechanical system. The physical constraints on the system are encoded in a connection on this principal bundle. Parallel transport over a trajectory in shape space gives the forward displacement effected by that trajectory. Finally, I will describe how a metric on shape space gives a measure of efficiency as a trajectory. On a robot model, this efficiency is given by forward displacement divided by the electrical energy consumed during the trajectory. This is a natural quantity to optimize, and I will discus robot experiments to verify optimal efficiency.

Novel mechanisms for detecting early warning signals of population outbreaks and extinction in a two-timescale predator-prey model

Susmita Sadhu
Georgia College & State University, USA
Saikat C. Thakur

In this talk, I will discuss methods of analyzing transient dynamics to detect early warning signals of population extinction or population outbreaks in a three-dimensional predator-prey model featuring two-timescales. The model under consideration studies the interaction between two species of predators competing for their common prey with explicit interference competition. We will consider two different scenarios in a parameter regime near *singular Hopf bifurcation* of the coexistence equilibrium. In one case, the system exhibits bistability between a periodic attractor and a boundary equilibrium state with long transients characterized by rapid oscillations and slow variation in amplitudes, while in the other case, the system exhibits chaotic *mixed-mode oscillations*, featuring concatenation of small and large-amplitude oscillations as long transients before approaching a stable limit cycle. To analyze the transients, the system is reduce to a suitable normal form. Exploiting the timescales separation and the underlying geometry of the normal form, the transient dynamics are analyzed. The analysis is then used to devise methods for identifying early warning signals of a large population transition leading to an outbreak or extinction of one of the species.

Principles of organization in the yeast genome

Paula Vasquez
University of South Carolina, USA

Chromosomes are not distributed randomly in the nucleus. Understanding the spatial dynamics of the genome is a crucial step in characterizing how DNA adopts and transitions between different functional states over the course of the cell cycle, facilitating vital functions such as gene expression, DNA replication, recombination, and repair.
In this talk we explore the organizational principles of the yeast genome using models derived from polymer physics and Brownian dynamics simulations. This work is a collaboration with Kerry Bloom's lab in the biology department at UNC-CH and Greg Forest's group in the math department at UNC-CH.

Special Session 65: Nonlinear Evolution Equations and Related Topics

Goro Akagi, Tohoku University, Japan
Michinori Ishiwata, Osaka University, Japan
Mitsuharu Otani, Waseda University, Japan

This session will focus on the recent developments in the theory of nonlinear evolution equations and related topics including the theory of abstract evolution equations in Banach spaces as well as the studies of several types of nonlinear partial differential equations (the existence, regularity and asymptotic behavior of solutions).

Rates of convergence to asymptotic profiles for fast diffusion on domains

Goro Akagi
Mathematical Institute, Tohoku University, Japan

This talk is concerned with rates of convergence to asymptotic profiles for (possibly sign-changing) weak solutions to fast diffusion equations on bounded domains. Our method of proofs relies on an energy method for nonlinear evolution equations along with a quantitative gradient inequality, which is newly developed in this work.

Existence of bounded classical solutions to a chemotaxis system modeling tumor angiogenesis

Yutaro Chiyo
Tokyo University of Science, Japan
Masaaki Mizukami

This talk deals with a chemotaxis system for tumor angiogenesis. In a simplified system with constant sensitivity, global existence and boundedness were proved by Tao and Winkler (Nonlinear Analysis; 2021;112324;16 pp). However, a non-simplified system with signal-dependent sensitivities seems not to have been studied yet. The purpose of this talk is to establish global existence and boundedness in a fully parabolic chemotaxis system for tumor angiogenesis with sensitivity functions.

Some remarks about the existence of nonglobal and global solutions of a class of evolution equations

Jorge Esquivel-Avila
Universidad Autónoma Metropolitana, México

We consider a class of second order evolution equations in time. In order to show the non existence of global solutions with high values of the initial energy, we analyze a differential inequality and present a positive invariant set for this purpose. On the other hand, the study of global solutions for high initial values of energy seems not to be a simple problem. We discussed the feasibility of the existence of a new invariant recently proposed in the literature. Our talk is based in the following papers.

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On fractional in time diffusion equations

Ciprian Gal
Florida International University, USA
Mahamadi Warma

This talk provides the introduction to fractional diffusion processes and provides a unified analysis and scheme for the existence and uniqueness of strong and mild solutions to such fractional (in time) kinetic equations. This class of equations is characterized by the presence of a nonlinear time-dependent source, generally of arbitrary growth in the unknown function, a time derivative in the sense of Caputo and the presence of a large class of diffusion operators.

Recent results for the Navier-Stokes-Cahn-Hilliard system with unmatched densities

Andrea Giorgini
Politecnico di Milano, Italy
Helmut Abels, Harald Garcke

We consider the initial-boundary value problem for the incompressible Navier-Stokes-Cahn-Hilliard system with non-constant density proposed by Abels, Garcke and Grün in 2012. This model arises in the diffuse interface theory for binary mixtures of viscous incompressible fluids. In particular, this system is a generalization of the well-known Model H in the case of fluids with unmatched densities. In this talk, I will present some recent results concerning the propagation of regularity of global weak solutions (for which uniqueness is not known) and their longtime convergence towards an equilibrium state in three dimensional bounded domains.

Multi-component Cahn-Hilliard and Allen-Cahn equations

Maurizio Grasselli
Politecnico di Milano, Italy

I intend to present some recent results on two nonlinear evolution phase field systems: the multi-component Cahn-Hilliard equation and the multi-component conserved Allen-Cahn equation. In the case of constant mobility matrix and singular potential, besides well-posedness and regularity, I will discuss the existence of global and exponential attractors as well as the convergence of any finite energy solution to a single stationary state. The validity of the strict separation property from pure phases will also be examined in both cases. All the results have been obtained in collaboration with C.G. Gal, A. Poiatti, J.L. Shomberg (Cahn-Hilliard equation) and A. Poiatti (conserved Allen-Cahn equation).

On critical phenomena for nonlinear heat equations on bounded domains characterized by nonlinear boundary conditions

Kosuke Kita
Waseda University, Japan
Mitsuharu Otani

We consider the existence and nonexistence of global solutions to the initial-boundary value problem of nonlinear heat equations on a bounded domain. For nonlinear heat equations in the whole space, it is well known that there exists the critical Fujita exponent which gives the threshold that divides the existence and nonexistence of positive global solutions. On the other hand, as for the same equation in bounded domains, there is no such critical exponent. In this talk, however, we show that a similar threshold phenomenon can occur in bounded domains, which is controlled according to boundary conditions but not to the exponent of a nonlinear term.

Non-linear evolution equations with nonlocal coefficients arised in chemotaxis models

Akisato Kubo
Fujita Health University, Japan

In this talk we investigate the global existence in time and asymptotic behaviour of solutions of nonlinear evolution equations with strong dissipation and nonlocal coefficient arising in chemotaxis type of mathematical models of biology and medicine. We consider the initial boundary value problem for the equations and show the desired result of it. For this purpose we deal with the problem applying the argument of the singular integral operator to the nonlocal term. Applying our result obtained in the above we study nonlocal chemotaxis models arising from biology and biomedicine and we show the global existence in time and asymptotic profile of the solution to the initial boundary value problem associated with the models.

Qualitative behavior of solutions to a class of Keller-Segel system

Monica Marras
University of Cagliari, Italy
T. Yokota, S. Vernier-Piro

We are interested in qualitative properties as blowup phenomena, decay in time, boundedness, global existence to solutions of some classes of parabolic systems. In particular we consider a chemotaxis system with flux limitation in a bounded and smooth domain $\Omega \subset \mathbb{R}^N$, $N \geq 3$ and we show a criterion which ensure that, under suitable condition on data, the solution blows up in finite time in $L^\infty(\Omega)$ and for some $p > \frac{N}{2}$ it also blows up in $L^p(\Omega) - norm$. Moreover we study the global existence and boundedness of the solution.

Homogenization problem with nonlinear boundary conditions and its applications to optimal design problems

Tomoyuki Oka
University of Tokyo, Japan
Kosuke Kita, Kei Matsushima

In this talk, we shall discuss a homogenization problem with nonlinear boundary conditions.
Main results consist of a homogenization theorem without assumptions for the periodicity of the oscillating coefficient, i.e., convergence of solutions as the oscillating parameter of coefficients goes to zero and characterization of the homogenized equation and an existence theorem for optimal two-phase domains

that minimize energies constructed by solutions of elliptic equations with nonlinear boundary conditions. Furthermore, we shall discuss such domains by employing a level set method based on nonlinear diffusion.

Numerical analysis of fractional nonlinear diffusion equation

Florian Salin
Institut Camille Jordan, France
Goro Akagi, Helene Hivert

This talk is concerned with the Cauchy-Dirichlet problem for a fractional nonlinear diffusion equation posed on a bounded domain. It is well-known that solutions to the standard (non-fractional) fast diffusion equation extinct in finite time, and that rescaled solution converges to asymptotic profiles which are solutions to the Dirichlet problem of a nonlinear elliptic equation. First, we introduce an extension of this result in the fractional case, using an energy method. It is based on precise energy decay estimates, but is only available when the exponent in the nonlinearity is below the Sobolev critical exponent. Then, we introduce a numerical scheme for the simulation of the fractional fast diffusion equation. The scheme is based on a discrete fractional Laplacian recently introduced by Huang and Oberman, which admits a variational structure very similar to the continuous fractional Laplacian, and it allows us to extend energy decay estimates to the numerical scheme. We use this scheme to qualitatively answer questions which are out of the frame of our theoretical result, such as precise estimation of the extinction, time, and convergence to asymptotic profiles when the exponent in the nonlinearity is in the Sobolev super-critical range.

On some evolution equation with irreversibility and energy-conservation arising from fracture mechanics

Kotaro Sato
Tohoku University, Japan
Goro Akagi

This talk deals with some doubly-nonlinear evolution equation, which is related to fracture mechanics and is also rewritten as an evolutionary variational inequality. This evolution equation is beyond the scope of general theory for doubly-nonlinear equations, since the nonlinear operator acting on the time-derivative is singular as well as degenerate due to irreversible and energy-conservative nature of the problem. The main result of this talk is concerned with the existence of strong solutions complying with three intrinsic qualitative properties, that is, irreversibility, unilateral equilibrium and energy conservation, which were originally introduced in a phase-field model for brittle fracture.

Blowup in a generalized diffusive Lotka–Volterra competition model with chemotaxis

Yuya Tanaka
Tokyo University of Science, Japan

This talk deals with a diffusive Lotka–Volterra competition model with chemotaxis. In 2012 Tello–Winkler (Nonlinearity; 2012: 25; 1413–1425) proposed a diffusive Lotka–Volterra competition model having chemotaxis terms. As to this model, boundedness of solutions was obtained under conditions that chemotactic effect is small by e.g. Tello–Winkler (2012) and Mizukami (Math. Methods Appl. Sci.; 2018; 41; 234–249). However, it is not clear whether solutions remain bounded or not in the case that chemotactic effect is large. The purpose of this talk is to discuss this question in a generalized diffusive Lotka–Volterra competition model with chemotaxis.

Solvability of doubly nonlinear parabolic equation with p-Laplacian

Shun Uchida
Oita University, Japan

We consider the initial boundary value problem of some doubly nonlinear parabolic equation with p-Laplacian ($p > 1$). For the case where the exponent p is sufficiently large, Barbu (1979) proved the solvability of this problem without any assumptions on the maximal monotone growth in the time-derivative term via some abstract problem in the Hilbert space. Main purpose of this talk is to show the existence of solution for every small p, where it is difficult to reduce the problem to the Hilbert setting.

Ground state solutions for quasilinear scalar field equations arising in nonlinear optics

Tatsuya Watanabe
Kyoto Sangyo University, Japan

We are interested in following quasilinear elliptic problem:

$$-\mathrm{div}\left\{\phi\left(\frac{u^2+|\nabla u|^2}{2}\right)\nabla u\right\}+\phi\left(\frac{u^2+|\nabla u|^2}{2}\right)u=g(u)$$

in $\mathbb{R}^N$, which appears in nonlinear optics. By using the mountain pass theorem together with a technique of adding one dimension of space and the theory of monotone operator, we prove the existence of a nontrivial weak solution for general nonlinear terms of Berestycki-Lions' type.

Global dynamics in a degenerate chemotaxis model for tumor invasion

Tomomi Yokota
Tokyo University of Science, Japan
Sachiko Ishida

We consider a degenerate chemotaxis model for tumor invasion in a bounded domain $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$, which consists of four equations for u, v, w, z. The first equation has the diffusivity f and the sensitivity g fulfilling $f(u, w) \geq u^{m-1}$ $(m > 1)$, $0 \leq g(u) \leq u^{\alpha}$. It is shown that if $\alpha + 1 < m + \frac{4}{N}$ $(N \geq 2)$, or if $\alpha + 1 = m + \frac{4}{N}$ $(N \geq 3)$ with small initial data, then the system possesses a global bounded weak solution which converges to the constant equilibrium in the weak* topology in $L^\infty(\Omega)$ as $t \to \infty$.

Special Session 66: Dynamics of Biological Materials Across Scales

M. Greg Forest, University of North Carolina Chapel Hill, USA
Qi Wang, University of South Carolina, USA

Studies of biological materials involve interdisciplinary collaborations across multiple disciplines and require experimental resolution and data across temporal and spatial scales. This special session invites a group of active researchers working at the frontier of modeling, analysis, and simulation of complex biological materials across scales, and how these studies are guided and benchmarked by data. Our aim is to showcase previous results and current progress in this rapidly moving field. The topics range from spatial-temporal models of intercellular protein dynamics and cell dynamics to cell aggregate dynamics in the cell self-assembly process in morphogenesis and tumor growth. The session also includes novel numerical algorithms developed specifically for simulations of biological materials.

Geometric structure guided nonnegative matrix factorization model for complete deconvolution of biological data

Duan Chen
University of North Carolina Charlotte, USA
Shaoyu Li, Xue Wang

For many human diseases, differential expression (DE) analysis is one of the important tools to unveil the gene expression profile (GEP) differences between patient and control groups. It will reveal novel insights into the genes and pathways, and is potentially helpful for drug targets and therapeutics. However, a fundamental knowledge gap still remains for DE, concerning whether disease-associated GEP changes in tissue samples are due to changes in their cellular compositions, or due to GEP changes in specific cells. It would be much more informative to study gene expression on specific cells, or identify cell-intrinsic differentially expressed genes (CI-DEGs). But for many complex biological mixtures, such as brain tissues, exhaustive knowledge of individual cell types and their specific markers is lacking. Although single-cell RNA sequencing (RNAseq) data can be used or serve as a reference, such approaches remain costly, cumbersome and limited in sample sizes. In contrast, computational tools can be used to leverage widely available large-scale bulk tissue RNAseq data sets. As a step prior to the DE analysis, bulk tissue GEP data can be de-convoluted as GEP in specific cell types and cellular composition of tissue samples. The basic mathematical model of complete deconvolution is nonnegative matrix factorization (NMF), which is also a major machine learning algorithm used in spectral unmixing in analytical chemistry, remote sensing, image processing and topic mining, etc. NMF is a well-known ill-posed problem and its solution is generally not separable, so a direct application will pose great challenges on interpretability of biological data. Based on the geometric properties of the GEPs in potential marker genes, we propose a geometric structure guided NMF model, for which the weak identifiability conditions of the NMF is partially satisfied. Computational algorithms for the resulting non-convex optimization are developed in the frame work of Alternating direction method of multipliers (ADMM). Our preliminary simulations on synthetic and biological data have shown improved solution separability.

The power of weak binding in biology

M. Greg Forest
University of North Carolina Chapel Hill, USA

An overview will be given of how weak, transient interactions are utilized in diverse biological materials to convey diverse functional properties. Many collaborators will be acknowledged.

Benchmarking immersed boundary models of viscoelastic flows in complex geometries

Cole Gruninger
University of North Carolina Chapel Hill, USA
Aaron Barret, Fuhui Fang, Boyce Griffith, Greg Forest

The immersed boundary (IB) method is widely used for the simulation of biological fluid dynamics and other problems where a structure is immersed in a Newtonian fluid. In the context of complex (viscoelastic) fluids, the IB method has enabled the study of a wide variety of dynamical problems including the settling of vesicles and the swimming of fibers in OldroydB fluids. However, to date, relatively little work has explored the accuracy or convergence properties of the numerical scheme. This talk will present benchmarking results for an IB solver for modeling viscoelastic flows in non-trivial geometries. To do so, I will present a variety of test cases, some inspired from experimental flow geometries and others based on numerical benchmarks obtained from more traditional finite element and finite volume methods.I will also compare the IB method against more complex finite element and finite volume viscoelastic flow solvers and, where possible, to real rheology experiments.

Parameter identifiability analysis and model reduction for data-driven models of biological soft tissues

Mansoor Haider
North Carolina State University, USA

The accurate estimation, physical interpretation, and elimination of parameters in mathematical models often depends on the quantities of interest (QoIs) for which data is available and the structure of the model with respect to its parameters. When data is limited, calibrating models with more than a few parameters can be challenging due to non-identifiabilty of a subset of the model parameters. We present techniques for combining local sensitivity and identifiability analysis with the numerical solution of inverse problems for parameter estimation and model reduction. These techniques are based on a decomposition involving the sensitivity matrix for QoIs tied to the data. Application of these techniques is illustrated for two mathematical models: (i) nonlinear elastic vessel wall deformation in large pulmonary arteries of the cardiovascular system in pulmonary hypertension, and (ii) reaction kinetics for enzyme-mediated polymerization of fibrinogen into insoluble fibrin matrix in a biomimetic wound healing system. In each case, the approach is tailored to the application and the capabilities to preserve or reduce objective cost in the optimization, while also producing reduced models, is demonstrated.

Data-driven modeling on Alzheimer's disease

Wenrui Hao
Penn State University, USA

With over 5 million individuals affected in the US, Alzheimer's disease (AD) has become a pressing concern. Personalized treatment plans for AD patients offer a promising new avenue for managing this disease but require novel approaches for analyzing the increasing amount of electronic brain data available. In this talk, we will introduce a mathematical modeling approach for describing the progression of AD clinical biomarkers and incorporating patient data to enable personalized prediction and optimal treatment. Specifically, we will validate this mathematical model on a multi-institutional dataset of AD biomarkers to provide personalized predictions for AD patients.

Effects of transmural pressure on clot structure and occlusion times

Karin Leiderman
University of North Carolina Chapel Hill, USA
David Montgomery

The size, formation time, and structure of a blood clot depends on the local hemodynamics, transmural pressure, and the nature and size of the injury where the clot is forming. Here we studied extravascular clotting, where blood leaks from a main vessel into the extravascular space. The goal was to better understand how transmural pressures across an injury and injury size affect clot structure and occlusion times. Our approach was using a mathematical model of platelet aggregation and coagulation. The model is based on a continuum approach to track the advection, diffusion, and aggregation of platelet densities in a dynamic fluid environment. We also developed a simplified model of coagulation that considers generation of coagulation enzymes at the injury walls and on the platelet surfaces. We used a finite volume method in a T-shaped geometry where blood flows through a main top channel and escapes through a downward injury channel. We found that higher pressures led to very dense homogenous clots and low pressures led to less dense heterogenous clots. Some clots that occluded in larger injuries continued to grow into the main channel where this was not seen with smaller injuries.

Generalized law of mass action (LMA) with energetic variational approaches (EnVarA)

Chun Liu
Illinois Institute of Technology, USA
Yiwei Wang, Bob Eisenberg

We'll present a systematic variational derivation to generalize the mass action kinetics of chemical reactions with detailed balance using an energetic variational approach. Our approach starts with an energy dissipation law for a chemical reaction system, which could be argued to carry all the information of the dynamics. The dynamics of the system is determined by both the choice of the free energy, as well as the dissipation, the entropy production. This approach enables us to capture the coupling and competition of various mechanisms, including mechanical effects such as diffusion, drift in an electric field, as well as the thermal effects. We will also discuss several practical examples under this approach, in particular, the modeling of wormlike micellar solutions.

Data-driven mathematical modeling, computation and experimental investigation of dynamical heterogeneity in breast cancer

Xinfeng Liu
University of South Carolina, USA

Solid tumors are heterogeneous in composition. Cancer stem cells (CSCs) are a highly tumorigenic cell type found in developmentally diverse tumors that are believed to be resistant to standard chemotherapeutic drugs and responsible for tumor recurrence. Thus understanding the tumor growth kinetics is critical for development of novel strategies for cancer treatment. For this talk, I shall introduce mathematical modeling to study Her2 signaling for the dynamical interaction between cancer stem cells (CSCs) and non-stem cancer cells, and our findings reveal that two negative feedback loops are critical in controlling the balance between the population of CSCs and that of non-stem cancer cells. Furthermore, the model with negative feedback suggests that over-expression of the oncogene HER2 leads to an increase of CSCs by regulating the division mode or proliferation rate of CSCs.

Modelling and dynamics of a three-dimensional virus capsid

Paolo Piersanti
Indiana University Bloomington, USA
Kristen White, Bogdan Dragnea, Roger Temam

In this talk, which is the result of a joint work of the speaker with Kristen White (IU), Bogdan Dragnea (IU) and Roger Temam (IU), we formulate a model describing the deformation of a virus capsid in the context of Atomic Force Microscope experiments.
Since the virus capsid is subjected not to cross a prescribed rigid surface, the problem under consideration is an obstacle problem.
To begin with, we derive the expression of the energy functional modelling the deformation of the virus capsid under consideration in the static case. Numerical experiments are subsequently carried out.
Finally, we examine the dynamical counterpart of the model under consideration. Unlike the static case, the concept of solution is a priori unknown in the time-dependent case. The rigorous concept of solution will be rigorously formulated upon completing the asymptotic analysis of a suitable penalised model.

Kinetic Monte Carlo simulations of multicellular aggregate self-assembly in biofabrication

Yi Sun
University of South Carolina, USA
Xiaofeng Yang, Qi Wang

We present a 3D lattice model to study self-assembly of multicellular aggregates by using kinetic Monte Carlo (KMC) simulations. This model is developed to describe and predict the time evolution of postprinting structure formation during tissue or organ maturation in a novel biofabrication technology–bioprinting. Here we simulate the self-assembly and the cell sorting processes within the aggregates of different geometries, which can involve a large number of cells of multiple types.

Macro-micro simulations in complex biological fluids using GPU Computing

Paula Vasquez
University of South Carolina, USA
Michael Cromer

Some of the most remarkable properties and functions served by biological fluids originate from the interplay between external fields and microstructural dynamics. From a computational point of view this generates a set of challenges related to the need of coupling dynamics at different length and times scales, sometimes spanning several orders of magnitude. Micro-macro simulations have gained a lot of recognition within the field because these methods allow capturing full dynamics at the macroscale without losing resolution at the microscale. In this talk, we will review our efforts to couple existing macroscopic solvers for the Navier-Stokes equations with microstructural dynamics described by Langevin-type equations. In particular, we will discuss dumbbells models -under viscometric and capillary thinning flows fields- and parallel computing using GPUs.

Collective motion of active particles on surfaces

Qi Wang
University of South Carolina, USA
Jun Li

We study collective motion of active particles on three prescribed surfaces with distinct topological and geometrical properties. Kinematics of the active particles on the surfaces is driven by selfpropelling, particle-particle interaction, surface constraining and under-damped stochastic forces described by Ornstein-Uhlenbeck processes. We demonstrate the prevailing collective patterns in the active particle systems on the three types of surfaces: a sphere, a torus and a hill and valley landscape with distinct topological and geometrical properties. We note that all the sustainable, spatial-temporal patterns are profoundly affected by the curvature of the surfaces as well as their symmetry. In particular, we find that the large magnitude of curvature in the hill and valley landscape coupled with certain surface symmetry warrants a spatial-temporal periodic traveling rings pattern which synchronizes the collective movement of the active particles with the symmetry in the landscape. However, the large magnitude of curvature alone without the necessary surface symmetry is not sufficient to sustain such a periodic, spatial-temporal pattern, instead collective motion settles into cyclic rotation.

How math and AI are transforming biosciences

Guowei Wei
Michigan State University, USA

Mathematics underpins fundamental theories in physics such as quantum mechanics, general relativity, and quantum field theory. Nonetheless, its success in modern biology, namely cellular biology, molecular biology, chemical biology, genomics, and genetics, has been quite limited. Artificial intelligence (AI) has fundamentally changed the landscape of science, engineering, and technology in the past decade and holds a great future for discovering the rules of life. However, AI-based biological discovery encounters challenges arising from the intricate complexity, high dimensionality, nonlinearity, and multiscale biological systems. We tackle these challenges by a mathematical AI paradigm. We have introduced persistent cohomology, persistent spectral graphs, persistent path Laplacians, persistent sheaf Laplacians, and evolutionary de Rham-Hodge theory to significantly enhance AI's ability to tackle biological challenges. Using our mathematical AI approaches, my team has been the top winner in D3R Grand Challenges, a worldwide annual competition series in computer-aided drug design and discovery for years. By further integrating mathematical AI with millions of genomes isolated from patients, we uncovered the mechanisms of SARS-CoV-2 evolution and accurately forecast emerging SARS-CoV-2 variants.

Modeling viscoelasticity and mechanical feedback on growth in tumor spheroids

Min Wu
Worcester Polytechnic Institute, USA
Nonthakorn Olaranont, Chaozhen Wei, John Lowengrub

Cell proliferation, apoptosis, and myosin-dependent contraction can generate elastic stress and strain in living tissues, which may be dissipated by internal rearrangement through cell topological transition and cytoskeletal reorganization. Moreover, cells and tissues can change their sizes in response to mechanical cues. We develop a thermodynamically consistent model to describe the above properties at the continuum level. By linearizing the model, we show that the stress follows the Maxwell-type viscoelastic relaxation. The rearrangement rate, which we call tissue fluidity, sets the stress relaxation time, and the ratio between the shear modulus and the fluidity sets the tissue viscosity.

By applying the model to tumor spheroid growth, we demonstrate the role of tissue mechanical properties, internal rearranging activities, and mechanical feedback on its size and mechanics regulation, in the context of differential growth induced by a field of growth-promoting chemical factors. We constrain the model by fitting experimental data of tumor spheroid growth and compare the results with previous modeling results.

A model for microtubule-mediated deformation of a cellular nucleus

Yuan Young
New Jersey Institute of Technology, USA
Reza Farhadifar, Michael J. Shelley

The cellular nucleus is enclosed by a permeable membrane mechanically supported by a meshwork of lamin fibers. The morphology and integrity of the nucleus are essential for the cell's function. Recent experiments show that loss of the lamin network results in nuclear deformations and rupture. To understand the mechanistic basis of this phenomenon, we developed a mathematical model that accounts for, and couples, the fluid flows around and through the permeable membrane, and the pulling on the membrane by membrane-bound, but mobile, molecular motors attached to impinging microtubules. Here the microtubules are assumed to nucleate from a cellular centrosome. We found that this model predicts the formation of a sharp corner in the vicinity of the centrosome, rather reminiscent of the Taylor cone for a surfactant-laden drop in an elongational flow. We analyze the equilibrium shape of the membrane in terms of the total number of motors and their mobility in the nuclear membrane. Our model provides a more mechanistic understanding of nuclear deformation in cells and can give insights into the correspondence between motor forces and membrane deformation leading to nuclear rupture, which has been observed in some cancer cells. Using numerical simulations we further investigate how the distribution of molecular motors couples to the deformation and shape dynamics of the nucleus.

Phase field Modeling of Dictyostelium Discoideum chemotaxis

Yanxiang Zhao
George Washington University, USA
Yunsong Zhang, Herbert Levine

A phase field approach is proposed to model the chemotaxis of Dictyostelium discoideum. In this framework, motion is controlled by active forces as determined by the Meinhardt model of chemical dynamics which is used to simulate directional sensing during chemotaxis. Then, the movement of the cell is achieved by the phase field dynamics, while the reaction-diffusion equations of the Meinhardt model are solved on an evolving cell boundary. This task requires the extension of the usual phase-field formulation to allow for components that are restricted to the membrane. The coupled system is numerically solved by an efficient spectral method under periodic boundary conditions. Numerical experiments show that our model system can successfully mimic the typically observed pseudopodia patterns during chemotaxis.

Special Session 68: (In)Stability and the Long Time Behaviour of Fluid Flows

Miroslav Bulìček, Charles University, Prague, Czech Republic
Vit Prusa, Charles University, Prague, Czech Republic

We focus on the dynamical systems driven by systems of nonlinear partial differential equations describing flows of fluids with complicated rheology (heat conducting viscoelastic fluids etc.). Recently several methods have been developed for studying the (non)uniqueness and/or the (in)stability of the solution. The main purpose of the special session is to present and discuss new results in the field, to give/propose an optimal criteria for uniqueness and stability or for non-uniqueness and instability of the solution and to present new optimal regularity results. The emphasis will be on the complicated models going much beyond the standard Navier-Stokes equations.

Stokes problem with dynamic boundary

Tomas Barta
Charles University, Prague, Czech Republic
P. Davies, P. Kaplicky

We study the Stokes problem with dynamic boundary conditions in a bounded domain with a smooth boundary. Since the problem combines evolutionary equations in the interior of the domain and on its boundary, the abstract reformulation works on a space which is a product of spaces in the interior and on the boundary. We show that the corresponding operator is the generator of an analytic semigroup and since we work in Hilbert spaces we obtain maximal $L^p - L^2$ regularity.

Viscoelastic rate-type fluids: Existence, regularity, stability

Miroslav Bulìček
Charles University, Czech Republic

Viscoelastic rate-type fluid models involving the stress and its observer-invariant time derivatives of higher order are used to describe the behaviour of materials with complex microstructure: geomaterials like asphalt, biomaterials such as vitreous in the eye, synthetic rubbers such as styrene butadiene rubber. A standard model that belongs to the category of viscoelastic rate-type fluid models of the second order is the model due to Burgers, which can be viewed as a mixture of two Oldroyd-B models of the first order. This viewpoint allows one to develop the whole hierarchy of generalized models of the Burgers type. We study such generalizations that can be viewed as a combination (mixture) of several classical viscoelastic models having in general two different relaxation mechanisms. We discuss the conditions guaranteeing the global in time and large data existence of a weak solution as well as the conditions for regularity and the (in)stability of the solution.

Stability of equilibria to generalized Navier-Stokes-Fourier system

Petr Kaplicky
Charles University, Czech Republic
A. Abbatiello, M. Bulìček

We consider a generalized Newtonian incompressible heat conducting fluid with prescribed nonuniform temperature on the boundary and with the noslip boundary conditions for the velocity. The fluid occupies a three dimensional domain. No external body forces are applied to the fluid. In dependence on the growth of the constitutively determined part of the Cauchy stress we identify different classes of proper solutions that converge to the equilibria exponentially in a suitable metric. Consequently, the equilibrium is nonlinearly stable and attracts all weak solutions from these classes. We also show that these classes are nonempty.

Stability result for the 2D Boussinesq equations with horizontal dissipation

Bataa Lkhagvasuren
Chonnam National University, Korea

The stability of smooth solutions of 2D anisotropic Boussinesq equations with horizontal dissipation remains open problem. In this work, we present a partial answer to this problem in a rougher function space. The main tools used are the Littlewood-Paley theory and standard techniques.

Long time behavior of Navier-Stokes equations with dynamic boundary condition

Dalibor Prazak
Charles University, Czech Republic
B. Priyasad, M. Zelina

There has been a recent interest in the study of fluids with unusual boundary conditions. In this talk, we will consider class of both Newtonian and non-Newtonian incompressible fluids coupled with a nonlinear dynamics on the boundary. – We will show that global attractors exist under general conditions both in 2D and 3D setting. Moreover, in 2D, we will provide sharp estimates of the attractor dimension.

Nonlinear stability and nonequilibrium thermodynamics – there and back again

Vit Prusa
Charles University, Czech Republic

We discuss the role of thermodynamics in nonlinear stability analysis of spatially distributed dissipative systems governed by nonlinear partial differential equations. We document profound interplay between various concepts in thermodynamics on one side and nonlinear stability analysis on the other side, and subsequently we summarise and comment on various results regarding the nonlinear stability of thermodynamically isolated as well as thermodynamically open systems.

On elastic solids with strain-gradient elastic boundary surfaces

Casey Rodriguez
University of North Carolina Chapel Hill, USA

Since the study of surface tension by Gibbs, it has become clear that surface stresses must be accounted for when modeling elastic bodies at small length scales. In this talk we report on a recently proposed mathematical model of a bulk solid containing a boundary surface with strain-gradient surface elasticity. The partial differential equations governing equilibrium states are the Euler-Lagrange equations associated to a Lagrangian energy functional with a novel surface energy density depending on the deformed surface's relative curvature, stretching, and stretching gradient. After considering its mathematical precursors and properties, we then discuss its promise for modeling fracture without the pathological singularities arising in more classical models.

On attractors and regularity for flows with dynamic slip boundary condition

Michael Zelina
Charles University, Czech Republic
Dalibor Pražàk

We consider an incompressible, non-Newtonian fluid of power-law type confined to a bounded domain in either $\mathbb{R}^2$ or $\mathbb{R}^3$. On its boundary, we consider the so-called dynamic slip boundary condition, i.e.

$$\partial_t \boldsymbol{u} + s(\boldsymbol{u}) + [\boldsymbol{S}(D\boldsymbol{u})\boldsymbol{n}]_\tau = 0,$$

where s has some non-linear, possibly even implicit, relation to the velocity field $\boldsymbol{u}$ and $\boldsymbol{S}$ is the Cauchy stress. Our primary goal is to study if such a model possesses a finite-dimensional attractor.
Firstly, we use an iteration scheme in Nikolskii-Bochner spaces to obtain additional fractional time regularity, provided that the power-law exponent is in the supercritical range $r \geq 11/5$. Thanks to this regularity, we can use a method of trajectories to obtain the attractor. Moreover, its dimension is finite, and it is even the exponential attractor, provided that s satisfies some reasonable growth condition.
In the two-dimensional setting, we are able to achieve also a better space regularity, namely $L^\infty_{\mathrm{loc}}(0,T;W^{2,q}(\Omega))$. With such regularity, it is now possible to show the differentiability of the solution operator. This opens the way to apply a method of Lyapunov exponents, which gives us a much better dimension estimate.

Special Session 70: Fractional Calculus: Theory, Methods and Applications

Khaled Furati, King Fahd University of Petroleum and Minerals, Saudi Arabia
Mokhtar Kirane, Khalifa University, United Arab Emirates

This session intends to cover a large spectrum of directions in fractional calculus not only in theory, in applications in various sciences, but also in their numerical analysis.

Qualitative properties of solutions to a nonlinear time-space fractional diffusion equation

Meiirkhan Borikhanov
Khoja Akhmet Yassawi International Kazakh-Turkish University, Kazakhstan
Michael Ruzhansky, Berikbol T. Torebek

In the present paper, we study the Cauchy-Dirichlet problem to a nonlocal nonlinear diffusion equation with polynomial nonlinearities

$$\mathcal{D}^{\alpha}_{0|t}u+(-\Delta)^s_p u = \gamma|u|^{m-1}u+\mu|u|^{q-2}u,\ \gamma,\mu\in\mathbb{R},\ m>0, q>1,$$

involving time-fractional Caputo derivative $\mathcal{D}^{\alpha}_{0|t}$ and space-fractional p-Laplacian operator $(-\Delta)^s_p$.
We give a simple proof of the comparison principle for the considered problem using purely algebraic relations, for different sets of γ, μ, m and q.
The Galerkin approximation method is used to prove the existence of a local weak solution. The blowup phenomena, existence of global weak solutions and asymptotic behavior of global solutions are classified using the comparison principle.

Numerical methods for stiff nonlinear fractional-order equations

Khaled Furati
King Fahd University of Petroleum and Minerals, Saudi Arabia

Nonlinear fractional-order equations are effective models for many complex phenomena in science and engineering. In general, these systems are stiff or have rapid oscillatory behavior, Furthermore, their solutions could be singular at the initial time. In this talk, we introduce high-order multistep and predictor-corrector generalized exponential time differencing schemes combined with graded mesh to mitigate the singularity effect. Theoretical results and numerical experiments will be presented.

Mathematical and numerical study of 1D fractional Schrödinger equations via semiclassical approximations

Songting Luo
Iowa State University, USA
Yijin Gao, Paul Sacks

In this talk, we will discuss the possibility of extending the WKB approximations for the standard Schrödinger equations to the fractional Schrödinger equations. We will conduct a preliminary study on 1D fractional Schrödinger equations, along with numerical experiments.

Finite volume method for solving fractional differential equations

Deepthika Senaratne
Fayetteville State University, USA
Champike Attanayake

We analyze the finite volume scheme for the Riemann-Liouville fractional differential equations. We produce error estimates in an energy norm and L_2 norm for the approximated solution. Numerical results are supplied to justify the theoretical work.

Decay estimates for the time-fractional evolution equations with time-dependent coefficients

Berikbol Torebek
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Asselya Smadiyeva

In this talk, the initial-boundary value problems for the time-fractional degenerate evolution equations are considered. Firstly, in the linear case, we obtain the optimal rates of decay estimates of the solutions. The decay estimates are also established for the time-fractional evolution equations with nonlinear operators such as: p-Laplacian, the porous medium operator, degenerate operator, mean curvature operator, and Kirchhoff operator. At the end, some applications of the obtained results are given to derive the decay estimates of global solutions for the time-fractional Fisher-KPP equation and the time-fractional porous medium equation with the nonlinear source.

Special Session 71: At the Edge of Ellipticity

Fabiana Leoni, Sapienza Università di Roma, Italy
Isabeau Birindelli, Sapienza Università di Roma, Italy
Sergio Polidoro, Università di Modena e Reggio Emilia, Italy

Diffusive phenomena are described through elliptic equations and typically the diffusive term is represented by the Laplacian that corresponds to a uniform diffusion. This need not be the case, for instance diffusion can happen along fixed directions: in the black death epidemic, contagion advanced along roads and from there spread inwards leading to a front like invasion of Western Europe roughly from South to North, and this effect of roads still matters in modern epidemics, as COVID-19 shows, [BRR]. Weakening the notion of ellipticity allows a more effective description of the reality, as occurs in models of mathematical finance, kinetic theory, mathematical physics, and engineering. In this Session we plan to tackle several problems that lie at the edge of ellipticity. Of course this edge is very vast, we will focus on:

- subelliptic operator
- fully nonlinear operators
- higher order operators

The concept of degeneracy serves as a bridge between the known phenomena typical of uniformly elliptic equations, and the development of new mathematical ideas needed to shed some light on some very unusual properties of degenerate PDEs.

FKPP equations with degenerate diffusion: Traveling waves and first considerations

Isabeau Birindelli
Sapienza Università di Roma, Italy

Introduced independently by Fisher and Kolmogorov, Petrovski and Piskunov in 1937 to study the spatial propagation of a genetic trait, this type of equations has since been used in various contexts, ranging from population dynamics to ecology, biology, epidemiology, phase changes, combustion theory, and to social sciences and other fields. In this project, we want to extend the questions described above to the case where the diffusion is not necessarily homogenous or linear i.e. when the diffusion operator is either degenerate elliptic or singular elliptic, or it could be degenerate elliptic and fully nonlinear. This is a very first attempt in this direction and a work in progress. In particular I will show some interesting new phenomena that arise when the operator is one of the so called truncated Laplacians that have been studied in recently in works in collaboration with Galise and Ishii. These would model diffusions that are only in the direction of some eigenvalues of the Hessian, i.e. the diffusion is in the directions where the growths is more decreasing or more increasing depending on the choice of the model.

The Nehari manifold for a degenerate logistic parabolic equation

Juliana Fernandes da Silva
Universidade Federal do Rio de Janeiro, Brazil
L. Maia

We analyse the behavior of solutions to a degenerate logistic equation with a nonlinear term of the form $b(x)f(u)$, where the weight function b is assumed to be nonpositive. We exploit variational techniques and comparison principle in order to study the evolutionary dynamics. A crucial role is then played by the Nehari manifold, as we note how it changes as the parameter λ in the equation or the function b vary, affecting the existence and non-existence of stationary solutions. We describe a detailed picture of the positive dynamics and also address the local behavior of solutions near a nodal equilibrium, which sheds some further light on the study of the evolution of sign-changing solutions.

Propagation of minima for nonlocal operators

Giulio Galise
Sapienza Università di Roma, Italy
Isabeau Birindelli, Hitoshi Ishii

I will present recent results concerning the geometry of the sets of minima for supersolutions of equations involving the k-th fractional truncated Laplacian and the k-th fractional eigenvalue which are fully nonlinear integral operators whose nonlocality is somehow k-dimensional. This is joint work with Isabeau Birindelli (Sapienza University, Italy) and Hitoshi Ishii (Tsuda University, Japan).

Symmetry and monotonicity results for solutions of semilinear PDEs in sector-like domains

Antonio Greco
University of Cagliari, Italy
Francesca Gladiali

In the field of PDEs, it is frequently observed that solutions whose Morse index is small have a simpler shape in comparison to others. I report on a recent work with Francesca Gladiali from the University of Sassari (Italy) concerning a mixed boundary-value problem for a semilinear elliptic equation with a convex nonlinearity in a sector-like domain. Using cylin-

drical coordinates (r, θ, z), we investigate the shape of possibly sign-changing solutions whose derivative in θ vanishes at the boundary. We prove that any solution whose Morse index does not exceed 1 must either be axially symmetric (i.e., radial in the plane), or strictly monotone with respect to θ. The proof is based on a rotating-plane argument. Classical references on Morse theory are those of Bott, Milnor, and Morse. More specific background on the talk's subject is found in papers by Gladiali, Pacella, Weth and in the book by Damascelli and Pacella.

Principal eigenvalues for fully nonlinear equations in punctured balls

Fabiana Leoni
Sapienza Università di Roma, Italy
I. Birindelli, F. Demengel

We present recent existence and regularity results for radial eigenfunctions and eigenvalues of fully nonlinear equations posed in punctured balls, in presence of singular potentials. We show cases of existence of bounded solutions, of existence of unbounded solutions and of non existence of solutions. Furthermore, we give for Pucci's operators and quadratic potential the explicit value of the eigenvalue, which generalizes the explicit value of the Hardy–Sobolev constant for the Laplacian.

Reverse Faber-Krahn inequality for a truncated Laplacian operator

Enea Parini
Aix Marseille Université, France

In this talk we will consider a reverse Faber-Krahn inequality for the principal eigenvalue $\mu_1(\Omega)$ of the fully nonlinear operator

$$\mathcal{P}_N^+ u := \lambda_N(D^2 u),$$

where $\Omega \subset \mathbb{R}^N$ is a bounded, open convex set, and $\lambda_N(D^2u)$ is the largest eigenvalue of the Hessian matrix of u. The result will be a consequence of the isoperimetric inequality

$$\mu_1(\Omega) \leq \frac{\pi^2}{\mathrm{diam}(\Omega)^2}.$$

Moreover, we will discuss the minimization of μ_1 under various kinds of constraints. The results have been obtained in collaboration with Julio D. Rossi and Ariel Salort (Buenos Aires).

Comparison principles for general potential theories and fully nonlinear PDEs with directionality

Kevin Payne
Università di Milano, Italy
Marco Cirant, Reese Harvey, Blaine Lawson, Davide Redaelli

We present some recent advances in the productive and symbiotic interplay between general potential theories (subharmonic functions associated to closed subsets $\mathcal{F} \subset \mathcal{J}^2(X)$ of the 2-jets on $X \subset \mathbb{R}^n$ open) and subsolutions of degenerate elliptic PDEs of the form $F(x, u, Du, D^2u) = 0$. We will describe the *monotonicity-duality* method begin by Harvey and Lawson [Comm. Pure Appl. Math, 2009] for proving comparison principles for potential theories where $\mathcal{F}$ has *sufficient monotonicity* and *fiberegularity* (in variable coefficient settings) and which carry over to all differential operators F which are *compatible* with $\mathcal{F}$ in a precise sense.
Particular attention will be given to *gradient dependent* examples with the requisite sufficient monotonicity of *proper ellipticity* and *directionality*. Examples operators we will discuss include those of *optimal transport* in which the target density is strictly increasing in some directions as well as operators which are parabolic in the sense of Krylov. Further examples, modeled on *hyperbolic polynomials* in the sense of Gårding, produce additional examples in which the comparison principles holds, but standard viscosity structural conditions fail to hold.

On a kinetic equation in special relativity

Sergio Polidoro
Università di Modena e Reggio Emilia, Italy
Francesca Anceschi, Annalaura Rebucci

We are concerned with the study of a second order degenerate kinetic operator in the framework of special relativity. In classical mechanics, the Fokker-Planck equation describes the diffusion of particles in the phase space, and satisfies a weak Hörmander condition. Despite of its many physical applilcations, an undesirable feature of its classical diffusion term is that it operates with infinite velocity, which is in conflict with special relativity. We consider the differential operator

$$\mathcal{L}f(p,y,t) = \sqrt{p^2+1}\,\frac{\partial}{\partial p}\left(\sqrt{p^2+1}\,\frac{\partial f}{\partial p}\right) - p\frac{\partial f}{\partial y} - \sqrt{p^2+1}\,\frac{\partial f}{\partial t}$$

which has the additional property of being invariant with respect to the Lorentz change of variable. Furthermore, $\mathcal{L}$ can be written in the Hörmander's form $\mathcal{L} = X^2 + Y$. This paves the way to a systematic study of the operator $\mathcal{L}$ in the framework of degenerate Hörmander's operators in Lie groups. Our main results are a Lorentz-invariant Harnack type inequality, and an accurate asymptotic lower bound for the fundamental solution to $\mathcal{L}f = 0$.

Asymptotics for a class of parabolic equations with critical nonlinearities

Federica Sani
University of Modena and Reggio Emilia, Italy
Michinori Ishiwata, Bernhard Ruf, Elide Terraneo

In this talk, we discuss the dichotomy between blowup and global existence for solutions of the Cauchy problem for a heat equation with initial data in $H^1(\mathbb{R}^N)$. We consider non-homogeneous nonlinearities with polynomial growth (when $N \geq 3$) and exponential growth (when $N = 2$) which are critical in the energy space $H^1(\mathbb{R}^N)$ according to the Sobolev and the Trudinger-Moser inequality respectively. By means of energy methods, we study the asymptotic behavior of solutions with low energies, and we show that the splitting between blowup and global existence is determined by the sign of a suitable functional.

Regularity estimates with optimized constants

Boyan Sirakov
PUC-Rio, Brazil
Philippe Souplet

We present several classical regularity estimates for general uniformly elliptic operators of second order in divergence form and unbounded coefficients, giving explicit and optimal dependence of the constants in these inequalities in terms of Lebesgue norms of the lower order coefficients of the operator, and the size of the domain. Among these estimates are the interior and global Harnack inequalities, the Hopf lemma, L^∞-estimates, and logarithmic gradient estimates. Applications include the Landis conjecture and the Vazquez strong maximum principle for operators with unbounded coefficients.

Special Session 72: Optimal Transport and Mean Field Games with Applications and Computations

Zixuan Cang, North Carolina State University, USA
Wuchen Li, University of South Carolina, USA
Yanxiang Zhao, George Washington University, USA

Optimal transport (OT) and mean field game (MFG) study strategic decision making in large populations where the individual players interact with each other and each individual is effected only by certain averaged quantities of all the other individuals. They have become central topics in the past decade both theoretically and numerically. While OT and MFG have been successfully applied in areas such as social sciences, image science and machine learning sampling problems, there is an urgent need to develop new model, design stable and efficient numerical schemes and find new applications for OT and MFG. Typical examples include cell biology, data science, geology, and epidemiology. This minisymposium aims to bring together researchers to discuss their recent advances in techniques, insights, and understanding to OT/MFG arising in several applications such as life science and data science.

High order spatial discretization for variational time implicit schemes: Wasserstein gradient flows and reaction-diffusion systems

Guosheng Fu
University of Notre Dame, USA
Siting Liu, Stanley Osher, Wuchen Li

We design and compute first-order implicit-in-time variational schemes with high-order spatial discretization for initial value gradient flows in generalized optimal transport metric spaces. We first review some examples of gradient flows in generalized optimal transport spaces from the Onsager principle. We then use a one-step time relaxation optimization problem for time-implicit schemes, namely generalized Jordan-Kinderlehrer-Otto schemes. Their minimizing systems satisfy implicit-in-time schemes for initial value gradient flows with first-order time accuracy. We adopt the first-order optimization scheme ALG2 (Augmented Lagrangian method) and high-order finite element methods in spatial discretization to compute the one-step optimization problem. This allows us to derive the implicit-in-time update of initial value gradient flows iteratively. We remark that the iteration in ALG2 has a simple-to-implement point-wise update based on optimal transport and Onsager's activation functions. The proposed method is unconditionally stable for convex cases. Numerical examples are presented to demonstrate the effectiveness of the methods in two-dimensional PDEs, including Wasserstein gradient flows, Fisher–Kolmogorov-Petrovskii-Piskunov equation, and two and four species reversible reaction-diffusion systems. This is a joint work with Siting Liu, Stanley Osher from UCLA and Wuchen Li from U. South Carolina.

Tangential Wasserstein projections with applications to causal inference

Florian Gunsilius
University of Michigan, USA
Meng Hsuan Hsieh, Myung Jin Lee

We develop a notion of projections between sets of probability measures using the geometric properties of the 2-Wasserstein space. In contrast to existing methods, it is designed for multivariate probability measures that need not be regular, is computationally efficient to implement via regression, and provides a unique solution in general. The idea is to work on tangent cones of the Wasserstein space using generalized geodesics. Its structure and computational properties make the method applicable in a variety of settings where probability measures need not be regular, from causal inference to the analysis of object data. An application to estimating causal effects yields a generalization of the synthetic controls method for systems with general heterogeneity described via multivariate probability measures.

Master equations for finite state mean field games with nonlinear activations

Jina-Guo Liu
Duke University, USA
Yuan Gao, Wuchen Li, Jian-Guo Liu

We formulate a class of mean field games on a finite state space with variational principles resembling continuous state mean field games. We construct a controlled continuity equation with a nonlinear activation function on graphs induced by finite reversible Markov chains. With this controlled dynamics on graph and the dynamic programming principle for value function, we derive the mean field game systems, the functional Hamilton-Jacobi equations and the master equations on the finite probability space for potential mean field games. We also give a variational derivation for the master equations of non-potential games and mixed games on a finite state space. Finally, several concrete examples of discrete mean field game dynamics on a two-point space are presented with closed formula solutions, including discrete Wasserstein distances, mean field planning, and potential mean field games.

Parametrization and computation of Wasserstein Hamiltonian flows

Shu Liu
UCLA, USA
Hao Wu, Xiaojing Ye, Haomin Zhou

In this work, we propose a numerical method to compute the Wasserstein Hamiltonian flow (WHF), which is a Hamiltonian system on the probability density manifold. Many well-known PDE systems can be reformulated as WHFs. We use parameterized function as push-forward map to characterize the solution of WHF, and convert the PDE to a finite dimensional ODE system, which is a Hamiltonian system in the phase space of the parameter manifold. We establish error analysis results for the continuous time approximation scheme. For the numerical implementation, we use neural networks as push-forward maps. We apply an effective symplectic scheme to solve the derived Hamiltonian ODE system while preseving some of its important quantities such as total energy. The computation is done by fully deterministic symplectic integrator without any neural network training. Thus, our method does not involve direct optimization over network parameters and hence can avoid the error introduced by stochastic gradient descent (SGD) methods, which is usually hard to quantify and measure. The proposed algorithm is a sampling-based approach that scales well to higher dimensional problems. In addition, the method provides an alternative connection between the Lagrangian and Eulerian perspectives of the original WHF through the parameterized ODE dynamics.

Approximations and learning in the Wasserstein space

Caroline Moosmueller
University of North Carolina Chapel Hill, USA
Alex Cloninger, Keaton Hamm, Harish Kannan, Varun Khurana, Jinjie Zhang

Detecting differences and building classifiers between distributions, given only finite samples, are important tasks in a number of scientific fields. Optimal transport (OT) has evolved as the most natural concept to measure the distance between distributions and has gained significant importance in machine learning. There are some drawbacks to OT: computing OT can be slow, and it often fails to exploit reduced complexity in case the family of distributions is generated by simple group actions.
If we make no assumptions on the family of distributions, these drawbacks are difficult to overcome. However, in the case that the measures are generated by push-forwards of elementary transformations, forming a low-dimensional submanifold in the Wasserstein space, we can deal with both of these issues on a theoretical and computational level. In this talk, we'll show how to embed the space of distributions into a Hilbert space via linearized optimal transport, and how linear techniques can be used to classify different families of distributions generated by elementary transformations and perturbations. The proposed framework significantly reduces both the computational effort and the required training data in supervised learning settings. We demonstrate the algorithms in pattern recognition tasks in imaging and provide some medical applications.

Optimally transporting active fluids

Suraj Shankar
Harvard University, USA

The classical problem of optimal mass transport has been explored for over two centuries with deep connections to economics, hydrodynamics, and machine learning. But much less is known about how to optimally transport physical materials, such as active fluids, that obey complex spatiotemporal and autonomous dynamics. Using minimal models to describe the fluid dynamics of active materials such as bacterial drops or motile cells, I will describe an optimal control framework for manipulating internal stresses in the fluid to transport active drops with the least amount of energy dissipated. By combining numerical solutions and analytical insight, I will highlight simple principles and characteristic trade-offs that govern the optimal policies, suggesting general strategies for optimal transportation in a wide variety of synthetic and biological active systems. Time permitting, I will also discuss more recent work on extending some of these ideas to more complex situations such as the transport of localized excitations in active fluids.

TrajectoryNet: A dynamic optimal transport network for modeling cellular dynamics

David van Dijk
Yale University, USA
Alexander Tong, Jessie Huang, Guy Wolf, Smita Krishnaswamy

It is increasingly common to encounter data from dynamic processes captured by static cross-sectional measurements over time, particularly in biomedical settings. Recent attempts to model individual trajectories from this data use optimal transport to create pairwise matchings between time points. However, these methods cannot model continuous dynamics and non-linear paths that entities can take in these systems. To address this issue, we establish a link between continuous normalizing flows and dynamic optimal transport, that allows us to model the expected paths of points over time. Continuous normalizing flows are generally under constrained, as they are allowed to take an arbitrary path from the source to the target distribution. We present TrajectoryNet, which controls the continuous paths taken between distributions to produce dynamic optimal transport.

We show how this is particularly applicable for studying cellular dynamics in data from single-cell RNA sequencing (scRNA-seq) technologies, and that TrajectoryNet improves upon recently proposed static optimal transport-based models that can be used for interpolating cellular distributions.

Special Session 73: Data-Driven Methods in Dynamical Systems

Ruhui Jin, University of Wisconsin Madison, USA
Shi Chen, University of Wisconsin Madison, USA
Qin Li, University of Wisconsin Madison, USA

Dynamical systems arise in a broad range of interdisciplinary fields, describing the evolutions of climate, networks, neuroscience and physical models. With advanced data-driven methodology, there has been significant development in learning the underlying governing equations from data. Theoretical and computational challenges accordingly arise. This session aims to present recent works in data-driven dynamical system learning and the related areas including machine learning, inverse problem and uncertainty quantification.

Global convergence of gradient descent for multi-layer ResNets with homogeneous activation functions in the mean-field regime

Shi Chen
University of Wisconsin Madison, USA
Zhiyan Ding, Qin Li, Stephen J. Wright

Finding the optimal configuration of parameters in ResNet is a nonconvex minimization problem, but first order methods often find the global optimum when the network is overparameterized and the training algorithm is run for sufficiently many iterations. We study this phenomenon in the mean-field regime, where the network can be described approximately by an ordinary integral equation (OIE) and the training process of ResNet becomes a gradient-flow partial differential equation (PDE).
Under the condition that the activation function is 2-homogeneous or partially 1-homogeneous, we show that this gradient-flow PDE converges to the global minimum.
This result suggests that if the ResNet is sufficiently large, first order optimization methods can find global minimizers that fit the training data as well.
Further, by controlling the generalization error, we prove that the gradient-flow PDE is stable with respect to its cost function. This result implies that a finite-sized but sufficiently large dataset drawn from the underlying data distribution is sufficient to exhibit the properties of the continuous limit.
We give lower bounds on the depth and width of the network for the gradient-flow approximation to hold. We also lower-bound the size of the training data set needed to attain accurate approximation.

Gradient flows for sampling: Affine invariance and numerical approximations

Yifan Chen
Caltech, USA
Daniel Huang, Jiaoyang Huang, Sebastian Reich, Andrew Stuart

Sampling a target distribution with an unknown normalization constant is a fundamental problem in data driven inference. Using dynamical systems to generate solutions to approach the target gradually has been a compelling idea. In this talk, we focus on probability gradient flows as the dynamical system and study several related foundational questions in sampling distributions. Any implementation of a gradient flow needs an energy functional, a metric, and a numerical approximation scheme. We show how KL divergence is a special and unique energy functional and how the affine invariant property in the metric can improve convergence. We also discuss numerical approximations that lead to implementable methods such as interacting particles, parametric variational inference, and Kalman approaches.

Deep operator learning lessens the curse of dimensionality for PDEs

Ke Chen
University of Maryland at College Park, USA
Chunmei Wang, Haizhao Yang

Deep neural networks (DNNs) have seen tremendous success in many fields and their developments in PDE-related problems are rapidly growing. Our work provides an estimate for the generalization error of learning Lipschitz operators over Banach spaces using DNNs with applications to various PDE solution operators. The goal is to specify DNN width, depth, and the number of training samples needed to guarantee a certain testing error.
Under mild assumptions on data distributions or operator structures, our analysis shows that deep operator learning can have a relaxed dependence on the discretization resolution of PDEs and, hence, lessen the curse of dimensionality in many PDE-related problems. We apply our results to various PDEs, including elliptic equations, parabolic equations, and Burgers equations.

Learning the evolution of unknown systems via deep neural networks

Victor Churchill
The Ohio State University, USA
Dongbin Xiu

Many phenomena in science and engineering are observable but not explainable. That is, we can observe solution data generated from many physical systems, but the actual physics, e.g. an ordinary or partial differential equation model, are unknown. In this case, developing a deep neural network based model that replicates the system's behavior is desirable. Hence in this talk, we will explore how to learn the time evolution of unknown ODE and PDE systems from their solution data using deep neural networks. The specific network architectures used are grounded in numerical methods for solving ODEs and PDEs. We also considering the case of partially observing the solution vector, where a time history of the observed variables are required.

Fast, low-memory methods for radiative transfer via hp-adaptive mesh refinement

Shukai Du
University of Wisconsin Madison, USA
Samuel N. Stechmann

Numerical solutions to the radiative transfer equation are typically computationally expensive. The large expense arises because the solution has a high dimensionality with NM degrees of freedom, where the N and M arise from spatial and angular degrees of freedom, respectively. Here, a numerical method is presented that aims for fast and low-memory calculations, in the sense of computational cost and memory requirements of only O(N). The method uses a discontinuous Galerkin (DG) spectral element method and hp-adaptive mesh refinement to reduce the number of spatial degrees of freedom from N to n, thereby reducing the total cost and memory to nM, with the aim of achieving nM approximately equal to N. After this reduction in memory to O(N), in order to ensure a computational cost of O(N), a suitable solver is identified and utilized. Numerical examples are presented showing large memory reduction ratios N/n and fast O(N) computational cost. A variety of examples is shown, including smooth spatial variations or steep gradients, and Rayleigh (isotropic) or Mie (anisotropic) scattering. The methods could enable more tractable computations for many applications, such as medical imaging and weather and climate prediction.

Bandlimited graph signal recovery from randomized space-time samples

Longxiu Huang
Michigan State University, USA
Deanna Needell, Sui Tang

Heat diffusion processes have found wide applications in modeling dynamical system over graphs. In this talk, I will talk about the recovery of a k-bandlimited graph signal that is an initial signal of a heat diffusion process from its space-time samples. In this work, we have proposed three random space-time sampling regimes, termed dynamical sampling techniques, that consist in selecting a small subset of space-time nodes at random according to some probability distribution. We show that the number of space-time samples required to ensure stable recovery for each regime depends on a parameter called the spectral graph weighted coherence, that depends on the interplay between the dynamics over the graphs and sampling probability distributions. Then, we propose a computationally efficient method to reconstruct k-bandlimited signals from their space-time samples. We prove that it yields accurate reconstructions and that it is also stable to noise. Finally, we test dynamical sampling techniques on a wide variety of graphs. The numerical results on support our theoretical findings and demonstrate the efficiency.

Tensor-structured sketching for constrained optimization

Ruhui Jin
University of Wisconsin Madison, USA
Ke Chen

Constrained least squares problems arise in many data-driven applications, for instance in dynamical system inference. The memory and computation costs are expensive in practice involving high-dimensional input data. We employ the so-called sketching strategy to project the least squares problem onto a space of a much lower sketching dimension via a random sketching matrix. In the meantime, the approximation accuracy is preserved.
Tensor structure is often present in the data matrices of least squares, including linearized inverse problems and tensor decompositions .In this work, we utilize a general class of row-wise tensorized sub-Gaussian matrices as sketching matrices in constrained optimizations for the sketching design's compatibility with tensor structures. We provide theoretical guarantees on the sketching dimension in terms of error criterion and probability failure rate. Our theories are demonstrated in a few concrete examples, including unconstrained linear regression and sparse recovery problems.

Advancements in reduced order modeling and physics-informed neural networks for solving large scale partial differential equations

Tulin Kaman
University of Arkansas, USA
Xuan Gu, Tulin Kaman

This presentation highlights recent advancements in reduced order modeling (ROM) and physics-informed neural networks (PINNs) for solving large scale partial differential equations (PDEs). Specifically, we compare several projection-based reduced order models (PROMs) with traditional finite element solvers on elliptic and parabolic problems, where the PDEs are first discretized and parameterized into a full order linear finite element system using NumPy/SciPy packages. The full order model is then projected onto a reduced space spanned by a reduced basis, which is constructed using strong greedy algorithms, proper orthogonal decomposition, and weak greedy algorithms. The coupled computation framework utilizes the discretization and reduced basis construction in the open-source pyMOR package, integrated with the finite element solver package FEniCS. Additionally, we apply physics-informed neural networks to incompressible and compressible Navier-Stokes equations to compare them with direct numerical solvers for both forward and inverse problems. Coarse grid solutions generated by numerical solvers are used to train and test PINNs using the open-source DeepXDE package with multiple parameter setups. Finally, we demonstrate the advantages and limitations of each ROM and PINNs approach under different conditions to illustrate their efficiency and accuracy in solving large-scale PDEs.

Scalable multi-species agent-based modeling with sparse GP

Charles Kulick
University of California Santa Barbara, USA
Sui Tang

We approach the data-driven learning problem for a general second order ODE agent-based system with multiple species. By modeling with interaction kernels, we can use a Gaussian Process approach to learn a nonparametric model for the dynamical system with built-in uncertainty quantification. In this talk, we develop the modeling system, present theoretical analysis on the learning methodology, and present empirical investigations into scalability and practicality using a biological predator-prey model.

Combining stochastic parameterized reduced-order models with machine learning for data assimilation and uncertainty quantification with partial observations

Changhong Mou
University of Wisconsin Madison, USA
Leslie M Smith, Nan Chen

A hybrid data assimilation algorithm is developed for complex dynamical systems with partial observations. The method starts with applying a spectral decomposition to the entire spatiotemporal fields, followed by creating a machine learning model that builds a nonlinear map between the coefficients of observed and unobserved state variables for each spectral mode. A cheap low-order nonlinear stochastic parameterized extended Kalman filter (SPEKF) model is employed as the forecast model in the ensemble Kalman filter to deal with each mode associated with the observed variables. The resulting ensemble members are then fed into the machine learning model to create an ensemble of the corresponding unobserved variables. In addition to the ensemble spread, the training residual in the machine learning-induced nonlinear map is further incorporated into the state estimation that advances the quantification of the posterior uncertainty. The hybrid data assimilation algorithm is applied to a precipitating quasi-geostrophic (PQG) model, which includes the effects of water vapor, clouds, and rainfall beyond the classical two-level QG model. The complicated nonlinearities in the PQG equations prevent traditional methods from building simple and accurate reduced-order forecast models. In contrast, the SPEKF model is skillful in recovering the intermittent observed states, and the machine learning model effectively estimates the chaotic unobserved signals. Utilizing the calibrated SPEKF and machine learning models under a moderate cloud fraction, the resulting hybrid data assimilation remains reasonably accurate when applied to other geophysical scenarios with nearly clear skies or relatively heavy rainfall, implying the robustness of the algorithm for extrapolation.

A causality-based learning approach for discovering the underlying dynamics of complex systems from partial observations with stochastic parameterization

Yinling Zhang
University of Wisconsin Madison, USA
Nan Chen

Discovering the underlying dynamics of complex systems from data is an important practical topic. Constrained optimization algorithms are widely utilized and lead to many successes. Yet, such purely data-driven methods may bring about incorrect physics in the presence of random noise and cannot easily handle the situation with incomplete data. In this paper, a new iterative learning algorithm for complex turbulent systems with partial observations is developed that alternates between identifying model structures, recovering unobserved variables, and estimating parameters. First, a causality-based learning approach is utilized for the sparse identification of model structures, which takes into account certain physics knowledge that is pre-learned from data. It has unique advantages in coping with indirect coupling between features and is robust to stochastic noise. A practical algorithm is designed to facilitate causal inference for high-dimensional systems. Next, a systematic nonlinear stochastic parameterization is built to characterize the time evolution of the unobserved variables. Closed analytic formula via efficient nonlinear data assimilation is exploited to sample the trajectories of the unobserved variables, which are then treated as synthetic observations to advance a rapid parameter estimation. Furthermore, the localization of the state variable dependence and the physics constraints are incorporated into the learning procedure. This mitigates the curse of dimensionality and prevents the finite time blowup issue. Numerical experiments show that the new algorithm identifies the model structure and provides suitable stochastic parameterizations for many complex nonlinear systems with chaotic dynamics, spatiotemporal multiscale structures, intermittency, and extreme events.

Special Session 74: Local and Nonlocal Fully Nonlinear Partial Differential Equations of Elliptic and Parabolic Type

Fernando Charro, Wayne State University, USA
Pablo Raul Stinga, Iowa State University, USA

This session is concerned with recent developments in the analysis of elliptic and parabolic fully nonlinear partial differential equations, both local and nonlocal. Equations in this class feature different regularization effects and include uniformly elliptic equations, degenerate equations such as Monge-Ampere, the p-Laplacian, and the infinity Laplacian, obstacle problems, transmission problems, among other. Topics under consideration are the regularity of solutions, its existence, uniqueness, and possible multiplicity. This session, that brings together a combination of experienced and young researchers, will contribute to the dissemination of recent results, and networking with the most qualified investigators in the field.

Perturbative results for parabolic optimal transport

Farhan Abedin
Lafayette College, USA
Jun Kitagawa

I will present results on a parabolic flow of Monge-Ampere type whose solution converges in the infinite-time limit to the Kantorovich potential of the optimal transport problem. Existing results pertaining to this flow assume the so-called Ma-Trudinger-Wang (MTW) regularity criterion on the cost function. I will discuss joint work with Jun Kitagawa (Michigan State University) where we show that, even when the MTW condition fails by a quantitative amount, the parabolic Monge-Ampere equation still exhibits desirable asymptotic behavior in the infinite-time limit.

Rectifiability for points with positive Alt-Caffarelli-Friedman limit

Mark Allen
Brigham Young University, USA
Dennis Kriventsov, Robin Neumayer

We study the interfacial regularity between a pair of nonnegative subharmonic functions with disjoint positivity sets. The portion of the interface where the Alt-Caffarelli-Friedman (ACF) monotonicity formula is asymptotically positive forms an $\mathcal{H}^{n-1}$ rectifiable set. Moreover, for $\mathcal{H}^{n-1}$-a.e. such point, the two functions have unique blowups, i.e. the Lipschitz rescaling converge in $W^{1,2}$ to a pair of non degenerate truncated linear functions whose supports meet at the approximate tangent plane.

Periodic solutions for critical fractional problems

Vincenzo Ambrosio
Università Politecnica delle Marche, Italy

In this talk, I will focus on the existence of 2π-periodic solutions to the following fractional critical problem:

$$\begin{cases} (-\Delta+m^2)^s u - m^{2s} u = f(x,u) + W(x)|u|^{2^*_s-2}u & \text{in } (-\pi,\pi)^N, \\ u(x+2\pi e_i) = u(x) \text{ for all } x\in\mathbb{R}^N, i=1,\dots,N, \end{cases}$$

where $s \in (0,1)$, $m \geq 0$, $N \geq 4s$, $2^*_s = \frac{2N}{N-2s}$ is the fractional critical exponent, $W(x)$ is a 2π-periodic positive continuous function, and $f(x,u)$ is a superlinear 2π-periodic (in x) continuous function with subcritical growth and (e_i) is the canonical basis in $\mathbb{R}^N$. When $m > 0$, I will show the existence of a nonconstant periodic solution by using the extension method in periodic setting and applying the Linking theorem. The case $m = 0$ will be studied by means of a limit procedure.

Nonlocal mean curvature with integrable kernel

Animesh Biswas
Iowa State University, USA
Mikil Foss, Petronela Radu

The focus of this talk will be on the recently introduced topic of nonlocal curvature, defined via an integral operator with a kernel J where J is a radially symmetric, integrable, nonincreasing and nonnegative convolution kernel. Several papers have studied the case of nonlocal curvature with nonintegrable singularity which requires the regularity of the boundary to be at least twice differentiable. Nonlocal curvature of this form appears in many different applications, such as image processing, curvature driven motion, deformations. Our results offer some generalizations and extensions to the constant mean curvature problem, where counterparts to Alexandrov theorem in the nonlocal framework with the highly singular kernel were established independently by two separate groups: Ciraolo, Figalli, Maggi, Novaga, and respectively, Cabré, Fall, Solá-Moreles, Weth. By using the concept of nonlocal curvature for integrable kernels J, as discussed by Mazón, Rossi, Toledo, we are able to lower requirements on the smoothness of the boundary.

Partial regularity for elliptic and parabolic systems with Orlicz growth

Teresa Isernia
Università Politecnica delle Marche, Italy

In this talk I will speak about partial regularity results for weak solutions to elliptic and parabolic systems satisfying ellipticity and growth conditions in terms of Orlicz functions. The main result is obtained by using a new $\mathcal{A}$-caloric approximation lemma compatible with an Orlicz setting.
This talk is based on joint works with M. Foss, C. Leone and A.Verde.

Interior gradient estimates for the special Lagrangian equation

Connor Mooney
University of California Irvine, USA
Arunima Bhattacharya, Ravi Shankar

The special Lagrangian equation is the potential equation for volume-minimizing Lagrangian graphs. Good regularity results are available when the Lagrangian phase is large, which corresponds to the equation being concave. On the other hand, little is known when the phase is small. For example, it is open whether interior gradient estimates hold in the case of small phase. We will discuss some results which are steps towards answering this question, including a rigidity result that rules out counterexamples with homogeneous structure, and an interior gradient estimate for large variable phase. The latter is joint work with A. Bhattacharya and R. Shankar.

Bifurcation in regularized two-phase, elliptic-parabolic free-boundary problems

Nurul Raihen
Stephen F Austin State University, USA
Fernando Charro, Alaa Haj Ali, Monica Torres, Peiyong Wang

In this talk, we will briefly introduce the free- or moving-boundary problems and their applications in many areas of mathematics and science. Then, we will focus on our recent work, where we showed a bifurcation phenomenon in a two-phase, singularly perturbed, free-boundary problem of phase transition. We show that the uniqueness of the solution breaks down for boundary values below a threshold and three solutions appear, the harmonic or trivial solution, a minimizer of the functional, and a mountain-pass solution. Then, we discuss the stability of the solutions by considering the convergence of a related evolution problem.

A diffuse interface soap film capillarity model

Daniel Restrepo
University of Texas Austin, USA
Francesco Maggi, Mike Novack

In this talk, we present a diffuse interface formulation of the well-known Plateau's problem (finding a minimal surface with a given boundary) with two parameters: thickness and volume. More precisely, we consider the family of problems

$$\inf\left\{\int_\Omega \epsilon|\nabla u|^2+\frac{1}{\epsilon}W(u)\mid\int_\Omega \mathcal{V}(u)=\delta\,,\ \text{“}u\text{ spans a given wire frame”}\right\}.$$

Where u represents the density distribution of the soap particles, the (unbounded) set Ω represents the exterior of a wire, δ the volume of the soap film, and ϵ its corresponding thickness. This point of view introduces the additional interfacial length scale to the Plateau's problem with volume from King-Maggi-Stuvard, which is recovered in the limit of vanishing interfacial thickness.
We will discuss existence of solutions for the diffuse interface problem for small thickness, convergence to the sharp interface problem introduced in King-Maggi-Stuvard, and partial regularity of minimizers. Additionally, we will discuss how the optimal regularity of minimizers u of the diffuse interface problem is closely related with the regularity of a new type of free boundary problem for semilinear elliptic equations.

SQG on bounded domains

Logan Stokols
Duke University, USA
Alexis Vasseur

The surface quasigeostrophic (SQG) equation is of interest both for its physical relevance to meteorology and climate science, and for its interesting mathematical properties. While well-posedness for this equation is well known on $\mathbb{R}^2$ and on the torus, more general domains Omega are only recently studied. We will discuss the SQG equation on bounded domains and the new difficulties introduced.

On a binary-ternary Boltzmann equation

Maja Taskovic
Emory University, USA
Ioakeim Ampatzoglou, Irene M. Gamba, Natasa Pavlovic

This talk will focus on a kinetic equation that models the evolution of a gas in which particles undergo binary and ternary interactions. We will discuss global well-posedness of the binary-ternary Boltzmann equation, and the generation and propagation of polynomial and exponential moments. Moment estimates, in particular, show that the presence of both binary and ternary collisions yields better results compared to the equations modeling purely binary or purely ternary interactions.

Special Session 75: Recent Developments in Nonlinear PDEs, Non-Uniformly Elliptic Problems and Related Topics

Alessio Fiscella, Università degli Studi di Milano-Bicocca, Italy
João Vitor da Silva, Universidade Estadual de Campinas, Brazil

The aim of this session is to present recent developments and future directions in the field of nonlinear elliptic PDEs with general structure, which embraces nonstandard growth conditions, non-uniform ellipticity and related topics. Besides analysis of classical nonlinear problems, the topics of the session include free boundary problems, calculus of variations and new trends on PDEs featuring non-standard growth conditions in the local and nonlocal scenarios. In particular, the existence, multiplicity, regularity and qualitative properties of solutions for modern nonlinear PDEs will be discussed.

A large class of nonlocal elliptic equations with singular nonlinearities

Rakesh Arora
Indian Institute of Technology at Varanasi, India
Phouc-Tai Nguyen, Vicentiu D. Radulescu

In this talk, we address the questions of existence, uniqueness, and boundary behavior of the positive weak-dual solution of $\mathbb{L}^s_\gamma u = \mathcal{F}(u)$, posed in a smooth bounded domain $\Omega \subset \mathbb{R}^N$ with appropriate homogeneous Dirichlet or outer boundary conditions. The operator $\mathbb{L}^s_\gamma$ belongs to a general class of nonlocal operators including typical fractional Laplacians, restricted fractional Laplacian (RFL), censored fractional Laplacian (CFL), and spectral fractional Laplacian (SFL). The nonlinear term $\mathcal{F}(u)$ covers three different amalgamation of singular nonlinearities with singular exponent $q > 0$, in particular, $F(u) \sim u^{-q}$ purely singular nonlinearity, $F(u) \sim u^{-q} + f(u)$ singular nonlinearity with a source term and $F(u) \sim u^{-q} - g(u)$ singular nonlinearity with an absorption term. Based on a precise analysis of the Green kernel, we develop a new unifying approach that empowered us to construct a theory for nonlocal elliptic equations involving singular nonlinearities. In particular, we show the existence of critical singular exponents $q^*_{s,\gamma}$ and $q^{**}_{s,\gamma}$ which provides a fairly complete classification of nonlocal elliptic equations with singular nonlinearities via subtle boundary behavior of the weak-dual solution. Various types of nonlocal operators are discussed to exemplify the wide applicability of our theory.

Maximal and minimal weak solutions for elliptic coupled systems with nonlinearity on the boundary

Shalmali Bandyopadhyay
University of North Carolina Greensboro, USA
Nsoki M Mavinga, Thomas Lewis

We consider the existence of weak solutions for semilinear elliptic coupled system with quasimonotone non decreasing nonlinearity on the boundary. We establish the existence of a maximal and a minimal weak solution between an ordered pair of sub- and supersolution. To prove the result, we utilize the surjectivity of a pseudomonotone and coercive operator, Zorn's lemma and a version of Kato's inequality.

A system of equations involving the fractional p-Laplacian and doubly critical nonlinearities

Mousomi Bhakta
Indian Institute of Science Education and Research, India
Kanishka Perera, Firoj SK

This paper deals with existence of solutions to the following fractional p-Laplacian system of equations

$$\begin{cases} (-\Delta_p)^s u = |u|^{p^*_s-2}u + \frac{\gamma\alpha}{p^*_s}|u|^{\alpha-2}u|v|^\beta & \text{in } \Omega, \\ (-\Delta_p)^s v = |v|^{p^*_s-2}v + \frac{\gamma\beta}{p^*_s}|v|^{\beta-2}v|u|^\alpha & \text{in } \Omega, \end{cases}$$

where $s \in (0,1)$, $p \in (1,\infty)$ with $N > sp$, α, $\beta > 1$ such that $\alpha + \beta = p^*_s := \frac{Np}{N-sp}$ and $\Omega = \mathbb{R}^N$ or any smooth bounded domains in $\mathbb{R}^N$. When $\Omega = \mathbb{R}^N$ and $\gamma = 1$, we show that any ground state solution of the above system has the form $(\lambda U, \tau\lambda V)$ for certain τ is a positive constant and U, V are two positive ground state solutions of $(-\Delta_p)^s u = |u|^{p^*_s-2}u$ in $\mathbb{R}^N$. When $\Omega = \mathbb{R}^N$, we also establish existence of positive radial solutions to the above system in various ranges of γ. On the other hand, when Ω is any ball, we show existence of a positive radial solutions to the above system for all $\gamma > 0$.

A transmission problem for first and second order operators

Héctor Chang-Lara
CIMAT, México
Arturo Arellano

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz boundary and let $\Omega_+ \subset \Omega$ be a sub-domain with the exterior ball property. We establish existence and uniqueness of viscosity solutions for a class of transmission problems governed by elliptic and eikonal type equations in Ω_+ and $\Omega_- := \Omega \setminus \Omega_+$ respectively. The main motivation is the Hamilton-Jacobi equation that results of the following optimal control problem: The goal is to minimize the expected time a particle takes from some initial position $x \in \Omega$ until it exits Ω for the first time. The controller is allowed to choose at each moment the direction that the particle takes whenever this is in the region Ω_-, being the speed equal to one. Over Ω_+ the particle performs instead a Brownian motion.

On logarithmic double phase problems

Angel Crespo-Blanco
Technische Universität Berlin, Germany
Patrick Winkert

In this talk I will speak about quasilinear elliptic problems involving the logarithmic double phase operator. Mainly I will focus on the properties of the appropiate functional space and of the differential operator.

Regularity properties in obstacle-type problems for higher-order fractional powers of the Laplacian

Donatella Danielli
Arizona State University, USA
Alaa Haj Ali, Arshak Petrosyan

In this talk we will discuss a sampler of obstacle-type problems associated with higher-order fractional Laplacians. Our goals are to establish regularity properties of the solution and to describe the structure of the free boundary. To this end, we combine classical techniques from potential theory and the calculus of variations with more modern methods, such as the localization of the operator and monotonicity formulas.

New regularity estimates for fully nonlinear elliptic equations

Thialita Nascimento
University of Central Florida, USA
Eduardo V. Teixeira

In this talk, we present new universal bounds for the Hessian integrability exponent of viscosity supersolutions of fully nonlinear, uniformly elliptic equations. Such estimates yield a quantitative improvement in the decay of this exponent with respect to the dimension. In particular, we solve, in the negative, the Armstrong-Silvestre-Smart Conjecture on the optimal exponent for the Hessian integrability. This is joint work with Eduardo Teixeira.

Boundary weak Harnack estimates and regularity for elliptic PDE in divergence form

Boyan Sirakov
PUC-Rio, Brazil
Fiorella Rendón, Mayra Soares

We obtain a global extension of the classical weak Harnack inequality which extends and quantifies the Hopf-Oleinik boundary-point lemma, for uniformly elliptic equations in divergence form. Among the consequences is a boundary gradient estimate, due to Krylov and well-studied for non-divergence form equations, but completely novel in the divergence framework. Another consequence is a new more general version of the Hopf-Oleinik lemma.

Almgren type monotonicity formulas

Mariana Smit Vega Garcia
Western Washington University, USA
Blair Davey

In this talk, we will explore the celebrated Almgrens monotonicity formula. This beautiful result with far-reaching consequences states that if u is harmonic in the unit ball, then a certain frequency function N(r) is non-decreasing. Moreover, N(r)=k for all r

$\mathcal{C}^1$-regularity for vicosity solutions of free boundary problems with gradient constraint

Aelson Sobral
UFPB/UCF, USA
D. Araùjo, E. Teixeira

In this talk, we will discuss $\mathcal{C}^1$ regularity results for viscosity solutions of certain free boundary problems with gradient constraint of the form

$$G(Dv, D^2v) \in L^\infty(\{|Dv| > \mu\}).$$

Due to the lack of appropriate structure, solutions are to be understood throughout a limiting procedure and we prove gradient regularity results that remain uniform in the process. To do so, we first get compactness by the method of doubling the variables and we combine it with a Bernstein-type argument. We provide some applications and, in particular, we are able to study the behavior of the family of normalized solutions $\{v_q\}$ to

$$|Dv_q|^q \Delta v_q = 1$$

as $q \to \infty$.

Surfaces of minimum curvature variation

Pablo Raul Stinga
Iowa State University, USA
Luis A. Caffarelli, Hernan Vivas

Surfaces whose curvature minimizes the Dirichlet energy are central in applications such as surface design in industry and architecture, and are generally constructed by using computer-aided design (CAD). We present the system of equations and prove the first result on existence of classical solutions. This is joint work with Luis A. Caffarelli (UT Austin) and Hernan Vivas (Universidad Nacional de Mar del Plata, Argentina).

Special Session 77: Analysis and Applications of Nonlinear Elliptic and Parabolic Equations

Yuanzhen Shao, University of Alabama, USA
Patrick Guidotti, University of California Irvine, USA
Hengrong Du, Vanderbilt University, USA

This special session will focus on the analysis of nonlinear elliptic and parabolic equations and their applications to mathematical physics and other applied sciences. Topics will include modeling and/or analysis of diffusion, phase transition, fluid dynamics as well as kinetic theory problems stemming from physics, biology, and image processing. We will put particular emphasis on the study of existence, uniqueness, regularity, global existence and stability, singularity formation and the motion of free interfaces in Euclidean space or on manifolds.

Cubic Schrödinger equation with randomized initial conditions

Juraj Foldes
University of Virginia, USA
Jean-Baptiste Casteras, Gennady Uraltsev

During the talk, we will discuss the local solutions of the super-critical cubic Schr odinger equation (NLS) on the whole space with general differential operator. Although such problem is known to be ill-posed, we show that the random initial data yield almost sure local well-posedness. Using estimates in directional spaces, we improve and extend known results for the standard Schrödinger equation in various directions: higher dimensions, more general operators, weaker regularity assumptions on the initial conditions. In particular, we show that in 3D, the classical cubic NLS is stochastically, locally well-posed for any initial data with regularity in H^{ε} for any $\varepsilon > 0$, compared to the known results $\varepsilon > \frac{1}{6}$.

Weak compactness property of simplified nematic liquid crystal flows in dimension two

Tao Huang
Wayne State University, USA
Hengrong Du, Changyou Wang

For any bounded smooth domain in dimension two, we establish the convergence of weak solutions of the Ginzburg-Landau approximation of the simplified Ericksen-Leslie system to a weak solution of the simplified Ericksen-Leslie system associated with either uniaxial or biaxial nematics, as the Ginzburg-Landau parameter tends to zero. We will also show the compactness property of weak solutions to the simplified Ericksen-Leslie system associated with either uniaxial or biaxial nematics.

Particle trajectories of large scale oceanic flow

Ning Ju
Oklahoma State University, USA

I will present my recent research results on global existence and uniqueness of particle trajectories for the vector field of viscous Primitive Equations for large scale oceanic flow.

Temperature effects and ideal gas Stokes flow

Chun Liu
Illinois Institute of Technology, USA
Jan-Eric Sulzbach

Abstract: In this work, we will introduce a general framework to derive thermodynamics of a mechanical system, which guarantee the consistence between the energetic variational approaches with the laws of thermodynamics. In particular, we will focus on the coupling between the thermal and mechnical forces. We will alose present the existences of weak solutions to these systems.

Supercritical partial differential equations and the calculus of variations on convex subsets

Abbas Momeni
Carleton University, Canada

In this talk, we shall provide a comprehensive variational principle that allows one to apply critical point theory on closed proper subsets of a given Banach space and yet, to obtain critical points with respect to the whole space. This variational principle has many applications in partial differential equations while unifies and generalizes several results in nonlinear Analysis such as the fixed point theory, critical point theory on convex sets and the principle of symmetric criticality.

The anisotropic Bernstein problem

Connor Mooney
University of California Irvine, USA
Yang Yang

The Bernstein problem asks whether entire minimal graphs in $\mathbb{R}^{n+1}$ are necessarily hyperplanes. It is known through spectacular work of Bernstein, Fleming, De Giorgi, Almgren, Simons, and Bombieri-De Giorgi-Giusti that the answer is positive if and only if $n < 8$. The anisotropic Bernstein problem asks the same question about minimizers of parametric elliptic functionals, which are natural generalizations of the area functional that both arise in many applications and offer important technical challenges. We will discuss the recent solution of this problem (the answer is positive if and only if $n < 4$). This is joint work with Y. Yang.

Quantitative rigidity of elliptic differential inclusions in two dimensions

Guanying Peng
Worcester Polytechnic Institute, USA
Xavier Lamy, Andrew Lorent

For any compact connected one-dimensional submanifold $K \subset \mathbb{R}^{2\times 2}$ without boundary which has no rank-one connection and is elliptic, we prove the quantitative rigidity estimate

$$\inf_{M\in K} \int_{B_{1/2}} |Du - M|^2 \, dx \leq C \int_{B_1} \operatorname{dist}^2(Du, K) \, dx, \quad \forall u \in H^1(B_1; \mathbb{R}^2).$$

This is an optimal generalization, for compact connected submanifolds of $\mathbb{R}^{2\times 2}$ without boundary, of the celebrated quantitative rigidity estimate of Friesecke, James and Müller for the approximate differential inclusion into $SO(n)$. Furthermore, no analogous result can hold true in $\mathbb{R}^{n\times n}$ for $n \geq 3$. This is joint work with Xavier Lamy and Andrew Lorent.

On the Cahn-Hilliard equation on manifolds with conical singularities

Pedro Tavares Paes Lopes
Universidade de São Paulo, Brazil
Nikolaos Roidos

We present new results on the Cahn-Hilliard equation on manifolds with conical singularities. We show how bounded imaginary powers and maximal regularity results obtained by Roidos and Schrohe can be used to prove that the solutions are globally defined in time and become instantaneously smooth in space and time. We use Mellin-Sobolev spaces to give precise information concerning the asymptotic behavior of the solutions close to the conical tips in terms of the local geometry. Finally we discuss some aspects of the dynamic behavior of the solutions on these spaces.
This a joint work with Nikolaos Roidos. This work is partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Processo número: 2019/15200-1

Stabilizing phenomenon for incompressible fluids

Weinan Wang
University of Arizona, USA
Wen Feng, Jiahong Wu

This talk presents several examples of a remarkable stabilizing phenomenon. For the Oldroyd-B model, we show that small smooth data lead to global and stable solutions. When Navier-Stokes is coupled with the magnetic field in the magneto-hydrodynamics system, solutions near a background magnetic field are shown to be always global in time. The magnetic field stabilizes the fluid. In all these examples the systems governing the perturbations can be converted to damped wave equations, which reveal the smoothing and stabilizing effect.

Heat flow of s-harmonic maps

Changyou Wang
Purdue University, USA

In this talk, I will discuss the problem of heat flow of s-harmonic maps from $\mathbb{R}^n$ to a Riemann manifold (N, h).

Existence and stability of a line-generating process modeled by a three component activator-substract-excitator system

Nung Kwan Yip
Purdue University, USA
Matthias Winter

We consider a morphogenetic mechanism generating filamentary lines by the trail of a traveling spike. The underlying model, suggested by Meinhardt, consists of three components: activator, substrate and excitator. We analyze this system in one space dimension for the singular limit of small activator diffusivity. Using Liapunov-Schmidt reduction, we rigorously prove the existence and (orbital) stability of a traveling spike solution with a trail, and most importantly, derive a formula for the pulse speed. The result is in contrast to two-component system where the spikes are often stationary in space. In the current case, the asymmetric far-field information of the excitator provides the key driving force for the motion of the spike. This is joint work with Matthias Winter.

Decay and vanishing of some D-solutions of the Navier-Stokes equations

Qi Zhang
University of California Riverside, USA
B. Carrillo, X.H. Pan, Na Zhao

An old problem since Leray asks whether homogeneous D solutions of the 3 dimensional Navier-Stokes equation in or some noncompact domains such as $\mathbb{R}^3$ or a slab are 0. In this paper, we give a positive solution to the problem in two special cases: (1) when the solution is axially symmetric and periodic in the vertical variable; (2) full 3 dimensional slab case with Dirichlet boundary condition. Other partial results will also be discussed. This is joint work with B. Carrillo, X.H. Pan and Na Zhao.

Special Session 79: Recent Advancements in the Numerical Analysis of Nonlinear Partial Differential Equations

Tom Lewis, University of North Carolina Greensboro, USA
Yi Zhang, University of North Carolina Greensboro, USA

This special session will highlight and address some of the current challenges for solving nonlinear partial differential equations (PDEs) numerically. PDEs have wide-ranging applications, and numerical methods remain an important tool for the understanding of solutions to PDEs. The emphasis in this session will be on theoretical results and computational results for reliably approximating solutions to nonlinear problems that are at the forefront of numerical PDEs; in particular, the rigorous study of convergence, admissibility, accuracy, and/or stability results or the formulation of highly efficient implementations and/or highly accurate methods for approximating application problems.

Numerical analysis of weak solutions for coupled elliptic systems with nonlinearity on the boundary

Shalmali Bandyopadhyay
University of North Carolina Greensboro, USA
Thomas Lewis, Nsoki M Mavinga

We consider approximating weak solutions for semilinear coupled elliptic system with nonlinearity on the boundary. We approximate existence of a weak solution in between an ordered pair of sub- and supersolution. We use finite difference method to discretize the solution and establish the stability of the solution.

Dual-wind discontinuous Galerkin method and its application to an optimal control problem and a parabolic variational inequality

Satyajith Boyana
University of North Carolina Greensboro, USA
Tom Lewis, Aaron Rapp, Yi Zhang

A symmetric DWDG method is used to discretize a control constrained elliptic optimal control problem with the PDE constraint as the Poisson's equation. We develop a scheme to obtain a finite dimensional optimization problem which is then solved with a primal-dual active set strategy. The convergence of the numerical solution pair $(\overline{y_h}, \overline{u_h})$ is proved. The rates of convergence are established in L_2 and energy norms.
Next, a fully discrete scheme to solve the parabolic variational inequality with a general obstacle function in $\mathbb{R}^2$ that uses a symmetric dual-wind discontinuous Galerkin discretization in space and a backward Euler discretization in time is proposed and analyzed. The convergence of numerical solutions in $L^\infty(L^2)$ and $L^2(H^1)$ like energy norms is established and the rates are computed. Several numerical tests are provided to demonstrate the robustness and effectiveness of the proposed methods. This is a joint work with Tom Lewis, Aaron Rapp and Yi Zhang.

Nonlocal variational inequalities for phase-field modeling in solidification

Olena Burkovska
Oak Ridge National Laboratory, USA

Phase-field models are a popular choice in computational physics to describe complex dynamics of substances with multiple phases and are widely used in applications including solidification of materials. In this talk, I will present nonlocal phase-field models of Cahn-Hilliard and Allen-Cahn types involving a nonsmooth double-well obstacle potential. Mathematically, in a weak form, the models translate to systems of variational inequalities, which are additionally coupled to a temperature evolution equation. I will demonstrate that under appropriate conditions on the nonlocal operator and the kernel we obtain a model that allows for sharp interfaces in the solution compared to a diffuse-interface local model. I will present an implicit-explicit time-stepping formulation for the model, well-posedness analysis and development of appropriate discretization methods that can be realized efficiently. Finally, several numerical experiments will be presented to support theoretical findings.

A new conservative discontinuous Galerkin method via implicit penalization for the generalized KdV equation

Yanlai Chen
UMass Dartmouth, USA
Bo Dong, Rebecca Pereira

We design, analyze, and implement a new conservative Discontinuous Galerkin (DG) method for the simulation of solitary wave solutions to the generalized Korteweg-de Vries (KdV) Equation. The key feature of our method is the conservation, at the numerical level, of the mass, energy and Hamiltonian that are conserved by exact solutions of all KdV equations. To our knowledge, this is the first DG method that conserves all these three quantities, a property critical for the accurate long-time evolution of solitary waves. To achieve the desired conservation properties, our novel idea is to introduce two stabilization parameters in the numerical fluxes as new

unknowns which then allow us to enforce the conservation of energy and Hamiltonian in the formulation of the numerical scheme. We prove the conservation properties of the scheme which are corroborated by numerical tests. This idea of achieving conservation properties by implicitly defining penalization parameters, that are traditionally specified a priori, can serve as a framework for designing physics-preserving numerical methods for other types of problems.

A conforming partition of unity method for a class of variational inequalities of the second kind

Christopher Davis
Tennessee Tech, USA
Yi Zhang

In this work, we consider the use of a flat-top partition of unity method to solve a class of fourth order variational inequalities of the second kind. Under the assumption that the solution is $H^3(\Omega)$ regular, optimal error estimates are made in the energy norm. Numerical examples are given to demonstrate the effectiveness of the proposed method.

Continuous data assimilation and long-time accuracy in a C0-IP method for the Cahn-Hilliard equation

Amanda Diegel
Mississippi State University, USA
Leo G. Rebholz

We propose a numerical approximation method for the Cahn-Hilliard equation that incorporates continuous data assimilation in order to achieve long time accuracy. The method uses a C0 interior penalty spatial discretization of the fourth order Cahn-Hilliard equation, together with a semi-implicit temporal discretization. We prove the method is long time stable and long time accurate, for arbitrarily inaccurate initial conditions, provided enough data measurements are incorporated into the simulation. Numerical experiments illustrate the effectiveness of the method on a benchmark test problem.

Numerical solutions of the nonlinear Korteweg-deVries equation

Daniel Guo
University of North Carolina Wilmington, USA

Applying One-step Semi-Lagrangian forward method to the nonlinear Kortewegde Vries (KdV) equation, we investigated the numerical solutions of the KdV equation with three sets of initial data. The main difficulty was the interpolations from the irregularly distributed Lagrangian grid to the regularly distributed Eulerian grid. Two treatments were studied as local four points cubic interpolation and the cubic spline interpolation. The numerical solutions generated by the Zabusky and Kruskal scheme and Semi-Lagrangian forward method were compared.

ISALT: Inference-based schemes adaptive to large time-stepping for locally Lipschitz ergodic systems

Xingjie Li
University of North Carolina Charlotte, USA
Fei Lu, Molei Tao, Xiaofeng Felix Ye

Efficient simulation of SDEs is essential in many applications, particularly for ergodic systems that demand efficient simulation of both short-time dynamics and large-time statistics. However, locally Lipschitz SDEs often require special treatments such as implicit schemes with small time-steps to accurately simulate the ergodic measures. We introduce a framework to construct inference-based schemes adaptive to large time-steps (ISALT) from data, achieving a reduction in time by several orders of magnitudes. The key is the statistical learning of an approximation to the infinite-dimensional discrete-time flow map. We explore the use of numerical schemes (such as the Euler-Maruyama, the hybrid RK4, and an implicit scheme) to derive informed basis functions, leading to a parameter inference problem. We introduce a scalable algorithm to estimate the parameters by least squares, and we prove the convergence of the estimators as data size increases.

A second order fitted operator finite difference scheme for a modified Burgers equation

Nana Adjoah Mbroh
North-West University, South Africa
S.C. Oukouomi Noutchie, R.Y. M'pika Massoukou

Burgers equation is a non-linear partial differential equation which occurs in various areas in applied mathematics and can be used to describe physical phenomena such as boundary layer theory. For large Reynolds numbers, the solution of the one-dimensional modified Burgers' equation is characterised by steep gradient and thus can be classified as a singularly perturbed problem. Due to the presence of the steep gradient, classical numerical methods are not able to mimic the behaviour of the exact solutions and thus yield unsatisfactory results.
In this talk, a numerical scheme which is able to resolve the inefficiencies of classical numerical schemes is proposed to solve the modified burgers equation. The stability of the scheme is established and the discretisation error is estimated. Numerical experiments will be conducted to validate any theoretical findings.

Numerical methods for approximating sublinear positone and semipositone boundary value problems using finite difference methods

Quinn Morris
Appalachian State University, USA
Thomas Lewis, Yi Zhang

Positone and semipositone problems have been of great interest to the PDE community for many years now due to the frequency with which they appear in reaction-diffusion models and the theory of nonlinear heat generation. While these problems pose many interesting theoretical challenges, they also pose particular challenges when attempt to find numerical approximations to solutions. We discuss recent results showing that a simple finite difference method, which adapts techniques from the method of sub- and supersolutions, can not only find approximate solutions, but also detects multiplicity and nonuniqueness for a wide class of sublinear problems in one or more dimensions.

Approximating the solutions to Hamilton-Jacobi equations with dual-wind discontinuous Galerkin methods

Aaron Rapp
University of the Virgin Islands, USA

A discontinuous Galerkin (DG) finite-element interior calculus is used as a common framework to describe various DG approximation methods for second-order elliptic problems. This framework allows for the approximation of both primal and variational forms of second order differential equations. In this presentation, we will study the error from using the dual-wind DG derivatives to approximate the the solution to stationary and time-dependent Hamilton-Jacobi equations. Some analytical results will be presented, along with numerical examples that support the convergence of this method.

A predictor-corrector strategy for adaptivity in dynamical low-rank approximations

Stefan Schnake
Oak Ridge National Laboratory, USA
Cory Hauck

In this talk, I present a predictor-corrector strategy for constructing rank-adaptive dynamical low-rank approximations of matrix-valued differential equation systems. Dynamical low-rank approximation (DLRA) is a nonlinear model reduction technique that evolves dynamical systems on a low-rank manifold, and has recently become popular in the approximation of linear and non-linear partial differential equations from kinetic theory. The strategy presented is a compromise between (i) low-rank step-truncation approaches that alternately evolve and compress solutions and (ii) strict DLRA approaches that augment the low-rank manifold using subspaces generated locally in time by the DLRA integrator. The strategy is based on an analysis of the error between a forward temporal update into the ambient full-rank space, which is typically computed in a step-truncation approach before re-compressing, and the standard DLRA update, which is forced to live in a low-rank manifold. This error is used, without requiring its full-rank representation, to correct the DLRA solution. A key ingredient for maintaining a low-rank representation of the error is a randomized singular value decomposition, which introduces some degree of stochastic variability into the implementation. The strategy is formulated and implemented in the context of discontinuous Galerkin spatial discretizations of partial differential equations and applied to several versions of DLRA methods found in the literature, as well as a new variant. Numerical experiments comparing the predictor-corrector strategy to other methods demonstrate robustness to overcome shortcomings of step truncation or strict DLRA approaches: the former may require more memory than is strictly needed while the latter may miss transients solution features that cannot be recovered. The effect of randomization, tolerances, and other implementation parameters is also explored.

A convergent monotone scheme for a nonlocal segregation model with free boundary

Xiaochuan Tian
University of California San Diego, USA

We consider a free boundary problem arising from segregation of two species with high competition. One species moves according to the classical diffusion and the other adopts a nonlocal diffusion strategy. Being a fully nonlinear nonlocal model, it is challenging to design effective ways to compute the solution, especially to capture the free boundary well. We propose an iterative scheme that constructs a sequence of monotone viscosity supersolutions that is shown to converge to the viscosity solution (in the sense of Crandall-Lions). The numerical method applies to general domains in all dimensions. Moreover, for simple domains it can be shown that the sequence of supersolutions converges with a precise rate. We will shown numerical experiments in the end. This is a joint work Luis Caffarelli and Irene Gamba.

Finite-difference methods for computing optimal transport PDE on the unit sphere

Axel Turnquist
University of Texas Austin, USA
Brittany Froese Hamfeldt, Richard Tsai

Here we present two approaches for computing the solution of Optimal Transport PDE on the sphere, for a wide variety of cost functions, such as the squared geodesic, logarithmic, and other exotic cost functions arising in various optics inverse problems. The first method is based on a monotone discretization that performs computations on wide-stencil neighborhoods projected on local tangent planes. This approach comes with convergence guarantees even for non-smooth $C^1(\mathbb{S}^2)$ solutions and is the most efficient provably convergent scheme over this class of problems. We show the success of this method in tackling moving mesh and reflector antenna problems and present work towards establishing convergence rates. The second method is derived by extending the Optimal Transport problem onto a thin shell containing the sphere, and then computing the solution for this new extended problem, which is consistent with the solution for the problem on the sphere. The benefit of this method is that discretization does not have to be done on point clouds, but rather on Cartesian grids, leading to a much simpler discretization.

Higher order time discretization method for a class of semilinear stochastic PDEs with multiplicative noise

Liet Vo
University of Illinois Chicago, USA
Yukun Li, Guanqian Wang

In this talk, we consider a new approach for the time discretization of a class of semilinear stochastic partial differential equations (SPDEs) with multiplicative noise. The nonlinearity in the diffusion term of the SPDEs is assumed to be globally Lipschitz while the nonlinearity in the drift term is only considered to satisfy a one-sided Lipschitz condition that has a broader application in reality. Our new strategy for the time discretization is based on the classical Milstein method from stochastic differential equations. In addition, for the spatial discretization, we also use a finite element interpolation technique to discretize the nonlinear drift term. We use the energy method to show a strong convergence order of at most 1 for the time discrete solution. The proof is based on new Hölder continuity estimates of the PDE solution and higher moment estimates for the H^1-norm of the numerical solution.

Convergent finite difference methods with higher order local truncation errors for stationary Hamilton-Jacobi equations

Xiaohuan Xue
University of North Carolina Greensboro, USA
Tom Lewis

A new non-monotone finite difference (FD) method for approximating viscosity solutions of stationary Hamilton-Jacobi problems with Dirichlet boundary conditions will be discussed. The new FD method has local truncation errors that are above the first order Godunov barrier for monotone methods. The method uses a stabilization term called a numerical moment to ensure that the proposed scheme is admissible, stable, and convergent. Numerical tests will be provided that compare the accuracy of the proposed scheme to that of the Lax-Friedrich's method.

Monotone meshfree methods for linear elliptic equations in non-divergence form via nonlocal relaxation

Qihao Ye
University of California San Diego, USA
Xiaochuan Tian

We design a monotone meshfree finite difference method for linear elliptic PDEs in non-divergence form on point clouds via a nonlocal relaxation method. The key idea is a combination of a nonlocal integral relaxation of the PDE problem with a robust meshfree discretization on point clouds. A major theoretical contribution is the existence of consistent and positive stencils for a given point cloud geometry. We provide sufficient conditions for the existence of positive stencils by finding neighbors within an ellipse (2d) or ellipsoid (3d) surrounding each interior point. It is well-known that wide stencils are in general needed for constructing consistent and monotone finite difference schemes for linear elliptic equations. Our result represents a significant improvement in the stencil width estimate for positive-type finite difference methods for linear elliptic equations in the near-degenerate regime (when the ellipticity constant becomes small), compared to previously known works in this area. Numerical algorithms and practical guidance are provided with an eye on the case of small ellipticity constant. Numerical results will be presented in both 2d and 3d, examining a range of ellipticity constants including the near-degenerate regime.

A general framework to derive linear, decoupled and energy-stable schemes for reversible-irreversible thermodynamically consistent models

Jia Zhao
Utah State University, USA

I will presents a general numerical platform for designing accurate, efficient, and stable numerical algorithms for incompressible hydrodynamic models that obey thermodynamical laws. The obtained numerical schemes are automatically linear in time. It decouples the hydrodynamic variable and other state variables such that only small-size linear problems need to be solved at each time marching step. Furthermore, if the classical velocity projection method is utilized, the velocity field and pressure field can be decoupled. In the end, only a few elliptic-type equations shall be solved in each time step. Several benchmark numerical examples are presented to further illustrate the proposed numerical framework's accuracy, stability, and efficiency.

Special Session 80: Inverse Problems and Imaging

Ru-Yu Lai, University of Minnesota, USA
Gunther Uhlmann, University of Washington, USA
Yang Yang, Michigan State University, USA

This special session will focus on the latest advances on inverse problems and related applications. Inverse problems arise from the need to find a cause for an observed effect from indirect measurements. The development of the field is strongly driven by practical applications and its rich methodologies are applied in broad fields, such as geophysics, medical imaging, biology, solar physics, and remote sensing. The objective of the session is to bring together researchers from different stages of career to share their recent research achievements, and provide a platform for professional networking. The topics of the talks will be centered around innovative mathematical theory and practical techniques developed in various classes of inverse problems, including geometric inverse problems, inverse boundary value problems, inverse scattering, integral geometry, and many others.

Detection of a thin waveguide in 2D Helmholtz equation

Matias Courdurier
Universidad Catolica de Chile, Chile
Eric Bonnetier, Faouzi, Triki, Axel Osses

We consider the solutions of the Helmholtz equation in the two-dimensional space in the presence of an unbounded thin waveguide layer with a high contrast index of refraction. We are interested in the identification of some parameters of the waveguide from its response to a localized excitation. This is related to some inverse problems appearing in seismology for layered media and in optical or sound probing of laminated media. In this study, for identification purposes, we will exploit an interesting phenomenon. When the medium is excited by an external point source in a wide range of frequencies, it can be observed that at some given frequencies the behavior of the solution abruptly changes. To properly characterize this phenomenon, we perform an asymptotic analysis of the explicitly solution of the Helmholtz equation, in the two-dimensional space, in the presence of an unbounded waveguide.

Recovering small volume corroded regions in EIT

Isaac Harris
Purdue University, USA
Govanni Granados

In this talk, we will consider the inverse shape problem coming from electrical impedance tomography with a Robin transmission condition. We consider the case where we have small volume regions. By writing the corresponding Dirichlet-to-Neumann mapping (i.e. the current gap operator) as an integral operator, we will use the small volume assumption to derive an asymptotic expansion for the mapping. From this, we will derive a MUSIC-type algorithm that can be used to recover the regions of interest. Numerical examples will be presented in two dimensions for the unit circle.

Single-excitation quantum optics: Analysis and algorithms

Jeremy Hoskins
University of Chicago, USA
Manas Rachh, John Schotland, Jason Kaye

Recent progress in experimental quantum optics has facilitated the physical construction of systems of increasing complexity. Of particular importance are experiments involving the scattering of one or two photons from a collection of atoms. In this context, a central question is to understand the time evolution of the entanglement between atoms, mediated by the field. In this talk we will discuss analytical results on the properties of these systems, and describe a related inverse problem.

The tensorial X-ray transform on asymptotically conic manifolds

Qiuye Jia
Stanford University, Peoples Republic of China
Andras Vasy

In this talk I will discuss the invertibility of the geodesic X-ray transform on one forms and 2-tensors on asymptotically conic manifolds, up to the natural obstruction, allowing existence of certain kinds of conjugate points. We use the 1-cusp pseudodifferential operator algebra and its semiclassical foliation version introduced and used by Vasy and Zachos, who showed the same type invertibility on functions.
The complication of the invertibility of the tensorial X-ray transform, compared with X-ray transform on functions, is caused by the natural kernel of the transform consisting of 'potential tensors'. We overcome this by arranging a modified solenoidal gauge condition, under which we have the invertibility of the X-ray transform.
This can be considered as a linearized version of the boundary rigidity problem.

Method-of-moments, machine learning, and cryo-electron microscopy

Yuehaw Khoo
University of Chicago, USA
Sounak Paul, Nir Sharon

We present a novel framework for the molecular structuring problem in cryo-electron microscopy via a method-of-moments, which can significantly reduce the per-iteration cost in solving the inverse problem. In this talk, we address the ill-conditioning of the moment-equations via a machine learning prior.

Fractional anisotropic Calderon problem on Riemannian manifolds

Katya Krupchyk
University of California Irvine, USA

We shall discuss some recent progress on the fractional anisotropic Calderon problem on closed Riemannian manifolds of dimensions two and higher. Specifically, we show that the knowledge of the local source-to-solution map for the fractional Laplacian, given on an arbitrary small open nonempty a priori known subset of a smooth closed Riemannian manifold, determines the Riemannian manifold up to an isometry. This can be viewed as a nonlocal analog of the anisotropic Calderon problem in the setting of closed Riemannian manifolds, which is wide open in dimensions three and higher. This is joint work with Ali Feizmohammadi, Tuhin Ghosh, and Gunther Uhlmann.

Unique determination of the variable coefficients in fractional equations

Li Li
University of California Irvine, USA

I will talk about the fractional Calderón problem and its evolutionary and nonlinear variants. The goal is to determine nonlinearities/coefficients in fractional equations from exterior partial measurements of the Dirichlet-to-Neumann map.

Acousto-electric inverse source problems

Wei Li
DePaul University, USA
John C. Schotland, Yang Yang, Yimin Zhong

We propose a method to reconstruct the electrical current density inside a conducting medium from acoustically-modulated boundary measurements of the electric potential. We show that the current can be uniquely reconstructed with Lipschitz stability. We also perform numerical simulations to illustrate the analytical results, and explore the partial data setting when measurements are taken only on part of the boundary. This method can also be applied to the reconstruction of the electrical current density from acoustically-modulated boundary measurements of time-harmonic electromagnetic fields.

Recover all coefficients in second-order hyperbolic equations from finite sets of boundary

Shitao Liu
Clemson University, USA
Antonio Pierrottet, Scott Scruggs

In this talk we consider the inverse hyperbolic problem of recovering all spatial dependent coefficients, which are the wave speed, the damping coefficient, potential coefficient and gradient coefficient, in a second-order hyperbolic equation defined on an open bounded domain with smooth enough boundary. We show that by appropriately selecting finite pairs of initial conditions and a boundary condition, we can uniquely and stably recover all those coefficients from the corresponding boundary measurements of their solutions. The proofs are based on sharp Carleman estimate, continuous observability inequality and regularity theory for general second-order hyperbolic equations.

Partial data inverse problem for the wave equation with time-dependent damping coefficients and potentials on confomally transversally anisotropic manifolds

Boya Liu
North Carolina State University, USA
Teemu Saksala, Lili Yan

Given (M, g), a compact Riemannian manifold of dimension $n \geq 3$, we study the inverse problem of determining time-dependent damping coefficient a and potential q appearing in the wave equation $\partial_t^2 u - \Delta_g u + a(t,x)\partial_t u + q(t,x)u = 0$ in $Q = (0,T) \times M$ with $T > 0$. More specifically, we are concerned with the case of conformally transversally anisotropic manifolds, i.e., compact Riemannian manifolds with boundary that are conformally embedded in a product of the Euclidean line and a transversal manifold M_0. Under the assumption that the attenuated geodesic ray transform on M_0 is injective, we prove that the knowledge of Cauchy data measured on certain subsets of ∂Q determines continuous time-dependent damping coefficient a and potential q uniquely.

X-ray transforms and degenerate elliptic operators

Francois Monard
University of California Santa Cruz, USA
Yuzhou Zou

The overarching theme of this talk is concerned with the mapping properties of X-ray transforms on manifolds with strictly convex boundary.
On certain symmetric manifolds with constant curvature and strictly convex boundary, recent functional relations between the X-ray transform and degenerate elliptic operators provide the framework to understand on what scales of spaces the X-ray transform satisfies tame estimates (i.e. finite-degree smoothing with finite-degree unsmoothing inverse) that holds all the way to the boundary. I will discuss these results, as well as recent results with Yuzhou Zou regarding a refined study of these degenerate elliptic operators.
Related recent works:
http://arxiv.org/abs/2112.14904
http://arxiv.org/abs/2203.09861
http://arxiv.org/abs/2302.08133

A new model for cardiac fiber identification

Axel Osses
Universidad de Chile, Chile
N. Barnafi

In the context of cardiac fiber identification, we propose a new PDE model for the construction of rule based model fibers. We perform its analysis, both from the theoretical point of view and for an efficient and robust numerical approximation for the direct and inverse problems involved. We compare our approach with other alternative modeling methods based on the manipulation of ad-hoc potentials, and show that our approach generalizes the state-of-the-art algorithms for ventricular fiber reconstruction.

Inverse problems arising in photoacoustic tomography

Benjamin Palacios
Pontificia Universidad Catolica de Chile, Chile

Photoacoustic Tomography (PAT) is a hybrid medical imaging modality that is able to generate high-resolution and high-contrast images by exploiting the coupling of electromagnetic pulses (in the visible region) and ultrasound waves via de photoacoustic effect. From its mathematical formulation, one obtains the inverse problem that consists of determining the initial source of acoustic waves from boundary data, however, in practice other coefficients such as the speed of sound and attenuation are unknown and of interest as well. In this talk, I will discuss current research on some of the inverse problems related to the imaging modality of PAT.

Fractional Dirac operators and geometric reconstruction

Hadrian Quan
University of Washington, USA
Gunther Uhlmann

I will discuss joint work with Gunther Uhlmann regarding the anisotropic fractional Calderon problem for Dirac operators on closed manifolds; these give fractional analogues of Maxwell systems. Namely we show that knowledge of the source-to-solution map of the fractional Dirac operator, for data sources supported in an arbitrary open set in a Riemannian manifold allows one to reconstruct the Riemannian manifold, its Clifford module structure, and the associated connection (up to an isometry fixing the initial set). Time permitting I will discuss on-going work regarding Caffarelli-Silvestre type extensions for fractional systems.

Three travel time inverse problems on simple Riemannian manifolds

Teemu Saksala
North Carolina State University, USA
Joonas Ilmavirta, Boya Liu

In this talk we will provide new proofs based on the Myers–Steenrod theorem to confirm that travel time data, travel time difference data and the broken scattering relations determine a simple Riemannian metric on a disc up to the natural gauge of a boundary fixing diffeomorphism. Our method of the proof leads to a Lipschitz-type stability estimate for the first two data sets in the class of simple metrics.

Analysis and reduction of metal artifacts in X-ray tomography

Yiran Wang
Emory University, USA

Due to beam-hardening effects, metal objects in X-ray CT often produce streaking artefacts which cause degradation in image reconstruction. As demonstrated by Seo et al in 2017, the nature of the phenomena is nonlinear. An outstanding inverse problem is to identify the nonlinearity which is crucial for reduction of the artefacts. In this talk, we show how to use microlocal techniques to analyze the artefacts and extract information of the nonlinearity. In particular, we will discuss the interesting connection between the strength of artefacts and the geometry of metal objects.

Inverse boundary problems for biharmonic operators and nonlinear PDEs on Riemannian manifolds

Lili Yan
University of Minnesota, USA
Katya Krupchyk, Gunther Uhlmann

In an inverse boundary problem, one seeks to determine the coefficients of a PDE inside a domain, describing internal properties, from the knowledge of boundary values of solutions of the PDE, encoding boundary measurements. Applications of such problems range from medical imaging to non-destructive testing. In this talk, starting with the fundamental Calderon inverse conductivity problem, we shall first discuss inverse boundary problems for first-order perturbations of biharmonic operators in the setting of compact Riemannian manifolds with boundary. Specifically, we shall present a global uniqueness result as well as a reconstruction procedure for the latter inverse boundary problem on conformally transversally anisotropic Riemannian manifolds of dimensions three and higher. Finally, we shall also discuss briefly inverse boundary problems for nonlinear magnetic Schrödinger operators on a compact complex manifold, illustrating the recent insight that the presence of nonlinearity may help when solving inverse problems.

Inverse problems arising in nonlinear acoustic imaging

Yang Zhang
University of Washington, USA
Gunther Uhlmann

Nonlinear ultrasound waves are widely used in medical imaging. The propagation of high-intensity ultrasound waves can be modeled by nonlinear wave equations. In this talk, we consider an inverse problem for a nonlinear wave equation with a general nonlinearity. We show the Dirichlet-to-Neumann map (DN map) determines the nonlinearity. We also consider inverse problems for nonlinear wave equations with a general nonlinear term and a damping term.

Inverse problems on multiphoton absorption

Yimin Zhong
Auburn University, USA

Multiphoton absorption is an important type of nonlinear effect in optics and has a variety of applications. In this talk, we will first discuss the modeling of multiphoton absorption in kinetic models. Then we will talk about two kinds of problems: (1) the related imaging problems regarding a finite number of internal radiance measurements; (2) the classical inverse transport problem with albedo data.

Stability and statistical inversion for travel time tomography

Hanming Zhou
University of California Santa Barbara, USA
Ashwin Tarikere

In this talk, we consider the travel time tomography for conformal metrics on a bounded domain which consists of determining the conformal factor of the metric from the length of geodesics joining boundary points. We establish forward and inverse stability estimates for simple conformal metrics under some a priori conditions. We then apply the stability estimates to show the consistency of the statistical inversion of the travel time tomography with discrete, noisy measurements.

Boundary triplets and Sobolev spaces associated to degenerate elliptic operators

Yuzhou Zou
Northwestern University, USA
Francois Monard

I will discuss machinery used to study both singularly weighted X-ray transforms on strictly convex manifolds with boundary (such as the Euclidean disk) as well as some degenerately elliptic differential operators associated to such transforms. Such machinery includes a scale of Sobolev spaces which take into account behavior up to the boundary, as well as generalizations of Dirichlet and Neumann traces called boundary triplets associated to degenerately elliptic operators which pick out the first and second most singular terms of a function near the boundary. Joint work with François Monard.

Special Session 81: Stochastic Modeling in Biological, Physical and Social Sciences: Theory and Applications

Wai-Tong (Louis) Fan, Indiana University Bloomington, USA
Krutika Tawri, University of California Berkeley, USA
Chuntian Wang, University of Alabama, USA
Roger Temam, Indiana University Bloomington, USA

Stochastic models play a significant role in mathematics, science and engineering. With ever-increasing applications in biology, climate science, physics, engineering and social science, these models offer insight into the dynamics, phase transitions and other phenomena of the complex system under study. Furthermore, lying at the intersection of probability, statistics, partial differential equations and many areas in mathematics, they provide a plethora of new and interesting mathematical challenges. Motivated by the need from within and beyond the mathematical community, our session will focus on recent advances of stochastic modeling in various application fields. We aim to bring together a diverse body of researchers to stimulate discussions about such models from both theoretical and applied points of view. Our topics include the regularity, properties and applications of stochastic partial differential equations, stochastic and statistical dynamics, stochastic numerical analysis, implementations, and simulations.

Synchronization of stochastic complex networks of reaction-diffusion equations

Hakima Bessaih
Florida International University, USA
Verena Kopp

We consider a network consisting of a system of stochastic reaction-diffusion equations with a special coupling. This includes a stochastic coupling. We prove that after a certain time, all elements of the network exhibit the same behavior. Hence, the system synchronizes.

Singularly perturbed differential operators and some stochastic analytic counterparts

Yu-Ting Chen
University of Victoria, Canada

Schrödinger operators with delta potentials are of longstanding interest for admitting solutions expressible in closed analytic forms, and they receive renewed interest for connections to the Kardar-Parisi-Zhang equation. Along with a review of the background, the talk will discuss recent results for a standard model of these operators in 2D and its counterpart in the form of the Feynman-Kac formula.

Global existence of stochastic heat equation in the superlinear-growth regime

Le Chen
Auburn University, USA
Jingyu Huang

In this paper, we study the *stochastic heat equation* (SHE) on $\mathbb{R}^d$ subject to a centered Gaussian noise that is white in time and colored in space. We establish the existence and uniqueness of the random field solution in the presence of locally Lipschitz drift and diffusion coefficients, which can have certain superlinear growth. This is a nontrivial extension of the recent work by Dalang, Khoshnevisan and Zhang [AOP'19], where the one-dimensional SHE on $[0, 1]$ subject to space-time white noise has been studied. This talk is based on a jointwork with Jingyu Huang.

Takeover, fixation and identifiability in finite neutral genealogy models

Eric Foxall
University of British Columbia Okanagan, Canada

For neutral genealogy models in a finite, possibly non-constant population, there is a convenient representation of the family trees, known as the lookdown representation, that arranges descendant subtrees in size-biased order. We give a simple, conceptual demonstration of the size-biasing property, and address the problem of identifiability: under what conditions can we infer some or all of the lookdown arrangement by examining the (unlabelled) descendant subtrees? We explain how this question is connected to two important properties of the graph: uniqueness of the infinite path, and existence of a dominant lineage, and give sufficient and sometimes necessary conditions for each. We also discuss connections to the spinal representation of size-biased Galton-Watson trees.

A framework for posterior consistency in PDE inverse problems

Christian Frederiksen
Tulane University, USA
Nathan Glatt-Holtz

One is often interested in estimating functional parameters in a partial differential equation given sparse and noisy observations of the solution. A Bayesian statistical methodology provides a comprehensive approach for such problems and establishing posterior consistency is an important step in validating this approach. In this talk I will introduce both posterior consistency and the Bayesian approach to inverse problems and present a newly developed abstract framework for establishing posterior consistency in PDE inverse problems. The abstract nature of the framework makes it easily adaptable to new problems unlike existing results which focus on specific problems. Additionally, and quite significantly, it allows for the use of fixed Gaussian priors and different observation types which are both absent in existing literature on PDE inverse problems. The talk should be readily accessible to anyone with a decent understanding of basic probability and PDE theory. This is joint work with Nathan Glatt-Holtz.

The lottery competition model in stochastic environments

Xiaoying Han
Auburn University, USA
Jiaqi Cheng, Peter Chesson, Ming Liao

The lottery model describes competition for recourses among ecological species under temporally varying stochastic environments. This talk includes two models derived from diffusion approximation: a two-species model in non-stationary stochastic environments described by non-autonomous stochastic differential equations, and an n-species model in stationary stochastic environments described by autonomous stochastic differential equations. Long-term dynamics will be discussed and conditions for co-existence or distinction will be developed.

Transitions in stochastic non-equilibrium systems: Efficient reduction and analysis

Honghu Liu
Virginia Tech, USA
Mickael D. Chekroun, James C. McWilliams, Shouhong Wang

A central challenge in physics is to describe non-equilibrium systems driven by randomness, such as a randomly growing interface, or fluids subject to random fluctuations that account e.g. for local stresses and heat fluxes in the fluid which are not related to the velocity and temperature gradients. For deterministic systems with infinitely many degrees of freedom, normal form and center manifold theory have shown a prodigious efficiency to often completely characterize how the onset of linear instability translates into the emergence of nonlinear patterns, associated with genuine physical regimes. However, in presence of random fluctuations, the underlying reduction principle to the center manifold is seriously challenged due to large excursions caused by the noise, and the approach needs to be revisited.
In this talk, we present an alternative framework to cope with these difficulties by exploiting the approximation theory of stochastic invariant manifolds, on the one hand, and energy estimates measuring the defect of parameterization of the high-modes, on the other. To operate for fluid problems subject to stochastic stirring forces, these error estimates are derived under assumptions regarding dissipation effects brought by the high-modes in order to suitably counterbalance the loss of regularity due to the nonlinear terms. As a result, the approach enables us to predict, from reduced equations of the stochastic fluid problem, the occurrence in large probability of a stochastic analogue to the pitchfork bifurcation, as long as the noise's intensity and the eigenvalue's magnitude of the mildly unstable mode scale accordingly. Application to a stochastic Rayleigh-Bénard model will also be presented.

Martingale solutions to stochastic nonlocal Cahn-Hilliard-Navier-Stokes Equations with multiplicative noise of jump

Aristide Ndongmo Ngana
North West University, South Africa
Theodore Tachim Medjo, Gabriel Deugoué

In this talk, we present an existence and uniqueness result for a mathematical system which models the dynamics of an incompressible isothermal mixture of two immiscible Newtonian fluids flowing in a two- or three-dimensional bounded domain under stochastic perturbations. This model can be seen as a stochastic version of Navier-Stokes-Cahn-Hilliard model. In fact, the Navier-Stokes-Cahn-Hilliard model consists of the Navier-Stokes equations for the velocity, nonlinearly coupled with a convective nonlocal Cahn-Hilliard equation for the order (phase) parameter. We prove the existence of weak martingale solution for both 2D and 3D cases. In addition, we prove the existence of a unique (probabilistic) strong solution in two-dimensional bounded domain.

Techniques of analyzing stochastic differential equation systems in biology

Tuan Phan
Institute for Modeling Collaboration and Innovation, University of Idaho, USA

In this presentation, I will discuss two commonly employed techniques for the theoretical analysis of stochastic differential equation (SDE) systems that frequently arise in the modeling of biological and medical problems. The first technique involves constructing Lyapunov functions to establish the existence of a unique stationary distribution of a SDE system within its invariant domain. The second technique involves studying the dynamics of a SDE system on the boundary of its invariant domain. This latter method offers several advantages and can yield precise conditions for both the persistence and extinction of a SDE system, which is of significant interest in the field of mathematical biology. To illustrate the integration of these techniques, I will utilize the cholera epidemic model and the human papillomavirus model.

Lattice Yang-Mills and SPDE limit in 2D

Hao Shen
University of Wisconsin Madison, USA
Ilya Chevyrev

Yang-Mills model is a quantum field theory which is fundamental in physics. Due to small scale singularity, various lattice regularized models such as the Wilson and Villain models were proposed. These are well-defined measures on the lattice. We prove a universality result which shows that these measures converge to the same limiting measure in the continuum limit, and the proof is based on stochastic PDE. In particular, the measure is invariant under the solution to the SPDE. Based on joint work with Ilya Chevyrev.

Finite Markov chains coupled to general Markov processes and an application to metastability

Jason Swanson
University of Central Florida, USA
Thomas G. Kurtz

We consider a diffusion given by a small noise perturbation of a dynamical system driven by a potential function with a finite number of local minima. The classical results of Freidlin and Wentzell show that the time this diffusion spends in the domain of attraction of one of these local minima is approximately exponentially distributed and hence the diffusion should behave approximately like a Markov chain on the local minima. By the work of Bovier and collaborators, the local minima can be associated with the small eigenvalues of the diffusion generator. Applying a Markov mapping theorem, we use the eigenfunctions of the generator to couple this diffusion to a Markov chain whose generator has eigenvalues equal to the eigenvalues of the diffusion generator that are associated with the local minima and establish explicit formulas for conditional probabilities associated with this coupling. The fundamental question then becomes to relate the coupled Markov chain to the approximate Markov chain suggested by the results of Freidlin and Wentzel. We provide a complete analysis of this relationship in the special case of a double-well potential in one dimension. More generally, the coupling can be constructed for a general class of Markov processes and any finite set of eigenvalues of the generator.

Hessian spectrum at the global minimum of locally isotropic Gaussian random fields

Qiang Zeng
University of Macau, Macau

Locally isotropic Gaussian random fields were first introduced by Kolmogorov in 1941. Such models were used widely in statistical physics. In particular, they were introduced to model a single particle in a random potential by Engel, Mezard and Parisi in 1990s. Using Parisi's award winning replica trick, Fyodorov and Le Doussal predicted the high dimensional limit of the Hessian spectrum at the global minimum of these models, and discovered phase transitions according to different levels of replica symmetry breaking. In this talk, I will present a solution in a strong sense to their conjecture in the so called replica symmetric regime. Our method is based on landscape complexity, or counting the number of critical points of the Hamiltonian. This talk is based on joint works with Antonio Auffinger (Northwestern University) and Hao Xu (University of Macau).

Special Session 83: Scientific Machine Learning for Dynamics Related Inverse Problems

Yanhzao Cao, Auburn University, USA
Feng Bao, Florida State University, USA
Guannan Zhang, Oak Ridge National Lab, USA

Scientific machine learning (SciML) has recently received significant attention in the research communities that involve large-scale data and complex models. Successfully trained neural networks, enabled by massive computing power and colossal amounts of data, have led to quantum leaps in artificial intelligence. Deep learning techniques are positioned to fundamentally change many sectors of society, by offering decision making capabilities which match and often exceed that of human experts. The power of SciML is not limited to learning forward models. It is also a powerful tool to solve the inverse problem, which is an important area in scientific research that aims to combine forward simulation with data to build-up inferences for target scientific models. In this special session we focus on the application of SciML to solving scientific inverse problems, especially the dynamical system itself given input data, and we bring researchers together that have developed methods and algorithms for solving inverse problems with the help of state-of-the-art SciML techniques. The topics of interest include but are not limited to learning dynamical systems, stochastic optimization, Bayesian inference, data assimilation, probabilistic machine learning, and scientific data analytics.

Federated learning for scientific facilities

Rick Archibald
Oak Ridge National Laboratory, USA

The US Department of Energy (DOE) makes substantial investments in the production and collection of massive amounts of scientific data through supporting the user facilities and scientific software. The high performance computing (HPC) resources supported by the Office of Advanced Scientific Computing Research (ASCR) provide an ideal platform for applying scientific machine learning (SciML) on these massive data to accelerate scientific discoveries. However, an efficient, scalable, federated algorithm is necessary to apply SciML to distributed data produced at scientific user facilities. There is a push at Oak Ridge National Laboratory (ORNL) to develop the next generation of smart laboratories (https://www.ornl.gov/intersect), locally developing connections between experimental and computational facilities at ORNL. This talk will focus on recent efforts by IBM/INTERSECT/ORNL/REDHAT/SLAC to connect experimental facilities at different laboratories using federated learning.

A stochastic maximum principle approach for reinforcement learning

Feng Bao
Florida State University, USA
Richard Archibald, Jiongmin Yong

In this work, we introduce a stochastic maximum principle (SMP) approach for solving the reinforcement learning problem with the assumption that the unknowns in the environment can be parameterized based on physics knowledge. For the development of numerical algorithms, we apply an effective online parameter estimation method as our exploration technique to estimate the environment parameter during the training procedure, and the exploitation for the optimal policy is achieved by an efficient backward action learning method for policy improvement under the SMP framework. Numerical experiments are presented to demonstrate that the SMP approach for reinforcement learning can produce reliable control policy, and the gradient descent type optimization in the SMP solver requires less training episodes compared with the standard dynamic programming principle based methods.

Controlling regularized conservation laws via entropy-entropy flux pairs

Wuchen Li
University of South Carolina, USA
Siting Liu, Stanley Osher

In this talk, we study variational problems for regularized conservation laws with Lax's entropy-entropy flux pairs. We first introduce a modified optimal transport space based on conservation laws with diffusion. Using this space, we demonstrate that conservation laws with diffusion are flux-gradient flows. We next construct variational problems for these flows, for which we derive dual PDE systems for regularized conservation laws. Several examples, including traffic flow and Burgers' equation, are presented. We successfully compute the control of conservation laws by incorporating both primal-dual algorithms and monotone schemes. This is based on joint work with Siting Liu and Stanley Osher.

Multidimensional quadrature rule for non-positive weight functions

Miroslav Stoyanov
Oak Ridge National Lab, USA
William Kong

The classical weighted quadrature rules rely on roots of orthogonal polynomial and work well in context when the roots are well defined and real, e.g., in the case when the weight is positive definite. In this work, we present a method for quadrature construction that extends the weights that are negative but bounded from below. The asymptotic convergence rate of our method trails that of the classical Gauss-Legendre quadrature; however, in the pre-asymptotic regime our exotic construction vastly outperformed that classical approach, which is further magnified when applied to a multidimensional context, such as many application of (Uncertainty Quantification and radiation transport).

High-dimensional optimization with a novel nonlocal gradient

Hoang Tran
Oak Ridge National Laboratory, USA

The problem of minimizing multi-modal loss functions with a large number of local optima frequently arises in machine learning and model calibration problems. Since the local gradient points to the direction of the steepest slope in an in infinitesimal neighborhood, an optimizer guided by the local gradient is often trapped in a local minimum. To address this issue, we develop a novel nonlocal gradient to skip small local minima by capturing major structures of the loss's landscape in black-box optimization. The nonlocal gradient is defined by a directional Gaussian smoothing (DGS) approach. The key idea of DGS is to conducts 1D long-range exploration with a large smoothing radius along d orthogonal directions in $\mathbb{R}^d$, each of which defines a nonlocal directional derivative as a 1D integral. Such long-range exploration enables the nonlocal gradient to skip small local minima. The d directional derivatives are then assembled to form the nonlocal gradient. We use the Gauss-Hermite quadrature rule to approximate the d 1D integrals to obtain an accurate estimator. We provide a convergence theory in the scenario where the objective function is composed of a convex function perturbed by a highly oscillating, deterministic noise. We prove that our method exponentially converge to a tightened neighborhood of the solution, whose size is characterized by the noise wavelength. The superior performance of our method is demonstrated in several high-dimensional benchmark tests, machine learning and model calibration problems.

Incorporating auxiliary data in parameter estimation through machine learning

Hans Werner van Wyk
Auburn University, USA
Chinedu Eleh, Lizzy Barlow

The estimation of spatially-dependent model parameters based on limited observations of related dynamic outputs is computationally difficult. When treated as a dynamics-constrained least squares problem, parameter identification suffers from the inherent ill-posedness of the inverse mapping, and in the Bayesian setting, the potential high-dimensionality of the parameter space limits its efficient traversal by a sampling scheme such as the one generated by a Markov Chain Monte Carlo (MCMC) sequence. Machine learning offers the potential of incorporating additional auxiliary information of the dynamical system to design regularization schemes, constrain the parameter space, or further condition the estimation problem. In this talk we investigate various machine learning tools and architectures that can be used to condition parameter estimation problems related to a simple model predicting the spread of pollutants in a slowly moving river.

On structure-preserving numerical methods for Stochastic Poisson systems

Lijin Wang
Chinese Academy of Sciences, Peoples Republic of China

Stochastic Poisson systems generalize stochastic Hamiltonian systems in dimension and structural matrices. In this talk we introduce some recent study on structure-preserving numerical methods for stochastic Poisson systems, including the approach of Darboux-Lie transformations, the midpoint-related methods, and the projection-based methods that can simultaneously preserve all invariant Hamiltonians. These methods are constructed to inherit the Poisson structure, the Casimir functions, or the invariant energy (resp. Hamiltonians) of the stochastic Poisson systems. Numerical tests are performed on some typical examples including the stochastically perturbed rigid body system and Lotka-Volterra system.

A pseudo-reversible normalizing flow for stochastic dynamical systems with arbitrary initial distributions

Minglei Yang
Oak Ridge National Laboratory, USA
Pengjun Wang, Diego del Castillo-Negrete, Yanzhao Cao, Guannan Zhang

We develop a pseudo-reversible normalizing flow to efficiently generate samples of the state of a stochastic differential equation (SDE) for various initial distributions. The goal of this work is to build an accurate and efficient sampler of the SDE to replace computationally expensive particle simulators. After training, the normalizing flow model can directly generate samples of the SDEs final state without simulating trajectories. Existing normalizing flows for SDEs depend on the initial distribution, meaning the model needs to be retrained when the initial distribution changes. The main novelty of our normalizing flow model is that it can learn the conditional distribution of the state, i.e., the distribution of the final state conditional on any initial state, such that the model only needs to be trained once and the trained model can be used to handle various initial distributions. This feature can provide a huge computational saving in studies of how the final state varies with the initial distribution. Numerical experiments are provided to demonstrate the effectiveness of the proposed normalizing flow mode.

Special Session 84: Recent Developments in Understanding of Nonlinear Phenomena in Fluid Dynamics, Biology, Statistical Mechanics and Optics

Rafail Abramov, University of Illinois Chicago, USA
Gregor Kovacic, Rensselaer Polytechnic Institute, USA

The purpose of this session is to highlight recent developments in applications of dynamical systems and differential equations to various disciplines, such as fluid dynamics, biology, statistical mechanics and optics. Our speakers will talk about stochastic parameterization in PDE, statistical mechanics in chemical and biological systems, light propagation, and also waves and turbulence in fluids.

Turbulence via intermolecular potential

Rafail Abramov
University of Illinois Chicago, USA

I will explain how turbulence develops in compressible flows at low Mach and high Reynolds numbers. There are two key conditions which have been overlooked in the conventional Navier–Stokes equations: first, the mean field effect of an intermolecular potential, and, second, compatibility of the pressure equation with convection. Once these two effects are accounted for, turbulence in my model emerges spontaneously from small fluctuations just like it does in nature. I will also show results of numerical simulations which support my hypothesis.

Effective thermal equilibrium for switching polymer model of chromosome dynamics

Anna Coletti
University of North Carolina Chapel Hill, USA

How the genome organizes and interacts within the cell can be understood through a polymer-like model of chromosome dynamics that combines Brownian motion with a stochastic switching force. The switching force, which follows a continuous-time Markov chain process, keeps the general overdamped Langevin system out of strict equilibrium. However, an effective energy landscape through the framework of quasipotentials helps us to understand the stability and transitions in the system. We show how this quasipotential changes as a function of switching rate which explains the difference in cluster-lifetimes observed in the large scale simulations of the experimental chromosome system.

Novel approach to finding stability of water waves

Sergey Dyachenko
State University of New York Buffalo, USA
Bernard Deconinck, Pavel M. Lushnikov, Anastassiya Semenova

The spectral stability of traveling waves in 2D ideal fluid of infinite depth is studied by linearization of the equations of motion for the free surface around a Stokes wave, and studying the spectrum of the associated Fourier-Floquet-Hill (FFH) eigenvalue problem. We developed a novel approach to studying the eigenvalue spectrum by combining the conformal Hamiltonian canonical variables, the FFH technique built into a matrix-free Krylov-Schur eigenvalue solver. The method has $N \log N$ numerical complexity and enjoys spectral accuracy.

Gibbs measures, limit shapes, and stochastic dynamics on partitions and their hydrodynamic limits

Ibrahim Fatkullin
University of Arizona, USA

I will talk about Gibbs measures on partitions of integers and sets and the corresponding limit shapes which appear in appropriate asymptotic limits. Models of this class are suitable for describing distributions of polymer sizes, fog droplets, or particle masses in colloidal suspensions. Then I will introduce various stochastic dynamics on these partitions and describe the corresponding hydrodynamic limits in terms of the PDEs governing the evolution of limit shapes for size distribution functions.

Slow pulse propagation in a damped, two-level, active optical medium

Gregor Kovacic
Rensselaer Polytechnic Institute, USA
Katelyn Leisman, Gino Biondini, David Cai

About two decades ago, an optical pulse traveling at the speed of a bicycle was observed in a ruby crystal. This pulse was modeled by two-level Maxwell-Bloch equations. However, a theoretical explanation on whether the pulse slowed down or traveled slowly since its injection in the crystal was missing. We present a computational study of the corresponding damped Maxwell-Bloch system explaining how the dynamics crucially depend on the amount of damping. We show that the pulse starts propagating slowly, and would eventually reach the speed of light in a longer crystal.

Collapse versus blowup and global existence in the generalized Constantin-Lax-Majda equation with dissipation

Pavel Lushnikov
University of New Mexico, USA
David Ambrose, Michael Siegel, Denis Silantyev

We analyze the dynamics of singularities and finite time blowup of generalized Constantin-Lax-Majda equation which corresponds to non-potential effective motion of fluid with competing convection and vorticity stretching terms. Both non-viscous fluid and fluid with various types of dissipation including usual viscosity are considered. An infinite families of exact solutions are found together with the different types of complex singularities approaching the real line in finite times. Both solutions on the real line and periodic solutions are considered. In the periodic geometry, a global-in-time existence of solutions is proven when the data is small and dissipation is strong enough. The found analytical solutions on the real line allow finite-time singularity formation for arbitrarily small data, even for various form of dissipation, thereby illustrating a critical difference between the problems on the real line and the circle. The analysis is complemented by accurate numerical simulations, which are able to track the formation and motion singularities in the complex plane. The computations validate and extend the analytical theory.

Nonlocal stochastic-partial-differential-equation limits of spatially correlated noise-driven spin systems derived to sample a canonical distribution

Katie Newhall
University of North Carolina Chapel Hill, USA

For a noisy spin system, we derive a nonlocal stochastic version of the overdamped Landau-Lipshitz equation designed to respect the underlying Hamiltonian structure and sample the canonical or Gibbs distribution while being driven by spatially correlated (colored) noise that regularizes the dynamics, making this Stochastic partial differential equation mathematically well-posed. We begin from a microscopic discrete-time model motivated by the Metropolis-Hastings algorithm for a finite number of spins with periodic boundary conditions whose values are distributed on the unit sphere. We demonstrate that for colored noise, the method used to project the noise onto the sphere and conserve the magnitude of the spins impacts the equilibrium distribution of the system, as coloring projected noise is not equivalent to projecting colored noise. In a specific scenario we show this break in symmetry vanishes with vanishing proposal size; the resulting continuous-time system of Stochastic differential equations samples the canonical distribution and preserves the magnitude of the spins while being driven by colored noise. Taking the continuum limit of infinitely many spins we arrive at the aforementioned version of the overdamped Landau-Lipshitz equation.

Non-exponential reversal times in models of bacterial aggregation

Ilya Timofeyev
University of Houston, USA
Mikhail Perepelitsa, Oleg Igoshin, Patrick Murphy

We consider prototype agent-based models of aggregation with reversals and a build-in chemotaxis mechanism. Following experimental data, we consider models where reversal times follow a non-exponential distribution. We derive a kinetic model for the behavior of the density and use Gamma distribution to approximate moments of reversal times in experimental data. We demonstrate that for non-exponential reversal times aggregates become more narrow compared with reversal times distributed exponentially.

Special Session 85: Interface Problems: Modelling, Analysis and Simulations

Xiaoping Wang, Hong Kong Uni. of Science and Technology / Chinese Uni. of Hong Kong Shenzhen, Hong Kong
Dong Wang, Chinese University of Hong Kong Shenzhen, Peoples Republic of China

Interfacial phenomena are ubiquitous. They define a boundary with the surrounding environment and influence the interactions with that environment. Interfaces play a vital role in science and technological applications in such diverse areas as biology, material science, image science and engineering design. The aim of this special session is to bring together experts in this area to present their recent research results and to explore new directions and topics, and to foster collaborations.

An efficient unconditionally stable method for computing Dirichlet partitions in arbitrary domains

Dong Wang
Chinese University of Hong Kong Shenzhen, Peoples Republic of China

A Dirichlet k-partition of a domain is a collection of k pairwise disjoint open subsets such that the sum of their first Laplace–Dirichlet eigenvalues is minimal. In this talk, we propose a new relaxation of the problem by introducing auxiliary indicator functions of domains and develop a simple and efficient diffusion generated method to compute Dirichlet k-partitions for arbitrary domains. The method only alternates three steps: 1. convolution, 2. thresholding, and 3. projection. The method is simple, easy to implement, insensitive to initial guesses and can be effectively applied to arbitrary domains without any special discretization. At each iteration, the computational complexity is linear in the discretization of the computational domain. Moreover, we theoretically prove the energy decaying property of the method. Experiments are performed to show the accuracy of approximation, efficiency and unconditional stability of the algorithm. We apply the proposed algorithms on both 2- and 3-dimensional flat tori, triangle, square, pentagon, hexagon, disk, three-fold star, five-fold star, cube, ball, and tetrahedron domains to compute Dirichlet k-partitions for different k to show the effectiveness of the proposed method. Compared to previous work with reported computational time, the proposed method achieves hundreds of times acceleration.

Nonlocal effects on a 1D generalized Ohta-Kawasaki model

Yanxiang Zhao
George Washington University, USA
Wangbo Luo

We propose a generalized Ohta-Kawasaki model to study the nonlocal effect on the pattern formation of some binary systems with general long-range interactions. While in the 1D case, the generalized Ohta-Kawasaki model displays similar bubble patterns as the standard Ohta-Kawasaki model, by performing Fourier analysis, we find that the optimal number of bubbles for the nonlocal model may have an upper bound no matter how large the repulsive strength is. The existence of such an upper bound is characterized by the eigenvalues of the nonlocal kernels. Additionally, we explore the conditions under which the nonlocal horizon parameter may promote or demote the bubble splitting, and apply the analysis framework to several case studies for various nonlocal operators.

Structure-preserving model order reduction for phase field models

Jia Zhao
Utah State University, USA

The phase field method is widely used in studying interfacial problems. However, solving phase field models can be computationally expensive and time-consuming, particularly for large-scale and 3D interfacial problems and long simulations. To overcome this challenge, model order reductions for phase field models can be designed to reduce computational costs while maintaining accuracy and feasibility. However, current reduced order model techniques do not preserve the energy variational structure that is crucial for phase field models to be thermodynamically consistent and produce accurate results. In this presentation, we propose a novel approach for designing thermodynamically consistent reduced order models for phase field PDEs. We will provide several examples using widely used phase field models to demonstrate the effectiveness of our proposed approach.

Special Session 87: Integrable Systems, Turbulence and Water Waves

Sergey Dyachenko, State University of New York Buffalo, USA
Anastassiya Semenova, University of Washington, USA
Denis Silantyev, University of Colorado Colorado Springs, USA

The special session will cover some recent advances in the field of dynamics in nonlinear systems with emphasis on phenomena in integrable and close-to-integrable systems. The main focus is given to the systems stemming from hydro- and gas dynamics, and will discuss novel results obtained by a wide range of approaches ranging from analysis and statistical physics to numerical methods.

Spontaneous development of turbulence in weakly compressible flow

Rafail Abramov
University of Illinois Chicago, USA

In my recent work, I proposed a model of weakly compressible flow, which relaxes the zero-divergence constraint in the incompressible flow model by replacing it with linear damping for the velocity divergence. I will show the results of numerical simulations with this model, where turbulence is created spontaneously via the mean field effect of the intermolecular potential.

Whitam modulation theory for multi-dimensional nonlinear wave equations and applications

Gino Biondini
State University of New York Buffalo, USA
Alexander Bivolcic, Alexandr Chernyavskiy, Mark Hoefer, Antonio Moro

Until recently, Whitham modulation theory had been mostly applied to systems in one spatial dimension, but in the last few years there have been several works aimed at generalizing and applying Whitham theory to systems in two and three spatial dimensions. This talk aims to present a survey of recent results, focusing on the Kadomtsev-Petviashvili (KP) equation (the integrable two-dimensional generalization of the KdV equation) and the two-dimensional nonlinear Schrödinger (NLS) equation. I will show how Whitham modulation theory can be generalized to multidimensional nonlinear evolution equations of KP type. I will also show how the KP-Whitham system can be successfully used to characterize analytically for the first time the evolution of a variety of initial conditions, including partial soliton stems and a combination of solitons and a mean flow. Time permitting I will discuss similarities and differences between the modulation theory for the KP equation and the two-dimensional NLS equation.

The instabilities of finite-depth Stokes waves

Eleanor Byrnes
University of Washington, USA
Bernard Deconinck, Sergey Dyachenko, Anastassiya Semenova

I will present a progress report on numerical results for the stability spectrum of Stokes wave solutions to the full water wave problem in finite depth. Following the work of Dyachenko and Semenova, we apply the Fourier-Floquet-Hill Method to a conformal mapping reformulation of the water wave problem to compute both sub- and co-periodic instabilities.

Rational solutions of KPI

Sarbarish Chakravarty
University of Colorado Colorado Springs, USA

A class of non-singular rational solutions of the Kadomtsev-Petviashvili (KP) I equation are explicitly constructed. The solutions have multiple peaks which form a complex two-dimensional wave pattern characterized by the partitions of a positive integer N – the number of peaks. The pattern formation are described by the roots of well-known polynomials arising in the study of rational solutions of Painlevé II and IV equations.

Whitham modulation theory for Zakharov-Kuznetsov equation

Alexander Chernyavsky
State University of New York Buffalo, USA
Gino Biondini

Whitham modulation theory for Zakharov-Kuznetsov (ZK) equation is presented. A system of four quasilinear first-order partial differential equations is derived. The system describes modulations of the periodic traveling wave solutions of the ZK equation. The system is then transformed to a hydrodynamic ZK-Whitham system, which is used to study stability properties of traveling wave solutions of the ZK equation. The results are validated by comparison with numerical simulations.

Traveling waves on a 2D fluid

Sergey Dyachenko
State University of New York Buffalo, USA
Alexander Korotkevich, Pavel Lushnikov, Anastassiya Semenova, Denis Silantiev

Traveling waves in 2D ideal fluid in infinite depth are also known as the Stokes wave. They range from small waves to the wave of the greatest height and present a challenge when one seeks waves whose crest is nearly angular. Some of the properties of the almost limiting waves will be discussed as well as numerical techniques for their computation. The singularities near the crests of these Stokes waves share some resemblance to singularities in steep waves observed in rough sea, and are conjectured to be significant for understanding of breaking ocean waves.

Direct verification of the kinetic description of wave turbulence for finite-size systems

Gregor Kovacic
Rensselaer Polytechnic Institute, USA
Jeffrey W Banks, Tristan Buckmaster, Alexander O Korotkevich, Jalal Shatah

The talk will present systems whose dynamics are governed by the nonlinear interactions among groups of 6 nonlinear waves, such as those described by the unforced quintic nonlinear Schrödinger equation. Specific parameter regimes in which ensemble-averaged dynamics of such systems with finite size are accurately described by a wave kinetic equation, as used in wave turbulence theory, are theoretically predicted. In addition, the underlying reasons that the wave kinetic equation may be a poor predictor of wave dynamics outside these regimes are also discussed. These theoretical predictions are directly verified by comparing ensemble averages of solutions to the dynamical equation with corresponding solutions of the wave kinetic equation.

Statistical properties and giant fluctuations for laser beam propagating in a turbulent medium

Pavel Lushnikov
University of New Mexico, USA

A statistical properties of a laser beam propagating in a turbulent medium are studied. It is proven that the intensity fluctuations at large propagation distances possess a Gaussian probability density function and establish quantitative criteria for realizing the Gaussian statistics depending on the laser propagation distance, laser beam waist, laser frequency, and turbulence strength. We calculate explicitly the laser envelope pair correlation function and corrections to its higher-order correlation functions breaking Gaussianity. At intermediate distances the laser intensity fluctuations follows the Poisson distribution (i.e. the amplitudes satisfies the Gaussian statistics) while the tail is strongly non-Gaussian with square-root dependence on the intensity in the exponent. The transition between the Poisson distribution and the non-Gaussian tail occurs at the values of intensity which quickly increases with the propagation distance. We find the explicit analytic expression for the fourth order correlation function vs. propagation distance and the turbulence strength which is determined by non-Gaussian tails. We finds that this correlation function is in excellent agreement with the large scale supercomputer simulations of laser wave propagation. We discuss also statistical properties of the brightest spots in the speckle pattern and find that the most intense speckle approximately preserves both the Gaussian shape and the diameter of the initial collimated beam while loosing energy during propagation. After propagating 7km through typical atmospheric conditions, approximately one in one thousand of atmospheric realizations produces an intense speckle with 20-30% of the initial power. Such optimal atmospheric realizations create an effective lens which focuses the intense speckle beyond the diffraction limit of the propagation in vacuum.

Nonlocal formulations, inverse problems and conservation laws for water waves

Katie Oliveras
Seattle University, USA

We consider a nonlocal formulation of the water-wave problem for a free surface with an irrotational flow, and show how the problem can be reduced to a single equation for the interface. The formulation is also extended to include constant vorticity and interfacial flows of different density fluids. We show how this formulation can be used to address various inverse problems related to water waves, and systematically derive Olver's conservation in terms of boundary variables. This framework easily lends itself to computing the related conservation laws for various asymptotic models.

Solitons and soliton interactions in the complex coupled short-pulse equation

Barbara Prinari
University at Buffalo, USA
Vincent Caudrelier, Aikaterini Gkogkou

The complex coupled short pulse equation (ccSPE) describes the propagation of ultra-short optical pulses in nonlinear birefringent fibers. The system admits a variety of vector soliton solutions: fundamental solitons, fundamental breathers, composite breathers (generic or non-generic), as well as so-called self-symmetric composite solitons.
In this talk, we will discuss interactions of ccSPE solitons. The investigation relies on the dressing method and the Darboux matrices corresponding to the various types of solitons, and it combines refac-

torization problems on generators of certain rational loop groups, and long-time asymptotics of these generators, as well as the main refactorization theorem for the dressing factors which leads to the Yang-Baxter property for the refactorization map and the vector soliton interactions. In particular, we derive explicit formulas for the polarization shift of fundamental solitons which are the analog of the well-known formulas for the interaction of vector solitons in the Manakov system. Our study also reveals that upon interacting with a fundamental breather, a fundamental soliton becomes a fundamental breather and, conversely, that the interaction of two fundamental breathers generically yields two fundamental breathers with a polarization shifts, but may also result into a fundamental soliton and a fundamental breather. Explicit formulas for the coefficients that characterize the fundamental breathers, as well as for their polarization vectors are obtained. New Yang-Baxter maps are obtained in the process.

Instability of Stokes waves in infinite depth fluid

Anastassiya Semenova
University of Washington, USA
Bernard Deconinck, Sergey A. Dyachenko, Pavel M. Lushnikov

We consider an ideal 2-dimensional fluid of infinite depth with potential flow and free surface. The equations of motion of free surface are linearized around Stokes waves, and we study the resulting eigenvalue problem. We discuss the numerical approach for computing the spectrum of the linearized operator by using a matrix-free Krylov methods and the Fourier-Floquet-Hill technique. We present the spectrum of Stokes wave of various steepnesses, discuss the maximum growth rate for these waves, and discuss the Benjamin-Feir, high-frequency and super-harmonic (localized) instabilities of these waves.

Global existence and singularity formation for the generalized Constantin-Lax-Majda equation with dissipation

Michael Siegel
New Jersey Institute of Technology, USA
David Ambrose, Pavel Lushnikov, Denis Silantyev

The question of global existence versus finite-time singularity formation is considered for the generalized Constantin-Lax-Majda (gCLM) equation with dissipation. This equation was first introduced by Constantin, Lax and Majda as a simplified model for singularity formation in the 3D incompressible Euler equations. It was later generalized to include dissipation and an advection term with parameter a, which allows different relative weights for advection and vortex stretching. There has been intense interest in the gCLM equation, and it has served as a proving ground for methods to study singularity formation in the 3D Euler equations. Despite significant effort, little is known about global existence versus singularity formation for general values of a. We use two complementary approaches to prove global-in-time existence of solutions for small data in the periodic geometry, when dissipation is strong enough. We also find a significant difference between the problems in the periodic and real-line geometries when dissipation is present.

Exact solutions of the generalized Constantin-Lax-Majda equation with dissipation

Denis Silantyev
University of Colorado Colorado Springs, USA
Pavel M. Lushnikov, Michael Siegel, David M. Ambrose

We consider the exact solutions of the generalized Constantin-Lax-Majda equation with dissipation $-\Lambda^\sigma$, where $\widehat{\Lambda^\sigma} = |k|^\sigma$, both for the problem on the circle $x \in [-\pi, \pi]$ and the real line. We analyze these solutions from the stand point of complex pole singularities and their motion in the complex space and find conditions for collapse in these solutions in a finite time for various advection parameter, dissipation coefficient and σ values.

Special Session 89: Recent Trends in Mathematical Fluid Mechanics

Milan Pokorny, Charles University, Czech Republic
Eduard Feireisl, Czech Academy of Sciences, Czech Republic
Antonin Novotny, University of Toulon, France

The session focuses on recent progress in the mathematical theory of fluids. The Euler and Navier Stokes systems are central but related models are also relevant. The emphasis is put on analysis and modeling with possible applications to numerical methods.

Conditional regularity for the Navier-Stokes-Fourier system with Dirichlet boundary conditions

Danica Basaric
Institute of Mathematics of the Czech Academy of Sciences, Czech Republic
Eduard Feireisl, Hana Mizerovà

We consider the Navier-Stokes-Fourier system governing the motion of a compressible, viscous, and heat conducting fluid confined to a bounded domain, on the boundary of which inhomogeneous Dirichlet boundary conditions for the velocity and the temperature are imposed. It is well-know that the system admits solutions in the classical sense; however, their existence can be guaranteed only on a maximal time interval. We show that a blowup will not occur as long as the density, the absolute temperature and the modulus of the fluid velocity remain bounded. The proof is based in deriving suitable a priori bounds.

From compressible Euler system to porous medium equation

Piotr Gwiazda
Institute of Mathematics Polish Academy of Sciences, Poland

This talk will be devoted to height friction and long time limit from compressible Euler to some target systems. In the classical case of isentropic Euler the example of target system is the porous medium equation. The limit will be done by relative entropy method, in the class of measure-valued solutions.

Energy conservation for the convective Brinkman-Forchheimer equations on bounded domains

Karol Hajduk
Institute of Mathematics Polish Academy of Sciences, Poland
Charles L. Fefferman, James C. Robinson

The flow of a fluid through a porous medium is classically described by Darcy's law. However, it typically applies for sufficiently slow viscous flows, e.g. for flows with small Reynolds numbers (laminar flows). When the flow is non-Darcian (e.g. turbulent flows), various modifications of Darcy's law are used to describe it. In this talk, we will discuss one such model, namely the convective Brinkman–Forchheimer equations (CBF). From the mathematical perspective, this model can be seen also as the incompressible Navier–Stokes equations with damping term $|v|^{r-1}v$, called the absorption term (or the Forchheimer term). We will give an overview of some available results for this model, focusing mainly on energy equality in the 'critical'case $r = 3$ on periodic domains. To handle more general bounded domains we use an abstract simultaneous approximation scheme which will be the main focus of this talk.

On the problem of singular limits

Sarka Necasova
Institute of Mathematics, Academy of Sciences, Czech Republic
Martin Kalousek, Tong Tang

Primitive Equations (PE) are an important model which is widely used in the geophysical research and the mathematical analysis. We give a rigorous mathematical derivation of inviscid compressible Primitive Equations from the Euler system in a periodic channel, utilizing the relative entropy inequality. It is a joint wotk with T.Tang.
Further, we deal with the asymptotic limit of the compressible Navier-Stokes system with a pressure obeying a hard–sphere equation of state on a domain expanding to the whole physical space R^3. Under the assumptions that acoustic waves generated in the case of ill-prepared data do not reach the boundary of the expanding domain in the given time interval and a certain relation between the Reynolds and Mach numbers and the radius of the expanding domain we prove that the target system is the incompressible Euler system on R^3. We also provide an estimate of the rate of convergence expressed in terms of characteristic numbers and the radius of domains. It is a joint work with M.Kalousek.

Two-phase compressible/incompressible Navier-Stokes system with inflow-outflow boundary conditions

Milan Pokorny
Charles University, Czech Republic
Aneta Wróblewska-Kamińska, Ewelina Zatorska

We prove the existence of a weak solution to the compressible Navier–Stokes system with singular pressure that explodes when density achieves its congestion level. This is a quantity whose initial value evolves according to the transport equation. We then prove that the "stiff" pressure limit gives rise to the two-phase compressible/incompressible system with congestion constraint describing the free interface. We prescribe the velocity at the boundary and the value of density at the inflow part of the boundary of a general bounded C^2 domain. For the positive velocity flux, there are no restrictions on the size of the boundary conditions, while for the zero flux, a certain smallness is required for the last limit passage.

Compressible magnetohydrodynamics driven by non-conservative boundary conditions

Agnieszka Swierczewska-Gwiazda
University of Warsaw, Poland
Eduard Feireisl, Piotr Gwiazda, Young-Sam Kwon

We propose a new concept of weak solution to the equations of compressible magnetohydrodynamics driven by large boundary data. The system of the underlying field equations is solvable globally in time in the out of equilibrium regime characteristic for turbulence. The weak solutions comply with the weak–strong uniqueness principle; they coincide with the classical solution of the problem as long as the latter exists. The choice of constitutive relations is motivated by applications in stellar magnetoconvection.

Special Session 90: Recent Advances in Wavelet Analysis, PDEs and Dynamical Systems

Emanuel Guariglia, Wenzhou-Kean University, China
Rodrigo Capobianco Guido, São Paulo State University, Brazil

This session will focus on some recent developments in wavelet analysis, including pure analysis and applications. In particular, this minisymposium is dedicated to the analysis of both PDEs and dynamical systems by wavelet analysis. Besides analysis of classical problems, the topics of the session include image processing, information theory, function spaces, calculus of variations and recent trends on local and nonlocal PDEs. Our goal is to bring together experts and young researchers in various subfields of wavelet analysis, PDEs and dynamical systems, in order to report the newest progress, exchange ideas, forge new cooperation and to discuss both classical and merging open problems. The following is a non-exhaustive list of topics to be discussed in this session:

- Wavelet methods for PDEs
- Fractal-wavelet characterization of dynamical systems
- Multiresolutional algorithms for operator equations
- Hamiltonian systems, Lyapunov functions and stability
- Time-frequency-shape joint analysis
- Variational image denoising

Solution of non-linear fractional Vander Pol equation through Boubaker wavelets

Sripathy Budhi
Vellore Institute of Technology, India
K.Tamilvannan, T.V.G. Shriprakash, R. Seethalakshmi

In this article, an efficient method based on Boubaker wavelets are applied for investigations of non-linear fractional order Vander Pol equations. Fractional order Vander Pol equation are defined in Caputo sense.For this investigation, operational matrix of derivative Boubake wavelets are constructed with the help of Boubaker polynomials. The operational matrix derivative is applied in Vander pol equations which converts the underlying differential equation into a system of algebraic equations. The unknowns in the algebraic equation can be found using Newton's method. Some numerical examples are presented to show the effectiveness and applicability of this wavelet based approach to justify in order to verify its validity.

Chebyshev wavelets and applications

Emanuel Guariglia
Wenzhou-Kean University, China

In this talk, we deal with Chebyshev wavelets. We analyze their properties computing their Fourier transform. Moreover, we discuss the differential properties of Chebyshev wavelets due the connection coefficients. The differential properties of Chebyshev wavelets, expressed by the connection coefficients are given by finite series in terms of the Kronecker delta. Moreover, we treat the p-order derivative of Chebyshev wavelets and compute its Fourier transform. Finally, we expand the mother wavelet in Taylor series with an application both in fractional calculus and fractal geometry.

Predictive algorithms for burst-like external force identification

Ilya Krishtal
Northern Illinois University, USA

I will describe two predictive algorithms for estimating bursts in the external force of an abstract initial value problem (IVP). The focus is on the special structure of the samples of the solution of the IVP that allows for quick and accurate estimation of Dirac or rapidly decaying bursts. The predictive nature of the algorithms manifests in using the current samples to estimate the measurements at the end of the following sampling period on the condition that no burst occurred within the period. The talk is based on joint work with A. Aldroubi, L. Gong, L. Huang, and K. Kornelson.

Comparing wavelet families for edge detection in digital images

Gabriel José Pellisser Dalalana
São Paulo State University, Brazil
Rodrigo Capobianco Guido

In this work, a brief presentation on the current edge detection problem will be presented, illustrating that this problem is far from being completely solved. Then, we will present a comparison between the different wavelet filter families for solving the problem of edge detection in digital images, particularly making use of the Two-Dimensional Discrete-Time Wavelet Transform.

The regular behavior in differential equations and related topics

Maria Alessandra Ragusa
University of Catania, Italy

We study the behavior of the highest order derivatives of solutions of partial differential equations of elliptic, parabolic and ultraparabolic type. Particular attention is posed on the coefficients of the highest order derivatives, that are supposed, in all the cases, discontinuous. A lots of different applications of these kind of equations are possible.

Approximate finite dimensional additive mappings in modular spaces

Kandhasamy Tamilvanan
R.M.K. Engineering College, Tamil Nadu, India

The purpose of this paper is to obtain the general solution and investigate an alternative H-U stability results for the finite dimensional additive functional equation in a Modular space χ_ρ via direct method.

Ulam Hyers stabilities of the Euler-Lagrange-Jensen (a, b)-orthogonal cubic functional equations in Banach spaces

Kandhasamy Tamilvanan
R.M.K. Engineering College, Tamil Nadu, India

We determine some stability results of the Euler-Lagrange-Jensen (a, b)-orthogonal cubic functional equations in Banach Spaces. Our result can be regarded as a generalization of the stability phenomenon in the framework of Banach Spaces.

Contributed Session 1: ODEs and Applications

Application of Laplace transform to cryptography using linear combination of functions

Emmanuel Adeyefa
Federal University Oye-Ekiti, Nigeria
Agbolade Oluwaferanmi David

The strength of a nation, organizations such as military, intelligence agency, etc is largely measured by its ability to keep sensitive information secured from intruders. Today, cryptography is used to protect mobile communication, internet services, bank details, etc from hackers and fraudsters. In this work, we present an algorithm for cryptography using Laplace transform of linearly combined functions for encryption and the inverse Laplace transform of the linearly combined functions for decryption.

A novel approach to analytic velocity profiles of an Oldroyd 6-constant fluid in a channel

Spencer Boebel
University of South Carolina Upstate, USA
Muhammad Hameed

The study concerns the application of a novel idea used to find analytical solutions of a nonlinear boundary value problem which arises in non-Newtonian channel flow. The velocity profile of a viscoelastic fluid between two planes is obtained with both slip and no-slip boundary conditions modeled by an Oldroyd 6 constant fluid. We present analytical solutions on two classical problems- the Poiseuille flow and the Couette flow- of a viscoelastic fluid between two parallel planes. We solved this equation by reducing it to a cubic equation in the derivative and the unknown. We used the cubic formula to solve this equation. Finally, we integrated this expression with binomial expansion to obtain a purely analytic series solution to this problem.

Dynamical analysis of a mathematical model of hepatitis C virus (HCV) infection with hepatocyte proliferation

Hossam Ezzat
Alexandria University, Egypt
Hesham A. Elkaranshawy

The basic mathematical model of hepatitis C virus infection was originally formulated in 1998. This basic model is unable to explain all the observed HCV RNA profiles under treatment e.g., a triphasic viral decay and a viral rebound to baseline values after the cessation of therapy. Though the extension of the basic model by including hepatocyte proliferation was constructed and became a more realistic model without any of these deficiencies, the dynamical analysis of its original form has not been considered. In this research work, a dynamical analysis for this model is treated. The basic reproduction number is obtained, and the equilibrium points are specified. The stability at the equilibrium points is analyzed based on Routh-Hurwitz criteria and Lyapunov invariance principle method. The results indicate that the uninfected equilibrium point is locally as well as globally asymptotically stable when the reproduction number is less than one. The infected equilibrium point of the model is locally and globally asymptotically stable when the basic reproduction number is greater than one. Numerical sensitivity analysis based on model parameters is performed and the result describes the influence of each parameter on the basic reproduction number.

Limit cycles and strange attractors of multi-parameter dynamical systems

Valery Gaiko
United Institute of Informatics Problems, National Academy of Sciences of Belarus, Belarus

A global bifurcation analysis of multi-parameter polynomial dynamical systems is carried out. To control the global bifurcations of limit cycles in planar systems, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcation-geometric methods based on the Wintner-Perko termination principle. Using these methods, we present, e.g., a solution of Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the Kukles cubic-linear system, the general Liénard polynomial system with an arbitrary number of singular points, the Euler-Lagrange-Liénard mechanical system, Leslie-Gower systems which model the population dynamics in real ecological or biomedical systems and a reduced planar quartic Topp system which models the dynamics of diabetes. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenarios in Lorenz type systems connecting globally the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of limit cycles.

Common attractors of generalized iterated function systems

Melusi Khumalo
University of South Africa, South Africa

In this talk, we develop some some new common attractors with the assistance of finite families of generalized contractive mappings, that belong to the special class of mappings defined on a partial metric space. Consequently, a variety of results for iterated function systems satisfying a different set of general-

ized contractive conditions are acquired. We present some examples to reinforce the results proved herein. These results generalize, unify and extend a variety of results that exist in current literature.

Scalar auxiliary variable schemes for Cahn-Hilliard systems with mass source

Kei Fong Lam
Hong Kong Baptist University, Hong Kong
Ru Wang

The scalar auxiliary variable (SAV) approach presents a novel way to discretize a large class of dissipative systems. We consider a general Cahn-Hilliard system with mass source that may not admit a known dissipative structure, and so the stability of discrete solutions is not immediate. With a bounded mass source, we can show stability and convergence of time discrete solutions for a first order SAV scheme. We present some qualitative simulations for some Cahn-Hilliard systems with mass source that appear in tumor growth, image inpainting and image segmentation.

Deterministic-statistical approach for moving sources with sparse partial data

Yanfang Liu
George Washington University, USA
Yukun Guo, Jiguang Sun

In this talk, we present a two-step deterministic-statistical approach to reconstruct the trajectories of moving sources using partial measured data. In the first step, an approximate direct sampling method is developed to obtain the locations of the sources at different times. Such information is coded in the priors, which is critical for the success of the Bayesian method in the second step. The well-posedness of the posterior measure is analyzed in the sense of the Hellinger distance. Both steps are based on the same physical model and use the same set of measured data. The combined approach inherits the merits of the deterministic method and Bayesian inversion as demonstrated by the numerical examples.

Geometric singular perturbation approach to glass networks

Benitho Ngwu
University of Nigeria Nsukka, Nigeria
G.C.E. Mbah, R.O. Ajewole, S.U. Isienyi

Glass Network is a class of gene regulatory networks used to model the interactions and controls existing among molecules, proteins and other transcription factors in a biological system. The activities of the genes and their regulatory effects are controlled through their concentration levels defined by threshold functions which are only defined at off threshold value. Qualitative and numerical studies of this network have revealed a lot of properties such as existence of periodic orbits, stable oscillation, chaos and steady states. This study applied the method of geometric singular perturbation to investigate the behaviour of the system within the threshold neighbourhood (known as singular states) of the networks. Some results such as transparency condition on non-sliding walls and manifold of sliding walls were obtained.

Revisiting the direct Fourier filtering technique for the boundary damped wave equation

Md Rafi as Sadeq ibn Emran
Western Kentucky University, USA
Ahmet Ozkan Ozer

A one-dimensional wave equation is considered with a boundary feedback controller, which is known to have exponentially stable solutions. However, the reduced models by the semi-discretized Finite Differences and Finite Elements lacks of exponential stability uniformly as the discretization parameter tends to zero. This is due to the loss of uniform gap among the high-frequency eigenvalues as the discretization parameter tends to zero. One remedy to overcome this discrepancy is the direct Fourier filtering technique where the high-frequency spurious eigenvalues are directly filtered. The exponential decay rate, mimicking the PDE counterpart, can be retained uniformly with the filtered solutions. However, the existing proof technique in the literature is solely based on an observability inequality. In this paper, exponential stability results for both filtered Finite Difference and Finite Element approximations are established by a Lyapunov-based direct approach. The maximal decay rate in terms of the filtering parameter and the optimal feedback gain is explicitly provided. Our results mimic the PDE counterpart. Several numerical tests are provided to support our results.

Propagation reversal for bistable differential equations on trees

Vladimir Svigler
University of West Bohemia, Czech Republic
Hermen Jan Hupkes, Mia Jukič, Petr Stehlìk

We study traveling wave solutions to the bistable differential equations on infinite k-ary trees in the form

$$\dot{u}_i = d(ku_{i+1} - (k+1)u_i + u_{i-1}) + g(u_i; a),$$

in which $i \in \mathbb{Z}$, $d > 0$ and $g : \mathbb{R} \to \mathbb{R}$ is a bistable nonlinearity of the Nagumo type, e.g.,

$$g(s; a) = s(1-s)(s-a), \quad a \in (0, 1).$$

In this talk, we discuss how comparison principles and construction of explicit lower and upper solution can be used to obtain information about the dependence of the wave speed $c \in \mathbb{R}$ on the parameters a, d, k.

We show that wave-solutions are pinned provided the diffusion parameter d is small. Upon increasing the diffusion d, the wave starts to travel with non-zero speed $c \neq 0$, in a direction that depends on the detuning parameter a. However, once the diffusion is sufficiently strong, the wave propagates in a single direction up the tree irrespective of the detuning parameter a.
As a consequence, we show that for certain range of the detuning parameter a the changes to the diffusion parameter d lead to a reversal of the propagation direction. This is in stark contrast to the behaviour of the standard lattice equation with $k = 1$.

Entropy-dissipation informed neural network for McKean-Vlasov PDEs

Zhenfu Wang
Peking University, Peoples Republic of China
Zebang Shen

We extend the concept of self-consistency for the Fokker-Planck equation (FPE) [Shen et al., 2022] to the more general McKean-Vlasov equation (MVE). While FPE describes the macroscopic behavior of particles under drift and diffusion, MVE accounts for the additional inter-particle interactions, which are often highly singular in physical systems. Two important examples considered in this paper are the MVE with Coulomb interactions and the vorticity formulation of the 2D Navier-Stokes equation. We show that a generalized self-consistency potential controls the KL-divergence between a hypothesis solution to the ground truth, through entropy dissipation. Built on this result, we propose to solve the MVEs by minimizing this potential function, while utilizing the neural networks for function approximation. We validate the empirical performance of our approach by comparing with state-of-the-art NN-based PDE solvers on several example problems.

Contributed Session 2: PDEs and Applications

Some new results and developments on the stabilization of swelling soils by employing different damping mechanisms

Adel Al-Mahdi
King Fahd University of Petroleum and Minerals, Saudi Arabia

Swelling soils are one of the environmental problems that have attracted many researchers due to their structural damage or destruction. It is characterized by a swell in soil volume when subjected to moisture. The clay minerals in them naturally attract and absorb water. When water is introduced to swelling soils, the water molecules are pulled into gaps between the soil plates. As more water is absorbed, the plates are forced further apart, leading to an increase in soil pore pressure. Consequently, swelling soils significantly lead to geotechnical and structural challenges to the environment and society. Swelling soils exist all over the world. More recently, the American Society of Civil Engineers estimates that one in four homes have some damage caused by swell soils. Typically, such soils cause property owners more significant financial loss than earthquakes, floods, hurricanes, and tornadoes combined. Consequently, it is crucial to study the practical ways and means to eliminate or minimize the damages caused by swelling soils. In this talk, we present some new results and developments on the stabilization of swelling soils by employing different damping mechanisms proven to be a very effective, economical, and friendly environment. These results are significant to engineers and architects as they might help swiftly attenuate the harmful effects of swelling soils. Furthermore, these results. will be a basis for further work and broaden the scope of damping technology.

Stabilization of some hyperbolic systems by means of viscoelastic damping or/and variable exponent frictional damping

Mohammad Algharabli
King Fahd University of Petroleum and Minerals, Saudi Arabia

Stabilization of some hyperbolic systems by means of viscoelastic damping or/and variable exponent frictional damping.
In our life, the natural supplies, although they are plentiful, could become scarce if they are not with prudence and wisdom. Within this perspective, it becomes very relevant to control the energy used for transferring a system from an initial to a finial state. The excessive expense of energy or the bad control of it can lead the plane to chaos.
Vibration has long been known for its capacity for disturbance, discomfort, damage and destruction. Since ancient times, mankind has tried to find ways to control this phenomenon. A damping effect may be caused by applying internal or boundary frictional mechanisms. Also, a damping effect may result from the nature of the material. In this talk, we will present some recent developments in stabilization of some systems using nonlinear frictional damping of variable exponent type and also the use of viscoelastic materials to stabilize some systems.

Stability analysis for a thermoelastic Timoshenko system with past history and general relaxation function

Johnson Daddy Audu
King Fahd University of Petroleum and Minerals, Saudi Arabia
Adel M. Al-Mahdi, Cyril D. Enyi, Soh E. Mukiawa

The purpose of our research is to investigate a thermoelastic Timoshenko system with an infinite memory term on the shear force while the bending moment is under the influence of a thermoelastic dissipation. We prove that the system's stability holds for a broader class of relaxation functions. Under this class of relaxation functions h at infinity, we establish a relation between the decay rate of the solution and the growth of h at infinity. Moreover, we drop the boundedness assumptions on the history data. We employ Neumann-Dirichlet-Neumann boundary conditions for our result. In comparison to the bulk of results in the literature, which frequently enforce the equal-wave-speed constraint, this result is of tremendous importance because our result does not require any conditions on the parameters.

Real line solitons of the BKP equation

Jen-Hsu Chang
National Defense University, Taiwan

The solitons solution of BKP equation can be constructed by the Pfaffian structure. Then one investigates the real line solitons structure of BKP equation using the totally non-negative Grassmannian. Especially, the N-soliton solution is studied and its self-dual Tau function is obtained. Also, one can construct the totally non-negative Grassmannian of the Sawada-Kotera equation for its real line solitons.

Memory effects for the heat conductivity of random suspension of spheres by using stochastic functional expansions

Abhinandan Chowdhury
Savannah State University, USA

Heat conduction of a suspension subjected to a time-dependent spatially constant temperature gradient is studied with the aid of stochastic functional expansions with random-point basis functions. It is argued that the basis function which is appropriate for modelling the chaotic behavior of nonlinear dynamical systems, e.g., turbulence, is not suited for studying the composite materials. It is shown that within the first order of approximation with respect to the concentration, the equation for the kernel of the 1st-order functional integral is the equation of the disturbance introduced by a single sphere (filler) in a matrix subjected to a time-dependent temperature gradient. After solving the resulting initial-boundary value problem, the effective correlation between the heat flux and temperature gradient is established. It turns out that the effective law involves a retardation (memory integral) of the temperature gradient. Approximate expression for the memory kernel is found by employing a method based on infinite series expansion. An interesting limiting case of filler material with infinite conductivity is discussed where memory integral becomes the Riemann-Liouville half integral.

Sectorial equations on fractional power scales

Tomasz Dlotko
University of Silesia, Poland
Radoslaw Czaja

Originating with the monograph by Dan Henry (1981), the semigroup approach to evolution problems having a positive sectorial operator in the main part allows us to settle them at various levels of the fractional power scale associated with the main linear operator. This translates into different regularity properties of local solutions to such equations. Specific applications of the abstract results to the *2D surface quasi-geostrophic equation* and the *fractional chemotaxis system* will be also presented. Presentation based on: R. Czaja, T. Dlotko, Sectorial equations on fractional power scales, submitted, January 2023.

Diffusion generated motion by mean curvature of a space curve

Gavin Glenn
Purdue University, USA
Aaron Yip

The Merriman-Bence-Osher Scheme, also known as Diffusion Generated Motion, is a simple algorithm for approximating mean curvature flow (MCF). The procedure alternates between diffusion and projection of a state function in such a way that its zero set approximates the solution of MCF. An advantage of this approach is that it naturally computes topological changes, like merging and breaking of solutions, without any additional steps. Originally proposed for hypersurfaces, the scheme is less well-understood when applied to submanifolds of higher codimension. Using a combination of rigorous analysis and numerical techniques, we characterize the convergence in the case of a space curve.

Generalized Choquard Schrödinger equation with vanishing potential in homogeneous fractional Musielak Sobolev spaces

Shilpa Gupta
BITS Pilani, Pilani Campus, India
Gaurav Dwivedi

In this work, we discuss the existence of a weak solution for the following problem:

$$(-\Delta)^s_{\mathcal{H}} u(x) + V(x)h(x,x,|u|)u(x) = \left(\int_{\mathbb{R}^N} \frac{K(y)F(u(y))}{|x-y|^\lambda} dy\right) K(x)f(u(x)) \text{ in } \mathbb{R}^N,$$

where $N \geq 1$, $s \in (0,1), \lambda \in (0,N)$,
$\mathcal{H}(x,y,t) = \int_0^{|t|} h(x,y,r)r\, dr$,
$h : \mathbb{R}^N \times \mathbb{R}^N \times [0,\infty) \to [0,\infty)$ is a generalized N-function and $(-\Delta)^s_{\mathcal{H}}$ is a generalized fractional Laplace operator. The functions $V, K : \mathbb{R}^N \to (0,\infty)$, non-linear function $f : \mathbb{R} \to \mathbb{R}$ are continuous and $F(t) = \int_0^t f(r)dr$.
First, we introduce the homogeneous fractional Musielak-Sobolev space and investigate their properties. After that, we pose the given problem in that space. To establish our existence results, we prove and use the suitable version of Hardy-Littlewood-Sobolev inequality for Lebesque Musielak spaces together with variational technique based on the mountain pass theorem. We also prove the existence of a ground state solution by the method of Nehari manifold.

Positivity of nonnegative solutions to a system of fractional Laplacian problems in a ball

Elliott Hollifield
University of North Carolina Pembroke, USA

We consider a cooperative system of equations involving the fractional Laplacian operator with zero Dirichlet external condition on a ball. We prove that nonnegative solutions of such problems, with semipositone nonlinearities, are positive and hence radially symmetric. We use the method of moving planes to establish our result.

Boundedness of blowup time to a wave equation with logarithmic variable-exponent nonlinearity

Mohammad Kafini
King Fahd University of Petroleum and Minerals, Saudi Arabia

In this talk, we are concerned with a nonlinear wave equation with variable exponents. In the presence of the logarithmic nonlinear source, we established a global nonexistence result under sufficient conditions on the initial data only without imposing the Sobolev Logarithmic Inequality. We established a blow up result with upper bound and lower bound.

Random dynamics of 2D stochastic Naiver-Stokes equations on the whole space

Kush Kinra
Indian Institute of Technology at Roorkee, India
Manil T. Mohan

In this talk, we consider the random dynamics and asymptotic analysis of the well known mathematical model,

$$\frac{\partial \boldsymbol{v}}{\partial t} - \nu\Delta\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \nabla p = \boldsymbol{f},\ \nabla\cdot\boldsymbol{v} = 0,$$

the Navier-Stokes equations. We consider the two-dimensional stochastic Navier-Stokes equations (SNSE) driven by a *linear multiplicative white noise of Itô type* on the whole space $\mathbb{R}^2$. Firstly, we will discuss that the non-autonomous 2D SNSE generates a bi-spatial $(\mathbb{L}^2(\mathbb{R}^2), \mathbb{H}^1(\mathbb{R}^2))$-continuous random cocycle. Due to the bi-spatial continuity property of the random cocycle associated with SNSE, we will address that if the initial data is in $\mathbb{L}^2(\mathbb{R}^2)$, then there exists a unique bi-spatial $(\mathbb{L}^2(\mathbb{R}^2), \mathbb{H}^1(\mathbb{R}^2))$-pullback random attractor for non-autonomous SNSE which is compact and attracting not only in $\mathbb{L}^2$-norm but also in $\mathbb{H}^1$-norm. At the end, we will discuss the existence of an invariant measure for the random cocycle associated with 2D autonomous SNSE. We will also address the uniqueness of invariant measures for $\boldsymbol{f} = \boldsymbol{0}$ and for any $\nu > 0$ by using the linear multiplicative structure of the noise coefficient and exponential stability of solutions.

On a thermoelastic Timoshenko system without the second spectrum: Existence and stability

Salim Messaoudi
University of Sharjah, United Arab Emirates
Ahmed Keddi, Mohamed Alahyane

In this work we study the well-posedness and the asymptotic stability of a linear thermoelastic Timoshenko system free of second spectrum, where the heat conduction is given by Cattaneo's law. We establish the well-posedness, using the semigroup theory. Then, we prove that the system is exponentially stable irrespective of the coefficients. We also illustrate our theoretical results by performing some numerical tests.

Difference in decay properties for symmetric hyperbolic system with memory-type diffusion and relaxation

Naofumi Mori
Tokyo University of Marine Science and Technology, Japan
Mari Okada, Shuichi Kawashima

We consider the decay property for a symmetric hyperbolic system with memory-type relaxation. We assume that the memory kernel is a strongly positive definite kernel and prove that the system has the decay property of regularity-loss type. Our proof is based on a technical energy method in the Fourier space which makes use of the structural condition (called Craftsmanship condition or Condition (K)) and the modified properties of strongly positive definite kernels.

Symmetry reduction of a gas dynamic system of PDEs with a special state function

Dilara Siraeva
North Carolina State University, USA

In this talk, I will present new results on the symmetry reduction of gas dynamic systems of PDEs following the general framework presented by Lev Ovsyannikov in his article The podmodeli program. Gas dynamics https://www.sciencedirect.com/science/article/pii/0021892894901376 The gas dynamics systems of equations, with an arbitrary state equation, has an 11-dimensional Lie algebra of symmetries which generates a group of space translations, time translation, rotations, Galilean translations, and uniform dilations. In this investigation, we consider a special state equation, where the pressure is the sum of two functions - the first function depends on density, and the second function depends on entropy. Such a system of equations has an additional symmetry, pressure translation, and thus admits a 12-dimensional Lie algebra of symmetries. I will present a classification of subalgebras of this 12-dimensional Lie algebra up to similarity and will demonstrate how using the process of symmetry reduction one can transform a gas dynamic system of this type into the systems with one or two independent variables, which can be solved exactly leading to new particular solutions of the original system. A description of the motion of particles for these particular solutions will be also presented.

Inverse dynamic problem for the 1-D Dirac system on finite metric tree graphs and the leaf peeling algorithm

Olha Sus
Tufts University, USA

In recent years the study of the Dirac operator on metric graphs has generated a growing interest. It has been widely used for modeling electronic properties of graphene, propagation of electromagnetic waves in graphene-like photonic crystals, ultracold matter in optical lattices and some other physical processes. In this talk, we consider the dynamic inverse problem for the one-dimensional Dirac system on finite metric tree graphs. The main goal is to recover the topology (connectivity) of a tree, lengths of edges, and a matrix potential function on each edge. We use the dynamic response operator as inverse data. In addition, we present a new dynamic algorithm to solve the forward problem for the 1-D Dirac system on general finite metric graphs. The talk is based on joint work with Sergei Avdonin and Nina Avdonina.

Contributed Session 3: Modeling, Math Biology and Math Finance

Analysis of neuronal oscillations of fractional-order Morris-Lecar model

Tahmineh Azizi
University of Wisconsin Madison, USA

Fractional calculus as a new approach for modeling has been used widely to study the nonlinear behavior of physical and biological systems with some degrees of fractionality or fractality using differential and integral operators with non integer orders. In this paper, to explore different dynamical classes of the Morris Lecar neuronal model with chloride channel, we extend its integer order domain into a new fractional order space using fractional calculus. The nonstandard finite difference (NSFD) method following the Grunwald Letnikov discretization may be applied to discretize the model and obtain the fractional order form. Fractional derivative order has been used as a new control parameter to extract variety of neuronal firing patterns that happen in real world application but the integer order operator may not be able to reveal them. To find the impact of chloride channel on dynamical behaviors of this neuronal model, the phase portrait and time series analysis have been performed for different fractional orders and input currents. Depending on different values for fractional derivative, the fractional order Morris Lecar model with a chloride channel reproduces quiescent, spiking and bursting activities the same as the fractional order Morris Lecar model without a chloride channel. We numerically discover the occurrence of hopf bifurcation, and homoclinic bifurcation for these two models.

Assessing potential insights of an imperfect testing strategy: Parameter estimation and practical identifiability using early COVID-19 data in India

Sarita Bugalia
Central University of Rajasthan, India
Jai Prakash Tripathi

A deterministic model with testing of infected individuals has been proposed to investigate the potential consequences of the impact of testing strategy. The model exhibits global dynamics concerning the disease-free and a unique endemic equilibrium depending on the basic reproduction number when the recruitment of infected individuals is zero; otherwise, the model does not have a disease-free equilibrium, and disease never dies out in the community. Model parameters have been estimated using the maximum likelihood method with respect to the data of early COVID-19 outbreak in India. The practical identifiability analysis shows that the model parameters are estimated uniquely. The consequences of the testing rate for the weekly new cases of early COVID-19 data in India tell that if the testing rate is increased by 20% and 30% from its baseline value, the weekly new cases at the peak are decreased by 37.63% and 52.90%; and it also delayed the peak time by four and fourteen weeks, respectively. Similar findings are obtained for the testing efficacy: if it is increased by 12.67% from its baseline value, the weekly new cases at the peak decrease by 59.05% and delay the peak by 15 weeks. Therefore, a higher testing rate and efficacy reduce the disease burden by tumbling the new cases, representing a real scenario. It is also obtained that the testing rate and efficacy reduce the epidemic's severity by increasing the final size of the susceptible population. The testing rate is found more significant if testing efficacy is high. Global sensitivity analysis using partial rank correlation coefficients (PRCCs) and Latin hypercube sampling (LHS) determine the key parameters that must be targeted to worsen/contain the epidemic.

Impact of prey induced dispersal on stability in predator-prey dynamics

Youngseok Chang
Korea University, Korea
Wonhyung Choi, Inkyung Ahn

In ecology, many evidences, such as the density of a species in a region and the interaction between individuals, can cause species dispersal, which involves species moving from one habitat to another. Thus, we focus on a situation in which a predator can change its motility speed according to its satisfaction with foraging prey in as predator–prey interaction. However, due to an inaccurate judgement, the predator sometimes determines their rate of movement. In this situation, we observe an effect of the predator's judgement about dispersal strategy on their invasion. We consider a predator–prey interaction with nonuniform dispersal, called prey-induced dispersal (PYID), in which the motility of predator is high when the prey density is lower than some specific value. Otherwise, if the prey density is higher than a certain value, the predator moves slowly. To analyze the effect of PYID, we observe a model with Holling-type II functional responses under zero-flux boundary conditions. We conclude that the predator under PYID sometimes plays a negative effect their invasion. However, in most cases, nonuniform dispersal of predators following PYID promotes predator fitness. This work contains a joint work with Wonhyung Choi and Inkyung Ahn.

Increasing complexity in sexually-transmitted disease models

Christine Craib
University of California Los Angeles, USA
Mason A. Porter

Sexually-transmitted disease (STD) transmission in a population depends on the underlying sexual network of that population. It is known that sexual behavior is heterogeneous and partially assortative, so informative models need to account for these complex attributes. We present a model of the transmission of bacterial STDs. This 3-disease SIS model describes the behavior of chlamydia, gonorrhea, and syphilis with coinfections and universal recovery in a fixed population. First, we assume homogeneous mixing and homogeneous sexual behavior in an 8-equation ODE model, with equations for each coinfection states. We study its equilibria, particularly emphasizing the behavior of the total prevalence of each disease with respect to the behavior of the ODE model. We then extend the ODE model by incorporating a time-dependent network structure. We examine the prevalence of each disease for both models, which we compare to determine the effects of increased complexity on forecasts of disease prevalence.

Mathematical modeling of pandemics in a metapopulation: New insights from the SIR-network model

Haridas Kumar Das
Oklahoma State University, USA
Lucas Martins Stolerman

In this work, we study the SIR-Network model proposed by Stolerman, Coombs, and Boatto in 2015, which describes the dynamics of an infectious disease in a city where each neighborhood is a node of a network, and the edges of a network represent the daily flux of people between the nodes. In their original study, Stolerman and colleagues established a flux-driven epidemic control by analyzing epidemic thresholds for fully-connected networks where a single node has a different infection rate. Inspired by this result, we analyze a larger class of networks establishing new epidemic thresholds using the basic reproduction number R_0 obtained from the classic next-generation matrix. We find a family of networks (star-class) with the same kind of epidemic control inspired by the star-shaped networks, providing analytical estimates in both general (any network size) and particular (fixed network size) cases. In addition, cycle-shaped networks exhibit a flux-driven epidemic control but with different epidemic thresholds compared to the star-class networks. Finally, we numerically integrate our system to gain an intuition of where the theoretical estimates are challenging and explore the temporal dynamical behavior for the different classes of networks.

Mathematical modeling of BK virus infection in kidney transplant recipients

Dana Droz
North Carolina State University, USA
Nicholas Myers, Hien Tran

Kidney transplant recipients are put on an immunosuppressant drug regimen to suppress their immune system which prevents allograft rejection. These drugs make the patient susceptible to infections. The opportunistic BK virus (BKV) infection has no effective antiviral treatment. The standard clinical practice to treat BKV is to reduce immunosuppression which in turn increases the risk of rejection. We use a mathematical model to predict the amount of BKV and develop an optical control method to provide individualized treatment to patients. We present the validation this model using data from the Duke Transplant center. We use multiple techniques for parameter estimation, including the Joint Unscented Kalman Filter.

A generalized ODE susceptible-infectious-susceptible compartmental model with potentially periodic behavior

Scott Greenhalgh
Siena College, USA
Anna Dumas

Differential equation compartmental models are crucial tools for forecasting and analyzing disease trajectories. Among these models, those dealing with only susceptible and infectious individuals are particularly useful as they offer closed-form expressions for solutions, such as the logistic equation. However, the logistic equation has limited ability to describe disease trajectories since its solutions must converge monotonically to either the disease-free or endemic equilibrium. Unfortunately, many diseases exhibit periodic cycles, and thus, do not converge to equilibria. To address this limitation, we developed a generalized susceptible-infectious-susceptible compartmental model capable of accurately incorporating the duration of infection distribution and describing both periodic and non-periodic disease trajectories. We characterized how our model's parameters influence its behavior and applied the model to predict gonorrhea incidence in the US, using AIC to inform on its merit relative to the classical SIS model. The significance of our work lies in providing a novel susceptible-infected-susceptible model whose solutions can have closed-form expressions that may be periodic or non-periodic depending on the parameterization. Our work thus provides disease modelers with a straightforward way to investigate the potential periodic behavior of many diseases and thereby may aid ongoing efforts to prevent recurrent outbreaks.

Generalized differential equation models for disease interventions: A novel approach for predicting sexually transmitted disease outbreaks

Scott Greenhalgh
Siena College, USA
Jack Farrell, Owen Spolyar

Sexually transmitted diseases pose a significant threat to the health and economic well-being of society. Hence, it is crucial to predict imminent outbreaks and identify effective interventions through the use of epidemiological tools like compartmental models. Unfortunately, traditional compartmental models, specifically the differential equation models developed by Kermack and McKendrick, require an exponential or Erlang distributed duration of infection, despite their biological invalidity. As a result, alternative approaches are required that capture the variability in the duration of infection, its associated effects on the disease trajectory, and the evaluation of disease interventions. To address this, we apply a novel family of differential equation compartmental models, based on the quantity "person-days of infection" that can accommodate non-exponential and non-Erlang distributed durations of infection. As proof of concept, we calibrate an SEAIR analog of our model to recent trends in chlamydia incidence in the United States and evaluate how awareness campaigns alter our model predictions of incidence averted, over a 5-year horizon.

Coexistence of predator-prey model with directional dispersal of predator

Kwangjoong Kim
Kookmin University, Korea
Wonhyung Choi

We study the effect of directional dispersal of a predator on a predator-prey model. The prey is assumed to have traits making it undetectable to the predator and difficult to chase the prey directly. Directional dispersal of the predator is described when the predator has learned the high hunting efficiency in certain areas, thereby dispersing toward these areas instead of directly chasing the prey. We investigate the stability of the semi-trivial solution and the existence of a coexistence steady-state. Moreover, we show that the predator that moves toward a high predation area may make the predators survive under the condition the predators cannot survive when they disperse randomly. The results are obtained through eigenvalue analysis and fixed-point index theory. Finally, we present the numerical simulation and its biological interpretations based on the obtained results.

Tractable signatures of evolutionary selection methods

Bertrand Ottino-Loffler
Rockefeller University, USA

Modeling population-level evolutionary dynamics is one of the oldest, and most enduring forms of mathematical biology. Key to many of these models is the notion of different strains having different "fitnesses," which represents their net competitive advantages in the environment. For example, in lymph nodes, we know that B cells have a competitive advantage when they can successfully neutralize pathogens. However, reducing this advantage to a singular 1D fitness is misleading, since there may be multiple mechanisms that lead to this advantage. Three simple, potential selection methods include encouraging the reproduction of high affinity cells ("birth selection"), encouraging cell death in low affinity cells ("death selection"), and adjusting the mutation rate based on cell affinity ("mutational selection"). Moreover, multiple methods can be active at the same time, and while all three forms of selection would lead to a net increase in affinity, different selection methods lead to distinct statistical outcomes. We discuss ramifications of different evolutionary models, and encourage thinking about evolutionary fitness as more than a singular number.

Poster Session

Peak Identification Using KS Test and Probability Distributions

Ashley Alfred
University of Texas Arlington, USA
Purnendu Dasgupta, Jianzhong Su

Chromatography is a technique for separating the chemical components of a mixture, and a chromatogram is used to identify the components based on retention time. Usually, peak identification is based on retention time, which is not ideal alone since retention time is prone to shifts. Additional detectors, such as, Mass spectrometers are commonly used to assist, but are a very expensive option. For analytes having nearly the same retention times, peak shape-based identification yields a correct prediction. Peak shape is a characteristic signature of the identity of an analyte, so there is a unique normalized peak shape for each analyte. Given the data, normalized peak shapes are obtained, and can be compared with unknown analytes to determine if analytes match and the goodness of the match. The significance of a match can be judged by several methods, such as, R2 values and absolute percent error. This project explores another method using the Kolmogorov-Smirnov (K-S) test and probability distributions.

Exploring Cow Grazing Preferences Using Remote Sensing and Neural Networks

Angela Avila
University of Texas Arlington, USA
A.J. Ashworth, H. Smith, T.C. Adams, E. Winzeler, P. Owens, D. Philipp, A. Thomas, T. Sauer, F. Kamangar, J. Su

This research project aims to explore factors that affect cattle grazing preferences in a controlled research field at the University of Arkansas Agricultural and Extension Center. Using unmanned aerial vehicles (UAV) images and tracking collars, the study monitored the grazing locations of cattle in land sample plots with varying grass types, fertilization levels, and availability of trees. A neural network model was trained using the UAV images and plot characteristics to predict expected cattle visits and identify favorable factors for grazing. The results of this study will contribute to a better understanding of cattle grazing behavior and inform land management practices.

Wolfram Demonstrations to Simulate Boundary Stabilization of PDEs for Piezoelectric Beams

Ahmet Kaan Aydin
University of Maryland Baltimore County, USA
Jacob Walterman, Md Zulfiqur Haider, Ahmet Ozkan Ozer

Novel reduced models of partial differential equations (PDE) for piezoelectric (smart material) single and multi-layer beam equations are developed by Finite Differences and Finite Elements. These reduced models accurately predict feedback-controlled vibrations traveling on the beam during the motion. The paired controller and sensor are collocated at the tip of the beam. First, it is shown that the feedback sensor placed at the tip of the beam can be designed by eliminating the short wavelength and high-frequency components of the solutions through the direct Fourier filtering technique. This way, the sensor becomes more able to distinguish one vibrational frequency from another. The filtered sensor data is then fed back to the controller, resulting in all of the vibrations on the beam being successfully suppressed exponentially fast, replicating the dynamics of the PDE case. Another approach, based on order reduction, accurately simulates the suppressed dynamics without the need for direct filtering. Finally, all Mathematica simulations are converted to the computational framework Wolfram's Computable Document Format (CDF), and they are published at the open-source-code website, Wolfram Demonstrations Project (WDP).

Improving Computational Efficiency for the Nearly Incompressible Elastography Inverse Problem

Rachel Hawks
Rochester Institute of Technology, USA
Basca Jadamba

The problem being analyzed is based on linear elasticity equations that describe displacement in soft tissue under applied body forces in biomedical applications, specifically in the case of identifying soft tissue cancers. The primary goal of this work is to develop an adaptive finite element solution framework for the identification of a distributed parameter in a system of partial differential equations where the inverse problem is formulated as an optimization problem. Description of a finite element discretization that fits the optimization framework and stochastic approximation iteration used in the numerical solution is given. We also propose an adaptive mesh refinement framework that provides the resolution needed for the recovery of the spatially varying parameter while improving the computational efficiency.

Honeybee population models with age structure and seasonality

Yun Kang
Arizona State University, USA
Jun Chen, Marisabel Rodriguez, Komi Messan, Gloria DeGrandi-Hoffman

Western honeybees (i.e., Apis Mellifera) serve extremely important roles in our ecosystem and economics as they are responsible for pollinating $215 billion dollars annually over the world. Unfortunately, honeybee population and their colonies have been declined dramatically. The purpose of this article is to explore how we should model honeybee population and valid the model by data so that we could identify varied factors that affect the survival and healthy of the honeybee colony. Our theoretical study combined with simulations and data validation suggests that the proper age structure incorporated in the model as well as seasonality are important for honeybee population modeling. The future model should include mites, viruses, and pesticides.

Population Dynamics in an Advective Environment

Ray Lee
Ohio State University, USA
King-Yeung Lam, Yuan Lou

We consider a one-dimensional reaction-diffusion equation describing single- and two-species population dynamics in an advective environment, based on the modeling frameworks proposed by Lutscher, Lewis, and McCauley in 2006 for modeling populations in rivers. We analyze the effect of rate of loss of individuals at both the upstream and downstream boundaries.

For a single species, we prove the existence of the critical domain size and provide explicit formulas in terms of model parameters. We further show that, in some cases, the critical domain size is either strictly decreasing over all diffusion rates, or monotonically increasing after first decreasing to a minimum.

We also consider competition between species differing only in their diffusion rates. For two species having large diffusion rates, we give a sufficient condition to determine whether the faster or slower diffuser wins the competition. We also note a link between these results and competition dynamics in species whose spatial niche is affected by shifting isotherms caused by climate change.

Comparison of iterative methods efficiency for solving 2D nonlinear problem of optical bistability realization in a semiconductor

Maria Loginova
Independent Researcher, USA
Vyacheslav Trofimov, Vladimir Egorenkov, Yongqiang Yang, Changhui Song, Di Wang, Changjun Han

We consider the 2D problem of femtosecond laser pulse interaction with a semiconductor under the condition of an optical bistability occurrence. It is a very promising phenomenon for creating and developing all-optical data processing. The mathematical model of this process represents the set of non-linear time-dependent PDEs concerning semiconductor characteristics and the equation describing optical intensity or optical amplitude evolution. The corresponding initial-boundary conditions are stated. The problem's numerical modeling complexity consists of the existence of non-linear feedback between the Poisson equation and the reaction-diffusion-convection type equation. In our study, we propose to solve the problem under consideration by using a conservative implicit finite-difference scheme written on the base of the Crank-Nicolson scheme. The main question consists of choosing the best technique for its implementation. For this purpose, we develop various two-stage iterative methods and study and compare their efficiency.

Irreversibility Analysis of Electromagnetic Hybrid Nanofluid Over a Stretchable Surface with Cattaneo-Christov Heat Flux Model

Muhammad Qureshi
King Fahd University of Petroleum and Minerals, Saudi Arabia

To get a better heat transmission capacity of ordinary fluids, new hybrid nanofluids (HNFs) with a considerably greater exponent heat than nanofluids (NFs) are being used. HNFs, which have a greater heat exponent than NFs, are being applied to increase the HT capacities of regular fluids. This work investigates the flow and HT features of HNF across a slick surface. As a result, the geometric model is explained by employing symmetry. The technique includes nanoparticles shape factor, Magnetohydrodynamics (MHD), porous media, Cattaneo-Christov, and thermal radiative heat flux effects. The governing equations are numerically solved by consuming a method known as the Galerkin finite element method (FEM). In this study, H2O-water was utilized as an ironic, viscous improper fluid, and HNF was investigated. Copper (Co) and Titanium Alloy (Ti6Al4V) nanoparticles are found in this fluid. The HT level of such a fluid (Ti6Al4V-Co/H2O) has steadily increased in comparison to ordinary Co-H2O

NFs, which is a significant discovery from this work. The inclusion of nanoparticles aids in the stabilization of a nanofluid flowing and maintains the symmetry of the flow form. The thermal conductivity is highest in the boundary-lamina-shaped layer and lowest in sphere-shaped nanoparticles. A system's entropy increases by three characteristics: their ratio by fractional size, their radiated qualities, and their heat conductivity modifications. The primary applications of this examination are the biological and medical implementations like dental and orthopedic implantable devices, as well as other devices such as screws and plates because they possess a favorable set of characteristics such as good biomaterials, corrosion resistance and wear, and great mechanical characteristics.

Inequalities for the Green's function in the existence results for a third-order boundary value problem

Gabriela Szajnowska
University of Rzeszow, Poland
Miroslawa Zima

We study the existence of positive solutions for the third order differential equation of the form

$$-u''' + m^2 u' = f(t, u, u'), \quad t \in [0,1], \tag{1}$$

subject to nonlocal boundary conditions

$$u(0) = 0,\ u'(0) = \alpha[u],\ u'(1) = \beta[u], \tag{2}$$

where $m > 0$ and α and β are the functionals (not necessarily linear) acting on the space $C^1[0,1]$. The purpose of this poster is to present the properties of the Green's function for the linear problem corresponding to (1)-(2). The properties we focus on are used to obtain sufficient conditions implying the existence of positive and increasing solutions of the above problem.

Approximation and non-parametric estimation of functions over high-dimensional spheres via deep ReLU networks

Tian-Yi Zhou
Georgia Institute of Technology, USA
Namjoon Suh, Xiaoming Huo

We develop a new approximation and estimation analysis of deep feed-forward neural networks (FNNs) with the Rectified Linear Unit (ReLU) activation. The functions of interest for the approximation and estimation are assumed to be from Sobolev spaces defined over the d-dimensional unit sphere with smoothness index $r > 0$. In the regime where r is in the constant order (i.e., $r = \mathcal{O}(1)$), it is shown that at most d^d active parameters are required for getting d^{-C} approximation rate for some constant $C > 0$. In contrast, in the regime where the index r grows in the order of d (i.e., $r = \mathcal{O}(d)$) asymptotically, we prove the approximation error decays in the rate $d^{-d^{\beta}}$.

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